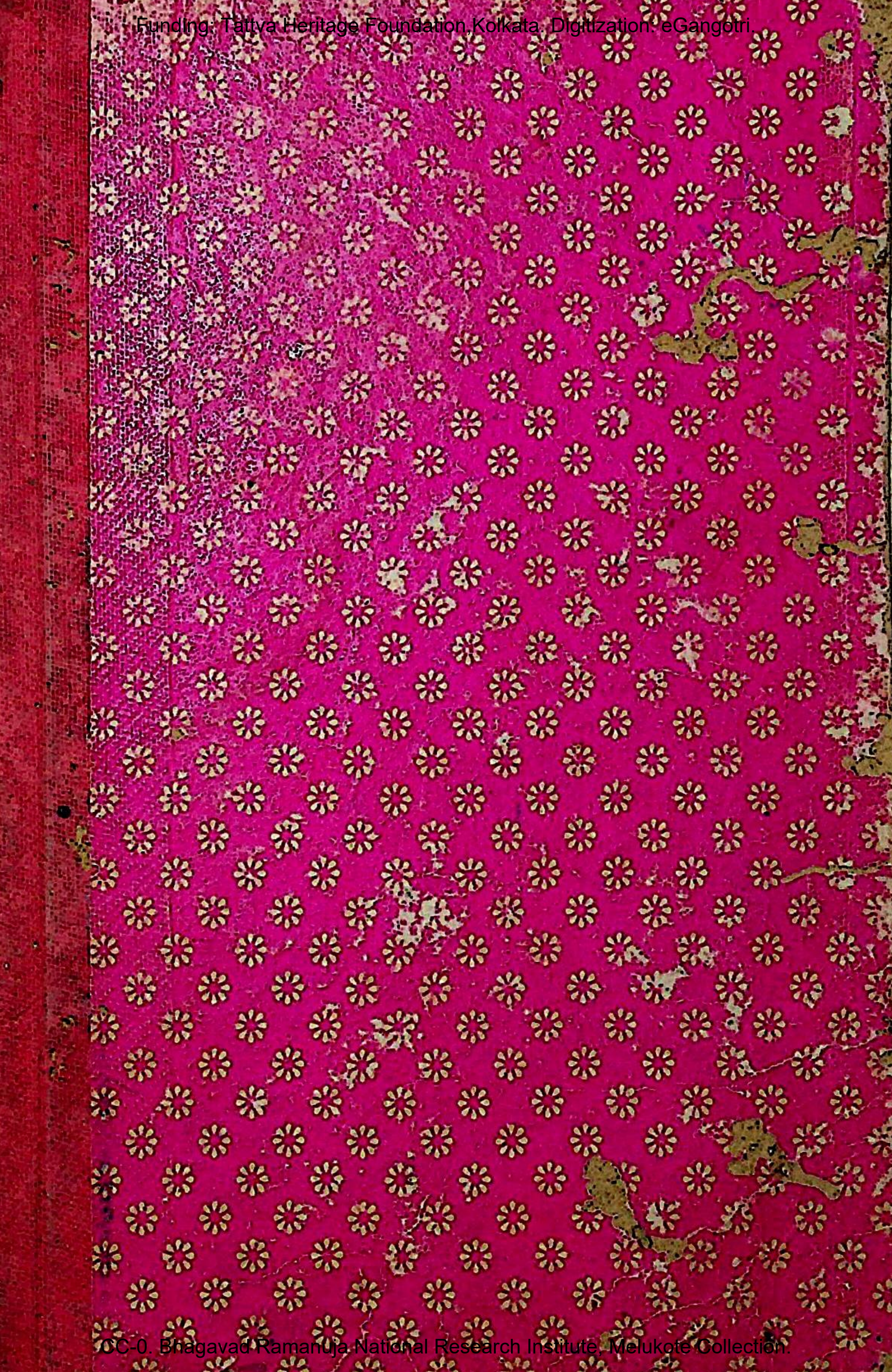

N.S.Rajesh

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Suma.B.V.

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SECTION 0: PRELIMINARIES

0.1 LIST OF FORMULAE (I Year)

0.11 ALGEBRA

1. (*i*) $a^2-b^2=(a+b)(a-b)$ (*ii*) $a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$ (*iii*) $(a\pm b)^2=a^2\pm 2ab+b^2$ (*iv*) $(a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$ (*v*) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$ (*vi*) $(x+a)(x+b)=x^2+(a+b)x+ab$

2. **Laws of Indices:** (*i*) $a^0=1,\ a\neq 0$ (*ii*) $a^{-m}=\frac{1}{a^m}$ (*iii*) $\sqrt[q]{a^p}=a^{\frac{p}{q}}$ (*iv*) $a^m a^n=a^{m+n}$ (*v*) $\frac{a^m}{a^n}=a^{m-n}$ (*vi*) $(a^m)^n=a^{mn}$ (*vii*) $(ab)^n=a^n\,b^n$.

3. **Surds:** (*i*) A rationalising factor of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$ and vice versa. (*ii*) $\sqrt{a+b\pm 2\sqrt{ab}}=\sqrt{a}\pm\sqrt{b},\ a>b$

4. **Logarithm:** (*i*) $\log_a mn=\log_a^m+\log_a^n$ (*ii*) $\log_a^{m/n}=\log_a^m-\log_a^n$ (*iii*) $\log_a m^n=n\ \log_a^m$ (*iv*) $\log_a^m=\log_b^m\div\log_b^a$ or $\log_b^m\ \log_a^b$ (*v*) $a^{\log_a^x}=x$ (*vi*) $\log_a 1=0$ (*vii*) $\log_a a=1$

5. **Quadratic Equations:** (*i*) $i=\sqrt{-1},\ i^2=-1$ (*ii*) A complex number is a number of the form $a+ib$ where a and b are real numbers (a–real part; b–imaginary part) (*iii*) The conjugate of a complex number $z=a+ib$ is $\bar{z}=a-ib$; $z+\bar{z}=2a=2\times$ the real part of z; $z-\bar{z}=2ib=2i\times$ the imaginary part of z; $z\bar{z}=|z|^2=a^2+b^2$ (*iv*) The *roots* of the quadratic equation $ax^2+bx+c=0$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (*v*) The *roots* of the quadratic equation will be *distinct real*, *equal* or *imaginary* according as its *discriminant* b^2-4ac is *positive*, *zero* or *negative* (*vi*) The quadratic *expression* ax^2+bx+c will be a *perfect square* if $b^2-4ac=0$ (*vii*) The *cube roots* of *unity* are 1, $\frac{-1\pm\sqrt{3}i}{2}$ or 1, ω, ω^2 where $\omega=\frac{-1\pm\sqrt{3}i}{2}$ (*viii*) The *fourth roots* of *unity* are $\pm 1,\ \pm i$

6. **Progressions and Series:** (*i*) t_n of an A.P. is $a+(n-1)d$ where a is the first term and d is the common difference (*ii*) t_n of a

G.P. is ar^{n-1} where a is the first term and r is the common ratio (*iii*) S_n of an A.P. is $\frac{n}{2}[2a+(n-1)d]$ (*iv*) S_n of a G.P. is $\frac{a(1-r^n)}{1-r}$ (*v*) S_∞ of a G.P. is $\frac{a}{1-r}$ if $|r| < 1$ (*vi*) $\Sigma n=\frac{n(n+1)}{2}$ (*vii*) $\Sigma n^2 =\frac{n(n+1)\ (2n+1)}{6}$ (*viii*) $\Sigma n^3=\frac{n^2(n+1)^2}{4}$ (*ix*) If a, b, c are in A.P, (G.P.), [H.P.], then $b=\frac{a+c}{2}$; ($b^2=ac$); $\left[b=\frac{2ac}{a+c}\right]$ respectively.

7. **Binomial Theorem:** When n is a +ve integer,
$(a+b)^n=nc_0\, a^n+nc_1\, a^{n-1}\, b+nc_2\, a^{n-2}\, b^2+\dots\dots\dots+nc_n b^n$.

8. **Partial fractions:** (*i*) Let $\frac{p(x)}{q(x)}$ be a rational function ($p(x)$ and $q(x)$ are polynomials in x). If the degree of $p(x) \geqslant$ the the degree of $q(x)$, then $\frac{p(x)}{q(x)}$ is called an *improper fraction*; otherwise a *proper fraction.*

(*ii*) While *resolving* $\frac{p(x)}{q(x)}$ into *partial fractions* (*a*) if $q(x)$ has $(ax+b)$ *as a non-repeated factor*, then the partial fraction corresponding to it is $\frac{A}{ax+b}$ (*b*) if $q(x)$ has $(ax+b)^2$ as a factor, then the partial fraction corresponding to it is $\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$ (*c*) if $q(x)$ has ax^2+bx+c (irreducible) as a factor, then the partial fraction corresponding to it is $\frac{Ax+B}{ax^2+bx+c}$.

9. **Set Theory:** (*i*) A *set* is a well-defined collection of objects; if the number of distinct elements of a set is finite, then it is called *finite*; otherwise *infinite*.

(*ii*) A set which contains no element is called the *empty set* or *null set* and it is denoted by ϕ.

(*iii*) A is called a subset of B i.e. $A \subset B$ if every element of A is also in B; two sets A and B are equal iff $A \subset B$ and $B \subset A$.

(*iv*) $A \cup B=\{x \mid x \in A$ or $x \in B\}$; $A \cap B=\{x \mid x \in A$ and $x \in B\}$; $A-B=\{x \mid x \in A$ and $x \notin B\}$.

0.12 TRIGONOMETRY

1. **Radian measure:** (*i*) $\pi^c = 180°$ (*ii*) Area of a sector of a circle of radius r containing an angle θ^c is $\frac{1}{2} r^2 \theta$ (*iii*) $l = r\theta$

2. **Trigonometric functions and fundamental relations:** (*i*) Let OX and OY be the initial and bounding lines of an acute angle θ. Take a point P on OY and draw PM $\perp$ OX. Then $\sin\theta = \frac{PM}{OP} = \frac{\text{opposite side}}{\text{hypotenuse}}$; $\cos\theta = \frac{OM}{OP} = \frac{\text{adjacent side}}{\text{hypotenuse}}$; $\tan\theta = \frac{PM}{OM} = \frac{\sin\theta}{\cos\theta}$; $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

(*ii*) $\sin^2\theta + \cos^2\theta = 1$; $\cos^2\theta = 1 - \sin^2\theta$; $\sin^2\theta = 1 - \cos^2\theta$

(*iii*) $1 + \tan^2\theta = \sec^2\theta$; $\sec^2\theta - 1 = \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$.

(*iv*) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$; $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$; $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

3. **Signs and ranges of trigonometric functions:**

θ	quadrant			
	I	II	III	IV
$\sin\theta$ and $\operatorname{cosec}\theta$	+	+	—	—
$\cos\theta$ and $\sec\theta$	+	—	—	+
$\tan\theta$ and $\cot\theta$	+	—	+	—

(*i*) $-1 \leqslant \sin\theta, \cos\theta \leqslant 1$

(*ii*) $-\infty < \tan\theta, \cot\theta < \infty$

(*iii*) $\operatorname{cosec}\theta, \sec\theta \leqslant -1$ or $\geqslant 1$

4. **Values of trigonometric functions of some frequently occurring angles:**

θ	$0°$	$30°$	$45°$	$60°$	$90°$	$15°$	$75°$	$(2n+1)\frac{\pi}{2}$	$n\pi$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$(-1)^n$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0	$(-1)^n$
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$2-\sqrt{3}$	$2+\sqrt{3}$	∞	0

5. **Allied angles:**—(*i*) The relation between *t*-functions of θ and its allied angles $n.90^\circ\pm\theta$ is obtained using the following two rules (*a*) Observe the quadrant in which $n.90^\circ\pm\theta$ lies and determine the sign (+*ve* or —*ve*) (*b*) If *n* is odd, the function will change into its co-function (i. e. $\sin\rightleftarrows\cos$; $\tan\rightleftarrows\cot$; $\sec\rightleftarrows\mathrm{cosec}$). For example: $\sin(180^\circ+\theta)=-\sin\theta$; $\cos(270^\circ+\theta)=\sin\theta$, (*ii*) Trigonometric functions of θ and $n.360^\circ+\theta$ are same (*iii*) $\sin(-\theta)=-\sin\theta$; $\cos(-\theta)=\cos\theta$; $\tan(-\theta)=-\tan\theta$

6. **Trigonometric functions of compound angles:**

(*i*) $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$ (*ii*) $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$ (*iii*) $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

(*iv*) $\tan\left(\dfrac{\pi}{4}\pm A\right)=\dfrac{1\pm\tan A}{1\mp\tan A}$ (*v*) $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$ (*vi*) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$

7. **Trigonometric functions of multiple angles:**

(*i*) $\sin 2A=2\sin A\cos A=\dfrac{2\tan A}{1+\tan^2 A}$

(*ii*) $\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A=\dfrac{1-\tan^2 A}{1+\tan^2 A}$ (*iii*) $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

(*iv*) $1+\cos 2A=2\cos^2 A$; $\cos^2 A=\frac{1}{2}(1+\cos 2A)$

(*v*) $1-\cos 2A=2\sin^2 A$; $\sin^2 A=\frac{1}{2}(1-\cos 2A)$

(*vi*) $1\pm\sin 2A=(\sin A\pm\cos A)^2$ (*vii*) $\sin 3A=3\sin A-4\sin^3 A$

(*viii*) $\cos 3A=4\cos^3 A-3\cos A$

(*ix*) $\tan 3A=\dfrac{3\tan A-\tan^3 A}{1-3\tan^2 A}$ (*x*) $\cot A-\tan A=2\cot 2A$

8. **Trigonometric functions of half angles:**

(*i*) $\sin\theta=2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}=\dfrac{2\tan\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}}$ (*ii*) $\cos\theta=\cos^2\dfrac{\theta}{2}-\sin^2\dfrac{\theta}{2}=2\cos^2\dfrac{\theta}{2}-1=1-2\sin^2\dfrac{\theta}{2}=\dfrac{1-\tan^2\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}}$

(*iii*) $1+\cos\theta=2\cos^2\frac{\theta}{2}$; $1-\cos\theta=2\sin^2\frac{\theta}{2}$

9. Formulae for conversion of sums (or differences) into products:

(*i*) $\sin C+\sin D=2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(*ii*) $\sin C-\sin D=2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(*iii*) $\cos C+\cos D=2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(*iv*) $\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

10. Formulae for conversion of product into sums (or differences):

(*i*) $\sin A\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$

(*ii*) $\cos A\sin B=\frac{1}{2}[\sin(A+B)-\sin(A-B)]$

(*iii*) $\cos A\cos B=\frac{1}{2}[\cos(A+B)+\cos(A-B)]$

(*iv*) $\sin A\sin B=-\frac{1}{2}[\cos(A+B)-\cos(A-B)]$

11. If A, B, C are angles of a triangle,

sin (sum of any two) = sin (third); eg. $\sin(B+C)=\sin A$

cos (sum of any two) = $-$cos (third); eg. $\cos(A+B)=-\cos C$

sin $\frac{1}{2}$ (sum of any two) = cos $\frac{1}{2}$ (third); eg. $\sin\frac{A+C}{2}=\cos\frac{B}{2}$

cos $\frac{1}{2}$ (sum of any two) = sin $\frac{1}{2}$ (third); eg. $\cos\frac{B+C}{2}=\sin\frac{A}{2}$

0.13 ANALYTICAL GEOMETRY

1. The *distance* between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$; the distance of a point (x, y) from origin is $\sqrt{x^2+y^2}$

2. (*i*) The coordinates of the point which *divides* the line joining the points (x_1, y_1) and (x_2, y_2) *internally* in the ratio $l:m$ are $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$ (*ii*) The coordinates of the *mid-point*

of the segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

3. The *area of the triangle* formed by the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2} \Sigma x_1 (y_2-y_3)$

4. The *slope or gradient* of a straight line is (*i*) $m=\tan\theta$ where θ is the angle made by the straight line with the $+ve$ direction of x—axis (*ii*) $m=\frac{y_2-y_1}{x_2-x_1}$ where (x_1, y_1) and (x_2, y_2) are two points on it (*iii*) $m=-{}^{a}/_{b}$ if the equation of the straight line is $ax+by+c=0$. The *slope* of a *horizontal* line (parallel to x-axis) is 0 and the *slope* of a *vertical line* is ∞

5. **Different forms of equation of a straight line:**

(*i*) *The slope-intercept form*: $y=mx+c$; (*ii*) The slope-point form: $y-y_1=m(x-x_1)$; In particular $y=mx$ is the equation of a straight line with slope m and passing through the origin (*iii*) *The two-point form*: $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ (*iv*) *Intercepts form*: ${}^{x}/_{a}+{}^{y}/_{b}=1$ (*v*) $y=k$ represents a *horizontal line* and $x=k$ represents a *vertical line* (*vi*) The equation of x—axis is $y=0$ and the equation of y—axis is $x=0$.

6. (*i*) To find the *point of intersection* of two straight lines (curves), solve the two equations.

(*ii*) Three lines $L_1=0$, $L_2=0$ and $L_3=0$ are *concurrent* if there exists constants l, m, n, (not all zeroes) such that $lL_1+mL_2+nL_3\equiv 0$.

7. (*i*) If θ is the *acute angle* between two lines with slopes m_1 and m_2, then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1 m_2}\right|$

(*ii*) The condition for two lines to be *parallel* is $m_1=m_2$

(*iii*) The condition for two lines to be *perpendicular* is $m_1m_2=-1$

(*iv*) *Any line parallel to the line* $ax+by+c=0$ is of the form $ax+by+k=0$ and *any line* $\perp$ *to it* is of the form $bx-ay+k=0$.

8. (*i*) The *length of the perpendicular distance* of a point (x_1, y_1) to a line $ax+by+c=0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

(*ii*) The *distance between the parallel lines*

$ax+by+c=0$ and $ax+by+c^1=0$ is $\left|\dfrac{c-c^1}{\sqrt{a^2+b^2}}\right|$

0.14 Area and Volume

1. The *area* of (*i*) a *triangle* $=\frac{1}{2}bh$ (*ii*) an *equilateral triangle* $=\dfrac{\sqrt{3}}{4}a^2$
2. The *area* of (*i*) a circle $=\pi r^2$ (*ii*) a square $=a^2$ (*iii*) a rectangle $=ab$
3. The *perimeter* of (*i*) a circle $=2\pi r$ (*ii*) a square $=4a$ (*iii*) a rectangle $=2(a+b)$
4. The *volume* of (*i*) a cube $=a^3$ (*ii*) a sphere $=\frac{4}{3}\pi r^3$ (*iii*) a cylinder $=\pi r^2h$ (*iv*) a cone $=\frac{1}{3}\pi r^2h$
5. The *surface area* of (*i*) a cube $=6a^2$ (*ii*) a sphere $=4\pi r^2$ (*iii*) a cylinder $=2\pi rh+2\pi r^2$ (closed) (*iv*) cone $=2\pi r+\pi rl$

0.2 SYLLABUS—II YEAR

ALGEBRA

1. **Set Theory:** Cartesian Product of sets, relations, equivalence relations, functions. Problems. (4 *hrs*)
2. **Elements of Number Theory:** Divisibility, Statement of the division algorithm and its application to find the G.C.D, Prime and composite Numbers, Proof of the following theorems:
   1. There are infinity of primes
   2. If c and a are relatively prime and $c \mid ab$, then $c \mid b$
   3. If p is a prime and $p \mid ab$, then $p \mid a$ or $p \mid b$.

   Problems. (4 *hrs*)

   Congruences: Definition, proofs of the following theorems:
   1. Congruence relation is an equivalence relation on the set of integers.
   2. $a \equiv b$ (*mod m*) implies $a \pm x \equiv b \pm x$ (*mod m*) and $ax \equiv bx$ (*mod m*)
   3. c is relatively prime to n and $ca \equiv cb$ (*mod n*) implies $a \equiv b$ (*mod n*). Problems. (4 hrs)

3. **Matrices and Determinants:** $m \times n$ matrix with $m, n \leqslant 3$, column, row and unit matrices. Algebra of matrices-addition, multiplication, transpose, adjoint and inverse of square matrices. Definition of determinants of 2×2, 3×3 matrices: If M (2, R) and M(3, R) are the sets of all 2×2 and 3×3 matrices (respectively) with real entries, then determinants are defined by $\Delta : M(2, R) \to R$

$$\Delta\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}\, a_{22} - a_{21}\, a_{12} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \text{— notation}$$

and $\Delta : M(3, R) \to R$

$$\Delta\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= a_{11}\,(a_{22}\, a_{33} - a_{32}a_{23}) - \ldots + \ldots = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Properties therefrom including $\Delta(AB) = \Delta(A)\,\Delta(B)$. Solutions of linear equations, Cramer's rule, Problems. (8 hrs)

4. **Groups:** Binary operations. Definition, Group and abelian group with simple examples from number system, modular systems, matrices of 2nd and 3rd order, permutation group S_3, square roots, cube roots and fourth roots of unity. Derivation of the properties:

   (*i*) the identity element is unique

   (*ii*) the inverse of an element is unique

   (*iii*) $(ab)^{-1} = b^{-1}\, a^{-1}$

   (*iv*) $(a^{-1})^{-1} = a$

   (*v*) Existence of solutions of equations $ax=b$ and $ya=b$ and their uniqueness. Definition of subgroups and examples. Problems. (12 *hrs*)

5. **2 and 3-Dimensional Vectors:** Definition of 2 and 3-dimensional vector as an ordered pair and triplet (a_1, a_2, a_3) of real numbers. Addition rule, rule for multiplying a vector by a real number and properties. The unit vectors: $i = (1, 0, 0)$, $j = (0, 1, 0)$ $k = (0, 0, 1)$.

   Representation $(a_1, a_2, a_3) = a_1\, i + a_2\, j + a_3\, k$.
   Scalar and Vector products $\Sigma\, a_i\, b_i$ and $\Sigma\, i\,(a_2\, b_3 - a_3\, b_2)$

Geometrical representation of a vector, geometric interpretation of addition of vectors, multiplication of a vector by a real number, geometrical meanings of scalar and vector products, Scalar triple product, volume and coplanarity. Problems. (10 *hrs*)

TRIGONOMETRY

Relations between sides and angles of a triangle. Solution of triangles. Inverse trigonometric functions. Solutions of simple trigonometric equations. Problems. Complex Numbers: Argand diagram, Polar representation of a complex number. Geometrical interpretation of addition and multiplication of complex numbers, De Moivre's theorem (with proof) Simple problems based on the cube roots of a complex number. (25 *hrs*)

ANALYTICAL GEOMETRY

Lines: Pair of lines, their homogeneous equation: $ax^2+2hxy+by^2=0$ and angle between them. Condition that $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent a pair of lines—finding the point of intersection and angle between them (numerical examples)., pair of parallel straight lines.

Circle: General equation of a circle; Equation and length of a tangent-Radical axis—condition for orthogonality.

Conic sections: Standard (Canonical) forms of parabola, ellipse and hyperbola—to find equation when focus and directrix are given—determining focus, directrix, and latus rectum, in simple numerical examples—equations of tangent and normal. Problems. (25 *hrs*)

CALCULUS

Differentiation: Definition of limit and continuity. Differentiability, statements of rules for differentiation of a sum, product and quotient of two functions. Differentiation of composite functions, polynomial, trigonometric, logarithmic, exponential and hyperbolic functions, implicit and parametric functions. Logarithmic differentiation. Tangent and normal to a curve. Angle between two curves. Subtangents and subnormals. Second order differentiation. Derivative as rate. Maxima, Minima. Problems.

Integration: Statement of fundamental theorem of calculus. Integrals of x^n, $\sin ax$, $\cos ax$, $\sec ax$ etc., and $[f(x)]^m f'(x)$. Integration by substitution and by parts. Problems.

Definite Integrals: Definite integral as limit of a sum. Area under a curve as a definite integral. Problems.

0.3 QUESTION PAPER PATTERN FOR PUC II YEAR

(Blue Print)

Topics	No. of teaching Hours	No. of objecitve type Multiple choice Questions 1 mark each	No. of short Answer Questions 2 marks each	No. of essay type Questions 4 marks each	No. of Problem solving type Questions 4 marks each	Total marks
Set theory	4	1	1	—	—	3
Elements of Number Theory	4	1	1	—	—	3
Congruences	4	1	1	—	—	3
Matrices and determinants	8	2	1	—	1	8
Groups	12	3	1	—	1	9
Vectors	10	2	2	—	—	6
Trigonometry	25	3	1	1	2	17
Analytical Geometry	25	3	1	1	2	17
Calculus	48	4	1	3	4	34
Total	140	20	10	5	10	100

0.4 MODEL QUESTION PAPER FOR PUC II YEAR

PART—A (Objective type) **20 Marks**

Note: (*i*) *Answer all questions.*

(*ii*) *Show your response to questions by ticking (√) in the appropriate box provided on the right margin.*

1. If A={(2, 3, 5} B= {4, 5, 6} then $(A \cap B) \times A$ is

(*a*) {(2, 5) (3, 5)} (*b*) {(5, 2) (5, 3)}

(*c*) {(5, 2) (5, 3) (5, 5)} (*d*) {(5, 2) (2, 5) (3, 5)}

2. If $(a, b) = 1$ and $a \mid bx$ then
   (a) $a \mid b$ (b) $a \mid x$ (c) $a \nmid x$ (d) none of these
3. The value of x satisfying $27x \equiv 3 \pmod 5$ is
   [✓ (a) 19] (b) 25 (c) 20 (d) 18
4. If $A = \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}$ then A^{-1} is
   [✓ (a) $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$] (b) $\begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$
5. $\begin{bmatrix} 77 & 78 & 79 \\ 75 & 74 & 73 \\ 76 & 75 & 73 \end{bmatrix} =$
   [✓ (a) even] (b) 133 (c) 21 (d) 7
6. The cube roots of unity form a group under
   (a) addition [✓ (b) multiplication] (c) subtraction (d) division
7. In the group $G = (1, 2, 3, 4, \times mod\ 5)$ the inverses of 2 and 4 are respectively
   (a) 2 and 3 (b) 2 and 4 (c) 3 and 4 (d) 3 and 1
8. If G is a non-commutative group and $a, b, c, d \in G$ then $(a\, b^{-1}\, c^{-1}\, d)^{-1}$ is (a) $a^{-1}\, bcd^{-1}$ (b) $bcd^{-1}\, a^{-1}$ (c) $d^{-1}\, cba^{-1}$ (d) $d^{-1}\, cb^{-1}\, a^{-1}$
9. If $\vec{a} = i + j - 2k$, $\vec{b} = -i + 2j + k$, $\vec{c} = i - 2j + 2k$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is
   (a) 9 (b) 12 (c) −6 (d) 0
10. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is
    (a) 0° (b) 180° (c) 90° (d) 60°
11. In a triangle ABC is $a = \sqrt{2}$, $c = \sqrt{3} - 1$ and $B = 135°$, then b is equal to
    (a) 1 (b) 2 (c) $2\sqrt{3}$ [✓ (d) 4]
12. $\sin\left(2 \cos^{-1} \frac{\sqrt{5}}{3}\right)$ is equal to
    (a) $\frac{4\sqrt{5}}{9}$ (b) $\frac{4\sqrt{5}}{3}$ (c) $4\sqrt{5}$ (d) $\frac{\sqrt{5}}{12}$
13. The real part of $\frac{1}{1 + \cos\theta + i \sin\theta}$ is
    (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

14. The angle between the lines $2x^2+3xy-2y^2=0$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{3}{4}\right)$

15. The centre of the circle $3x^2+3y^2-6x+27y-2=0$ is
[✓ (a)] $(1, -9/2)$ (b) $(2, 3)$ (c) $(9/2, 1)$ (d) $(-2, 3)$

16. The length on the transverse axis of the hyperbola $9x^2-4y^2=25$ is
(a) 5/3 (b) 10/3 (c) 5/2 (d) 5

17. $\lim_{x \to 1}\left(\frac{x^m-1}{x^n-1}\right)$ is
(a) mn [✓ (b)] $\frac{m}{n}$ (c) $\frac{n}{m}$ (d) 1

18. If $y=\log\sqrt{\sin x}$ then $\frac{dy}{dx}=$
(a) $\frac{\cos x}{\sqrt{\sin x}}$ [✓ (b)] $\frac{1}{2}\cot x$ (c) $\frac{1}{2}\tan x$ (d) $\sqrt{\sin x \cos x}$

19. $\int \frac{\sin x}{1+4\cos x}\,dx$ is
(a) $\log(1+4\cos x)+c$ (b) $-4\log(1+\cos 4x)+c$
(c) $-\log(\sin x)+c$ [✓ (d)] $-\frac{1}{4}\log(1+4\cos x)+c$

20. $\int_0^{\pi/3} \tan x\,dx$ is
[✓ (a)] $\log 2$ (b) 0 (c) 1 (d) $1-\log 2$

PART—B **Marks 80**

I Answer any TEN questions ***(Two Marks each)***

(1) Define Equivalence relation. Give an example of a relation which is transitive but not reflective.

(2) Find the G.C.D. of 657 and 963. If it is equal to $657\,x+963\times(-15)$ find x. 22

(3) If $a \equiv b \pmod{m}$ and n is a positive factor of m prove that $a \equiv b \pmod{n}$

(4) If $A=\begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix}$ find AB'. $\begin{bmatrix} 12 & -7 \\ 12 & 22 \end{bmatrix}$

(5) Given $A=\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$ find *adj A.* $\begin{bmatrix} -1, & 5 & -1 \\ 2 & -1 & 2 \\ 4 & -2 & 5 \end{bmatrix}$

(6) If the operation $*$ is defined by $a * b=\frac{ab}{7}$ on the set of rational numbers show that $*$ is associative.

(7) Define subgroup of a group. Give an example.

(8) If $\vec{a}=i-2j+3k$ and $\vec{b}=2i+j+k$ find

(*i*) unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$

(*ii*) sine of the angle between $\vec{a}$ and $\vec{b}$.

(9) If the vectors $i+2j+5k$, $2i+xj+10k$ and $3i+9j-2k$ are coplanar, find x.

(10) Solve the triangle ABC, given that $b=2$, $c=4$, $A=60°$.

(11) Find the eccentricity of the ellipse $25x^2+16y^2=400$.

(12) If $y=\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, show that $\frac{dy}{dx}=\frac{1}{2}$.

II (1) If Q_1 is the set of all rational numbers except 1 with binary operation $*$ defined by $a * b=a+b-ab$, $\forall\, a, b \in Q_1$ show that $(Q_1, *)$ is a group. 4

OR

Prove that a non-empty subset H of a group G is a subgroup of G if and only if $\forall\, a, b \in H$, $ab^{-1} \in H$. 4

(2) Find the inverse of $\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \\ -1 & 2/3 & 1/3 \end{bmatrix}$ 4

OR

Show that $\begin{vmatrix} a^2 & a^2+ab & ab \\ bc & b^2 & b^2+bc \\ ac+c^2 & ac & c^2 \end{vmatrix}=4a^2b^2c^2$ 4

III (1) Find the general solution of $\sin x-\sqrt{3}\cos x=1$ 4

OR

Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ 4

(2) Find the modulus and amplitude of $\frac{(1+i)\ (1+3i)}{1+5i}$ 4

(3) Show that $(1+i)^n+(1-i)^n=2^{n/2+1}\cos\frac{n\pi}{4}$ 4

OR

Find the cube roots of $1+i$ and represent them in Argand diagram 4

IV (1) Find k so that $3x^2-11xy+10y^2-7x+13y+k=0$ may represent a pair of lines. Find their point intersection. 4

OR

Find the equation of the circle which cuts orthogonally the circles $x^2+y^2-10x-4y+21=0$ and $x^2+y^2-4x-6y+11=0$ and has its centre on $2x+3y-7=0$ 4

(2) Derive the equation of the parabola in the form $y^2=4ax$ 4

(3) Find the centre, foci of $4x^2+9y^2+16x-18y=11$ 4

OR

Find the equations of the tangent and normal to the hyperbola $9x^2-4y^2=36$ at $\left(\frac{2\sqrt{10}}{3},\ 1\right)$ on it. 4

V (1) Prove that $\lim\limits_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=na^{n-1}$ when n is a rational number 4

(2) Find the derivative of $\operatorname{cosec} x$ w.r.t. x from first principles. 4

OR

Differentiate w.r.t. x $\tan^{-1}\left[\sqrt{\frac{1+x}{1-x}}\right]$ 4

(3) If $x=3\cos\theta-\cos^3\theta$, $y=3\sin\theta-\sin^3\theta$
find $\frac{d^2y}{dx^2}$ at $\theta=\pi/4$ 4

OR

Show that the curves $y=6+x-x^2$ and $y=\left(\frac{x+2}{x-1}\right)$ touch each other at (2, 4). Find the equation of the common tangent. 4

VI (1) If $y=\cos(m\sin^{-1}x)$ prove that $(1-x^2)y_2-xy_1+m^2y=0$ 4

(2) In the curve $xy=c^2$, show that the subnormal at any point varies as the cube of the ordinate at that point. 4

OR

Find the angle of intersection of the curves $x^2+y^2=5$ and $y^2=4x$ 4

(3) What is the largest size rectangle that can be inscribed in a semicircle of radius 1 so that 2 vertices lie on the diameter. 4

OR

Show that $\int_0^a f(x)dx=\int_0^a f(a-x)\,dx$ and hence find the value of $\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ 4

(4) Integrate w.r.t. x (*i*) $\frac{(\tan^{-1}x)^3}{1+x^2}$ (*ii*) $\frac{1}{x\sqrt{1-\log x}}$ 4

OR

Find the area enclosed between the parabolas $y^2=4ax$ and $x^2=4ay$ 4

SECTION I: ALGEBRA

CHAPTER I

CROSS PRODUCTS, RELATIONS AND FUNCTIONS

1. Define *cross product* of two sets A and B. Illustrate with examples.

2. (*i*) If $A=\{1, 2, 3\}$ and $B=\{3, 4\}$, find $A\times B$ and $B\times A$. Are they equal?
   (*ii*) If $A=\{a, b\}$, find $A\times A$.
   (*iii*) If $A=\{0, 1, 2\}$, $B=\{0, 4, 5\}$ and $C=\{4, 5\}$, find $A\times B$, $A\times C$, $B\times C$, $(A\cap B)\times C$, $A\times(B\cup C)$ and the number of elements in each cross product.
   (*iv*) If $A=\{x \mid x^3-x=0\}$, $B=\{x \mid x^2-3x+2=0\}$ and $C=\{x \mid x^2-1=0\}$, find $B\times C$ and $B\times A$. Is $B\times C$ a subset of $B\times A$?
   (*v*) If $A=\{a, b\}$, $B=\{b, c\}$ and $C=\{c, d\}$, find $A\times(B\cup C)$, $(A\times B)\cup(A\times C)$, $A\times(B\cap C)$ and $(A\times B)\cap(A\times C)$.
   (*vi*) If $A=\{1, 2\}$, $B=\{1, 2, 3, 4, 5\}$ and $C=\{3, 5, 7, 9\}$ find $(A\times B)\cap(A\times C)$ and $A\times(B-C)$.
   (*vii*) If $A=\{2, 3, 5\}$ and $B=\{4, 5, 6\}$, find $A\times(A\cap B)$, $(A\cap B)\times A$, $(A\cap B)\times(A\cup B)$ and $(B-A)\times(A-B)$.

3. Explain how $A\times B$ can be represented geometrically and find a geometrical representation for $R\times R$ where R is the set of real numbers.

4. (*a*) Define a relation from a set A to a set B. Illustrate with examples.
   (*b*) Define a relation on A. Illustrate with examples.

5. (*i*) Let $A=\{2, 4, 5\}$ and $B=\{1, 3, 7\}$. Find the relation $R=\{(x, y) \mid x\in A,\ y\in B \text{ and } x<y\}$.
   (*ii*) Let $A=\{2, 3, 4\}$ and $B=\{3, 4, 5, 6\}$ and R be a relation from A to B defined by xRy iff x divides y. Find R.
   (*iii*) Find the relation R defined on $A=\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ by $(x, y)\in R$ if (*a*) y is twice x (*b*) y is more than x by 5 (*c*) x and y are relatively prime to each other.

6. Define the domain and the range of a relation R from A to B and find the same for relations in problem no: (5)

7. (*i*) Define a reflexive relation on A. Illustrate with examples
   (*ii*) Define a symmetric relation on A. Illustrate with examples.
   (*iii*) Define a transitive relation on A. Illustrate with examples
   (*iv*) Define an equivalence relation on A. Illustrate with examples.
   (*v*) Define an anti-symmetric relation on A. Illustrate with examples.
   (*vi*) Give an example of a relation on a set which is
   (*a*) reflexive only (*b*) symmetric only (*c*) transitive only
   (*d*) reflexive and symmetric but not transitive
   (*e*) symmetric and transitive but not reflexive
   (*f*) reflexive and transitive but not symmetric
   (*g*) equivalence (*h*) symmetric and anti-symmetric
   (*i*) none of reflexive, symmetric and transitive.

8. State whether or not each of the following relations is
   (*a*) reflexive (*b*) symmetric (*c*) transitive
   (*d*) equivalence (*e*) anti-symmetric. Give reasons.
   (*i*) Let A be the set of lines in a plane and let R be the relation in A defined by "parallel to"
   (*ii*) Let A be as in (*i*) and R be "perpendicular to"
   (*iii*) Let N be the set of natural numbers and R be "relatively prime to"
   (*iv*) on N, let R be "the square of"
   (*v*) on N, let R be "a factor of"
   (*vi*) Let A be the set of triangles in a plane and R be "has the same area as"
   (*vii*) Let A be as in (*vi*) and R be "similar to"
   (*viii*) Let A be as in (*vi*) and R be "congruent to"
   (*ix*) Let A be as in (*vi*) and R be "has same perimeter as"
   (*x*) Let A be the set of all subsets of a set S and R be "the subset of".

9. Let A={1, 2, 3}. State whether or not each of the following relations is (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) equivalence (*e*) anti-symmetric. Give reasons.
   (*i*) { (1, 2), (3, 2), (2, 2), (2, 3) } (*ii*) { (1, 2), (2, 3), (1, 3) }
   (*iii*) { (1, 1), (2, 2) } (*iv*) { (1, 1), (3, 2), (2, 3) } (*v*) { (1, 1) }
   (*vi*) A×A

10. Define a function f: A→B as a correspondence and establish that it is a relation from A to B.

11. Let $A=\{1, 2, 3, 4\}$. Consider the following relations on A: (*i*) $R_1=\{(1, 2), (2, 3), (3, 4), (4, 1)\}$ (*ii*) $R_2=\{(1, 1), (1, 2), (1, 3), (1, 4)\}$ (*iii*) $R_3=\{(1, 1), (2, 1), (3, 1), (4, 1)\}$ (*iv*) $R_4=\{(1, 2), (3, 4), (4, 1)\}$ (*v*) $R_5=\{(2, 1), (4, 4), (3, 1), (2, 3)\}$.
State whether or not each of these relations is a function from A to B. If so, find its domain and range.

12. (*i*) Define constant function and identity function. Give examples.
(*ii*) Define one-one function. Give examples.
(*iii*) Define onto function. Give examples.

13. Let $A=\{1, 2, 3, 4, 5\}$. State whether or not each of the following functions is (*a*) one-one (*b*) onto function of A into itself
(*i*) $f(1)=1, f(2)=2, f(3)=3, f(4)=5, f(5)=4$
(*ii*) $f(1)=2, f(2)=3, f(3)=1, f(4)=3, f(5)=4$
(*iii*) $f(1)=f(2)=f(3)=5, f(4)=4, f(5)=2$
(*iv*) $f(1)=f(2)=f(3)=f(4)=f(5)=5$

14. Let f: R→R be defined by
(*i*) $f(x)=2x-1$ (*ii*) $f(x)=x^2$ (*iii*) $f(x)=|x|$ (*iv*) $f(x)=\sin x$ (*v*) $f(x)=x^3$ (*vi*) $f(x)=\tan x$. Examine whether they are one-one and onto or not.

15. Define the composite function of f:A→B and g:B→C. Illustrate with example.

16. The following functions are defined from R→R
(*i*) If $f(x)=x^2$ and $g(x)=x+3$, find (fog) (x) and (gof) (x). Are they equal?
(*ii*) If $f(x)=x^2+2x-3$ and $g(x)=3x-4$ find (gof) (2) and (fog) (2)
(*iii*) If $f(x)=2x-3$ and $g(x)=\cos x$ find $f(g(0))$ and $g(f(0))$

17. Let f; A→B be a function. Define $f^{-1}(b)$ (inverse of an element $b \in$ B) and the inverse function f^{-1}; B→A.

18. Let f: R→R defined by $f(x)=x^2$. Find (*i*) $f^{-1}(25)$ (*ii*) $f^{-1}(-3)$. Does the inverse function exist?

19. Consider the functions (*i*) $f(x)=2x-3$ (*ii*) $g(x)=x^3+5$ defined from R→R. Observe that they are both one-one and onto and therefore they have inverses. Find the formulae that define the inverse functions.

Answers

2. (*i*) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$; $\{(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)\}$; no. (*ii*) $\{(a, a), (a, b), (b, a), (b, b)\}$ (*iii*) $\{(0, 0), (0, 4), (0, 5), (1, 0), (1, 4), (1, 5), (2, 0), (2, 4), (2, 5)\}$; $\{(0, 4), (0, 5), (1, 4), (1, 5), (2, 4), (2, 5)\}$; $\{(0, 4), (0, 5), (4, 4), (4, 5), (5, 4), (5, 5)\}$; $\{(0, 4), (0, 5)\}$; $A \times B$; 9, 6, 6, 2, 9. (*iv*) $\{(1, -1), (1, 1), (2, -1), (2, 1)\}$; $\{(1, -1), (1, 0), (1, 1), (2, -1), (2, 0), (2, 1)\}$; yes (*v*) $A \times (B \cup C) = (A \times B) \cup (A \times C) = \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)\}$; $A \times (B \cap C) = (A \times B) \cap (A \times C) = \{(a, c), (b, c)\}$. (*vi*) $\{(1, 3), (1, 5), (2, 3), (2, 5)\}$; $\{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4)\}$ (*vii*) $\{(2, 5), (3, 5), (5, 5)\}$; $\{(5, 2), (5, 3), (5, 5)\}$; $\{(5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$; $\{(4, 2), (4, 3), (6, 2), (6, 3)\}$-

5. (*i*) $\{(2, 3), (2, 7), (4, 7), (5, 7)\}$ (*ii*) $\{(2, 4), (2, 6), (3, 3), (3, 6), (4, 4)\}$ (*iii*) (*a*) $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$. (*b*) $\{(1, 6), (2, 7), (3, 8), (4, 9)\}$ (*c*) $\{(2, 3), (2, 5), (2, 7), (2, 9), (3, 2), (3, 4), (3, 5), (3, 7), (3, 8), (4, 3), (4, 5), (4, 7), (4, 9), (5, 2), (5, 3), (5, 4), (5, 6), (5, 7), (5, 8), (5, 9), (6, 5), (6, 7), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (7, 8), (7, 9), (8, 3), (8, 5), (8, 7), (8, 9), (9, 2), (9, 4), (9, 5), (9, 7), (9, 8)\}$

6. (*i*) domain$=$A; range$=\{3, 7\}$ (*ii*) A; $\{3, 4, 6\}$ (*iii*) (*a*) $\{1, 2, 3, 4\}$; $\{2, 4, 6, 8\}$ (*b*) $\{1, 2, 3, 4\}$; $\{6, 7, 8, 9\}$ (*c*) $A - \{1\}$; $A - \{1\}$

8. (*i*) *a, b, c, d* (*ii*) *b* (*iii*) *b* (*iv*) *e* (*v*) *a, c, e* (*vi*), (*vii*), (*viii*), (*ix*) *a, b, c, d* (*x*) *a, c, e*

9. (*i*) none of *a, b, c, d, e* (*ii*) *c, e* (*iii*) *b, c, e* (*iv*) *b* (*v*) *b, c, e* (*vi*) *a, b, c, d*

11. (*i*) and (*iii*) only; domain is A; range of (*i*)$=$A; range of (*iii*)$=\{1\}$

13. (*i*) one-one; onto (*ii*), (*iii*) and (*iv*) neither one-one nor onto

14. (*i*) and (*v*) one-one; onto (*ii*), (*iii*) and (*iv*) neither one-one nor onto (*vi*) onto but not one-one.

16. (*i*) x^2+6x+9; x^2+3; no (*ii*) 11, 5 (*iii*) -1; $\cos 3$

18. (*i*) $\{-5, 5\}$ (*ii*) ϕ;dosn't exist. 19. (*i*) $\frac{x+3}{2}$ (*ii*) $\sqrt[3]{x-5}$.

CHAPTER 2

ELEMENTS OF NUMBER THEORY

2.1 Divisibility

1. Define the divisibility relation in N and Z.
2. Are the following true? Give reasons
   (*i*) $3 \mid 24$ (*ii*) $5 \nmid 31$ (*iii*) $4 \mid 250$ (*iv*) $3 \nmid 420$ (*v*) $-2 \mid 2$
   (*vi*) $16 \nmid -800$
3. Prove that the divisibility is (*i*) reflexive, antisymmetric and transitive in N (*ii*) reflexive and transitive but not symmetric, not antisymmetric in Z.
4. Prove that in Z (*i*) $a \mid 0$, $a \neq 0$ (*ii*) if $a \mid b$, then $a \mid -b$, $-a \mid b$ and $-a \mid -b$ (*iii*) if $a \mid b$ and $b \mid a$, then $a = \pm b$ (*iv*) if $a \mid b$ and $a \mid c$, then $a \mid pb+qc$, $p, q \in Z$ (*v*) if $a \mid b$, then $a \mid cb$, $c \in Z$ (*vi*) if $a \mid b$, then $ac \mid bc$, $c \neq 0$ (*vii*) if $ac \mid bc$, then $a \mid b$ (*viii*) if $a \mid b$ and $c \mid d$, then $ac \mid bd$
5. State division algorithm.
6. Define g.c.d. of two integers and explain the method of finding the same by successive application of division algorithm (Euclidean algorithm)
7. Find by Euclid's algorithm the g.c.d. of each of the following pairs of numbers and express it as a linear combination of them.
   (*i*) 32 and 54 (*ii*) 81 and 237 (*iii*) 963 and 657 (*iv*) 506 and 1155 (*v*) 595 and 252 (*vi*) 4824 and 2072. Is it unique?
8. Define prime and composite numbers, giving an example for each. Is '0' a composite number? What about 1?
9. Illustrate with examples that a composite number can be expressed as a product of primes.
10. When are two integers *a* and *b* said to be relatively prime to each other? Examine whether the following pairs of integers are relatively prime or not.
    (*i*) 14; 49 (*ii*) 18; 15 (*iii*) 220; 229
11. Prove that the number of primes is infinite.
12. If p is a prime, prove that $p \mid ab$ implies $p \mid a$ or $p \mid b$.
13. If $(c, a)=1$ and $c \mid ab$, then prove that $c \mid b$.

3 Write down the orders of the following matrices

(*i*) $\begin{pmatrix} 1 & 6 \\ 2 & 3 \end{pmatrix}$ 2×2 (*ii*) $\left\| \begin{matrix} 1 & 4 & 6 \\ 2 & 3 & 1 \end{matrix} \right\|$ 2×3 (*iii*) $\begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$ 3×1

(*iv*) $[\,2\,]$ 1×1 (*v*) $[\,2\ 3\ 5\ 4\,]$ 1×4 (*vi*) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ 3×3

4 Define equality of matrices, multiplication of a matrix by a scalar, addition and subtraction of matrices; illustrate with examples.

5 Define transpose of a matrix and illustrate with examples.

6 Write down the orders of the transposes of matrices in question (3).

7 (*i*) If $A=\begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ find $A+B$ and $2A-3B$.

(*ii*) If $A=\begin{pmatrix} 2 & 5 \\ 0 & 7 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -1 \\ 6 & 2 \end{pmatrix}$ and $C=\begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix}$ find $-A$, $A+B$, $A-B$, $3A-2B$, $2B+C$, $2A-3C$ and $2A+3B+4C$.

8 (*i*) If $A=\begin{pmatrix} 3 & 5 & 2 \\ 4 & 7 & 6 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 2 & 5 \\ 4 & 0 & 3 \end{pmatrix}$ and $C=\begin{pmatrix} 0 & 3 & 4 \\ 9 & 8 & -7 \end{pmatrix}$ find $5A-4B+3C$ and verify that $(A+B)'=A'+B'$ and $(A')'=A$

(*ii*) If $A=\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 6 \end{bmatrix}$, show that $(A')'=A$

(*iii*) If $A=\begin{pmatrix} 1 & 2 \\ 6 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ and $C=\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ find $3A'+4B'-5C'$.

9. (*i*) If $B+A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $B-A=\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \end{pmatrix}$, find A and B

(*ii*) If $2A+B=\begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 0 \end{pmatrix}$ and $3A+2B=\begin{pmatrix} 4 & 6 & 1 \\ 2 & 3 & 5 \end{pmatrix}$, find A and B

(*iii*) If $A=\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$, find a matrix C such that $2A+B+2C=0$

(*iv*) If $\begin{pmatrix} 2x & y \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 1+y & -2+x \\ 7 & -6 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 9 & z \end{pmatrix}$,
find x, y and

(*v*) If A$=\begin{pmatrix} 1+x & x \\ 5+y & 3 \end{pmatrix}$, B$=\begin{pmatrix} x & y \\ 7 & 3 \end{pmatrix}$ and C$=\begin{pmatrix} -4 & 5 \\ 35 & y \end{pmatrix}$
and 2A+B=C, find x and

10. When do we say that two matrices are conformable fo multiplication.

11. (*i*) If A is a matrix of order $m \times n$ and B is a matrix of orde $n \times p$, do AB and BA exist? If so what are their orders?

(*ii*) If A is a 3×2 matrix, B is a 4×3 matrix, do AB, A′B, AB′ and A′B′ exist? If so what are their orders?

(*iii*) If A is a 3×4 matrix and B is a 4×3 matrix, does (AB+BA)′ exist? If so what is its order?

(*iv*) If A is a 3×2 matrix and B is a 5×3 matrix do A+B, A–B, AB and BA exist? If so what are their orders?

(*v*) If A is matrix of order $m \times n$ and B is a matrix of order $n \times m$, what is the order of (AB)′?

12. Define matrix multiplication and illustrate with examples.

13. (*i*) If A$=\begin{pmatrix} 1 & 3 \end{pmatrix}$ and B$=\begin{pmatrix} 3 & -2 & 1 & -3 \\ -1 & 4 & -3 & 2 \end{pmatrix}$, find AB.

(*ii*) If A$=\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ and B$=\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ find AB and BA,
if they exist.

(*iii*) If A$=\begin{pmatrix} 4 & 3 & 2 \\ 1 & -1 & 3 \end{pmatrix}$ find AA′ and A′A. Are they equal?

(*iv*) If A$=\begin{bmatrix} 1 & 2 \\ 4 & -3 \\ -5 & 6 \end{bmatrix}$ and B$=\begin{bmatrix} 0 & 3 & 5 \\ 4 & 2 & 6 \end{bmatrix}$ find AB and BA.

(*v*) If A$=\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and B$=\begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}$ find AB′ and A′B.

(*vi*) If A$=\begin{pmatrix} -1 & 2 \\ 2 & 4 \end{pmatrix}$ and B$=\begin{pmatrix} 3 & 5 \\ 6 & -7 \end{pmatrix}$ verify that
(AB)′=B′A′.

(*vii*) Find $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

(*viii*) (*a*) Find $\begin{pmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$

(*b*) If $A=\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 2 \\ -2 & 0 & 5 \end{bmatrix}$, find A^2.

(*c*) If $X=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, find X^3.

(*ix*) If $A=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and

$C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{bmatrix}$ verify that $A(B+C)=AB+AC$.

(*x*) If $A=\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ and $C=\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ verify that $A(BC)=(AB)\,C$.

(*xi*) If $A=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, prove that

$A^2=\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$ and $A\,A'=I$.

(*xii*) If $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ verify that $(AB)'=B'A'$.

(*xiii*) If $A=\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ find A^2-B^2; Is $A^2-B^2=(A+B)\,(A-B)$?

(*xiv*) If $A=\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$, $B=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $C=\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ where $i=\sqrt{-1}$ prove that $A^2=B^2=C^2=-I$ and $AB=-C=-BA$.

(*xv*) If $A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ show that $A^2=3A$ and deduce that $A^3-A=2A\,(A+I)$.

14. (*i*) If $A=\begin{pmatrix} 1 & 3 \\ 1 & 0 \end{pmatrix}$ prove that $A^2-A-3I=0$.

(*ii*) If $A=\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ prove that $A^2-4A+3I=0$.

(*iii*) If $A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ prove that $A^2-4A-5I=0$.

(*iv*) If $A=\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} -4 & 6 \\ 2 & -3 \end{pmatrix}$ prove that $AB=0$.

(*v*) If $A=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$ show that $A^2=0$.

(*vi*) If $A=\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ prove that $AB=BA=0$.

15. (*i*) Solve for x:— $\begin{pmatrix} x^2 & 1 \\ 2 & 3 \end{pmatrix}+\begin{pmatrix} 2x & 3 \\ 1 & 4 \end{pmatrix}=\begin{pmatrix} 3 & 4 \\ 3 & 7 \end{pmatrix}$.

(*ii*) Solve:— $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(*iii*) Solve:— $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(*iv*) Find x and y if $\begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Answers

3. 2×2; 2×3; 3×1; 1×1; 1×4; 3×3

6. orders of transposes: 2×2; 3×2; 1×3; 1×1; 4×1; 3×3.

7. (*i*) $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 1 & 6 \end{pmatrix}$; $\begin{pmatrix} -4 & 4 & -11 \\ -11 & -3 & -8 \end{pmatrix}$ (*ii*) $\begin{pmatrix} -2 & -5 \\ 0 & -7 \end{pmatrix}$; $\begin{pmatrix} 3 & 4 \\ 6 & 9 \end{pmatrix}$; $\begin{pmatrix} 1 & 6 \\ -6 & 5 \end{pmatrix}$; $\begin{pmatrix} 4 & 17 \\ -12 & 17 \end{pmatrix}$; $\begin{pmatrix} 2 & -2 \\ 11 & 6 \end{pmatrix}$; $\begin{pmatrix} 4 & 10 \\ 3 & 8 \end{pmatrix}$; $\begin{pmatrix} 7 & 7 \\ 14 & 28 \end{pmatrix}$.

8. (*i*) $\begin{pmatrix} 19 & 26 & 2 \\ 31 & 59 & -3 \end{pmatrix}$ (*iii*) $\begin{pmatrix} 7 & 16 \\ 1 & -7 \end{pmatrix}$.

9. (*i*) $A=\begin{pmatrix} -\frac{1}{2} & -2 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}$; $B=\begin{pmatrix} \frac{3}{2} & 1 & 2 \\ \frac{5}{2} & \frac{7}{2} & 3 \end{pmatrix}$ (*ii*) $A=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 5 & -5 \end{pmatrix}$; $B=\begin{pmatrix} 2 & 3 & -1 \\ 1 & -6 & 10 \end{pmatrix}$ (*iii*) $\begin{pmatrix} -\frac{5}{2} & -\frac{3}{2} & 2 \\ -7 & -1 & -\frac{9}{2} \\ -2 & 1 & -\frac{5}{2} \end{pmatrix}$.

(*iv*) $x=1$, $y=1$, $z=-3$ (*v*) $x=-2$, $y=9$.

11. (*i*) order of AB is $m \times p$; BA does not exis. ($\because p \neq m$) (*ii*) only A′B′ exists and order is 2×4 (*iii*) no (*iv*) only BA exists; order 5×2 (*v*) $m \times m$ (*vi*) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (*vii*) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

13. (*i*) $(0 \;\; 10 \;\; -8 \;\; 3)$ (*ii*) $AB = \begin{pmatrix} -7 \\ 2 \end{pmatrix}$; BA dosn't exist

(*iii*) $\begin{pmatrix} 29 & 7 \\ 7 & 11 \end{pmatrix}$; $\begin{pmatrix} 17 & 11 & 11 \\ 11 & 10 & 3 \\ 11 & 3 & 13 \end{pmatrix}$; unequal (*iv*) $AB = \begin{pmatrix} 8 & 1 & 7 \\ -12 & 6 & 2 \\ 24 & -3 & 11 \end{pmatrix}$;

$BA = \begin{pmatrix} -13 & 21 \\ -26 & 38 \end{pmatrix}$ (*v*) $AB' = \begin{pmatrix} 33 & 43 \\ 59 & 77 \end{pmatrix}$; $A'B = \begin{pmatrix} 44 & 50 \\ 56 & 66 \end{pmatrix}$

(*vii*) $(ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx)$

(*viii*) (*a*) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (*b*) $\begin{pmatrix} 4 & -5 & 14 \\ 1 & 3 & 16 \\ -16 & 2 & 21 \end{pmatrix}$ (*c*) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(*xiii*) $\begin{pmatrix} -1 & 16 \\ -8 & -1 \end{pmatrix} \neq \begin{pmatrix} -9 & 16 \\ -8 & 7 \end{pmatrix}$.

15. (*i*) 1, -3 (*ii*) $x=1$, $y=2$, $z=3$ (*iii*) $x=-4$, $y=5$, $z=3$ (*iv*) $x=1$, $y=1$.

3.2 Determinants

1. Define determinant of a 2×2 matrix; a 3×3 matrix.
2. Define a minor of an element of a determinant and express the value of a determinant interms of the elements of a row and their minors
3. Evaluate:

(*i*) $\begin{vmatrix} -1 & 3 \\ 5 & -8 \end{vmatrix}$ (*ii*) $\begin{vmatrix} 0 & 3 \\ -2 & 5 \end{vmatrix}$ (*iii*) $\begin{vmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ (*iv*) $\begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & 2 \\ 1 & -3 & -3 \end{vmatrix}$

(*v*) $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ (*vi*) $\begin{vmatrix} \sec x & \sin x & \tan x \\ 0 & 1 & 0 \\ \tan x & \cot x & \sec x \end{vmatrix}$ (*vii*) $\begin{vmatrix} 2 & 0 & -1 \\ 0 & 1 & 5 \\ 1 & 3 & -4 \end{vmatrix}$

(*viii*) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (*ix*) $\dfrac{|D|}{2}$ where $D = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$.

4. **Prove that:—**

(*i*) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ (*ii*) $\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = 4abc.$

5. **Prove** that (*i*) the value of a determinant is unaltered if its rows and columns are interchanged (*ii*) If any two rows (or columns) of a determinant are interchanged, then the sign of the determinant changes (*iii*) If any two rows (or columns) of a determinant are identical, then the determinant vanishes (*iv*) If every element of any row (or column) of a determinant is multiplied by *k*, then the value of the determinant is multiplied by *k* (*v*) If one row (or column) of a determinant is a multiple of another row (or column), then the determinant vanishes (*vi*) If each element of one row (or column) of a determinant is the sum two elements, then determinant can be expressed as a sum of two determinants (*vii*) The value of a determinant is unaltered if to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column) are added.

6. Without expanding the determinants prove the following:—

(*i*) $\begin{vmatrix} 2 & 11 \\ 5 & 7 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 11 & 7 \end{vmatrix}$ (*ii*) $\begin{vmatrix} 1 & 2 & -5 \\ 2 & 3 & 4 \\ 5 & 7 & 8 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & -5 \\ 5 & 7 & 8 \\ 2 & 3 & 4 \end{vmatrix}$

(*iii*) $\begin{vmatrix} -2 & 3 & -2 \\ -1 & 2 & -1 \\ 4 & 6 & 4 \end{vmatrix} = 0$ (*iv*) $\begin{vmatrix} 8 & 10 & 50 \\ 2 & 3 & 4 \\ 4 & 5 & 25 \end{vmatrix} = 0$ (*v*) $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 2 & 4 & 6 \end{vmatrix} = 0$

(*vi*) $\begin{vmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 5 & 6 & 7 \end{vmatrix} = 0$ (*vii*) $\begin{vmatrix} -1 & 0 & -1 \\ 2 & 4 & 2 \\ 5 & 5 & 5 \end{vmatrix} = 0$ (*viii*) $\begin{vmatrix} 2 & -3 & 5 \\ 4 & -6 & 1 \\ 6 & -9 & 15 \end{vmatrix} = 0$

(*ix*) $\begin{vmatrix} 2 & 0 & 1 \\ 4 & 0 & 2 \\ 6 & -10 & 3 \end{vmatrix} = 0.$

7. **Evaluate:—**

(*i*) $\begin{vmatrix} 91 & 92 & 93 \\ 94 & 95 & 95 \\ 97 & 98 & 99 \end{vmatrix}$ (*ii*) $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 4^2 & 5^2 & 6^2 \\ 7^2 & 8^2 & 9^2 \end{vmatrix}$ (*iii*) $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix}$

(*iv*) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

8. Prove that:—

(i) $\begin{vmatrix} 5 & 5^2 & 5^3 \\ 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \end{vmatrix} = 0$ (ii) $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$ where w is a cube root of unity

(iii) $\begin{vmatrix} a-x & a-y & a-z \\ b-x & b-y & b-z \\ c-x & c-y & c-z \end{vmatrix} = 0$ (iv) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

(v) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$ (vi) $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(vii) $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$ (viii) $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

(ix) $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$ (x) $\begin{vmatrix} bc & a & 1 \\ ca & b & 1 \\ ab & c & 1 \end{vmatrix} = \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$

9. Prove that:—

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

(v) $\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(vi) $\begin{vmatrix} a^2+bc & a & 1 \\ b^2+ca & b & 1 \\ c^2+ab & c & 1 \end{vmatrix} = -2(a-b)(b-c)(c-a).$

10. Prove that:—

(*i*) $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

(*ii*) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(*iii*) $$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(*iv*) $$\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = (x-p)\,(x-q)\,(x+p+q)$$

(*v*) $$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$$

(*vi*) $$\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = 1+a+b+c$$

(*vii*) $$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

(*viii*) (4) (*i*) and (*ii*) using the properties of determinants.

11. If $\begin{vmatrix} a^3+1 & a^2 & a \\ b^3+1 & b^2 & b \\ c^3+1 & c^2 & c \end{vmatrix} = 0$ and a, b, c are all different, show that $abc+1=0$

12. Solve:—

(*i*) $\begin{vmatrix} x & 5 \\ 5 & x \end{vmatrix} = 0$ (*ii*) $\begin{vmatrix} x+2 & 2x+5 \\ 3x-1 & x-3 \end{vmatrix} = 8$ (*iii*) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$

(*iv*) $\begin{vmatrix} 1 & 2 & 5 \\ 2 & x & 10 \\ 3 & 9 & -2 \end{vmatrix} = 0$ (*v*) $\begin{vmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{vmatrix} = 0$ (*vi*) $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

(*vii*) $\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix} = 0$ (*viii*) $\begin{vmatrix} x+3 & 5 & 7 \\ 3 & x+5 & 7 \\ 3 & 5 & x+7 \end{vmatrix} = 0$

(*ix*) $\begin{vmatrix} 1+x & 1-x & 1-x \\ 1-x & 1+x & 1-x \\ 1-x & 1-x & 1+x \end{vmatrix}=0$ (*x*) $\begin{vmatrix} 4x+5 & 4x+7 & 4x+9 \\ 4x+9 & 4x+5 & 4x+7 \\ 4x+7 & 4x+9 & 4x+5 \end{vmatrix}=0.$

13. Prove the Cramer's rule for solving linear equations.

14. Solve the following using Cramer's rule:

(*i*) (*a*) $2x+3y+7=0;\ x+y=8$ (*b*) $3x+4y=7;\ 7x=y+6$

(*ii*) $x+y+z=7$, $2x+3y+2z=17$, $4x+9y+z=37$

(*iii*) $4x+y=7$, $3y+4z=5$, $3z+5x=2$

(*iv*) $3x+2y+5z=32$, $2x+5y+3z=31$, $5x+3y+2z=27$

(*v*) $4(y-x)=5z-2$, $3z+4x=6y+2$, $z-3y=14-10x$

(*vi*) $7x+6y-5z=30$, $3x-4y+z=0$, $x+2y-3z=10$

(*vii*) $2(x-y)=3z-2$, $x-3z=3y-1$, $2x+3y=4(1-z)$

(*viii*) $5y+2x+z=-1$, $x+7y-6z=-18$, $3y+6z=9$

(*ix*) $y+z=4$, $z+x=5$, $x+y=3$

Answers

3. (*i*) -7 (*ii*) 6 (*iii*) 30 (*iv*) 42 (*v*) $abc+2fgh-ab^2-bg^2-ch^2$ (*vi*) 1 (*vii*) -37 (*viii*) $3\,abc-a^3-b^3-c^3$ (*ix*) -1.

7. (*i*) 0 (*ii*) -216 (*iii*) $2a^3b^3c^3$ (*iv*) $4a^2b^2c^2$.

12. (*i*) ± 5 (*ii*) $-\frac{9}{5}$, -1 (*iii*) $\frac{5}{2}$ (*iv*) 4 (*v*) -6 (*vi*) 1, 2, -3 (*vii*) 0, -6 (*viii*) 0, -15 (*ix*) 0, 3 (*x*) $-7/4$.

14. (*i*) (*a*) $x=31,\ y=-23$ (*b*) $x=1=y$ (*ii*) $x=2,\ y=3,\ z=2$ (*iii*) $x=1,\ y=3,\ z=-1$ (*iv*) $x=2, y=3, z=4$ (*v*) $x=2, y=4, z=6$ (*vi*) $x=2,\ y=1,\ z=-2$ (*vii*) $x=\frac{5}{13},\ y=-\frac{18}{13},\ z=\frac{24}{13}$ (*viii*) $x=1, y=-1,\ z=2$ (*ix*) $x=2,\ y=1,\ z=3$.

3.3 Algebra of matrices (contd.)—adjoint and inverse

1. Define cofactor of an element of a determinant.

2. Prove that (*i*) if the elements of any row (or column) are multiplied by their corresponding cofactors and added, then their sum is the value of the determinant and (*ii*) if the elements of a row (or column) are multiplied by the cofactors of the corresponding elements of any other row (or column) and added, then the sum is zero.

3. Define adjoint of a square matrix.

4. Find the adjoints of the following matrices:—

(*a*) (*i*) $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$ (*ii*) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(*b*) (*i*) $\begin{pmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{pmatrix}$ (*ii*) $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$ (*iii*) $\begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$

(*iv*) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (*v*) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$ (*vi*) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$

5. If $A=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix}$ verify that $A\ (\text{adj } A)=|A|\ I$

6. For a 3×3 matrix A, prove that
$A\ (\text{adj } A)=|A|\ I=(\text{adj } A)\ A.$

7. Define singular and non-singular matrices and give examples for each.

8. Which one of the following is singular?
$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$, $\begin{pmatrix} 2 & 3 \\ -2 & -3 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

9. Define inverse of a square matrix.

10. Find the inverses of the following matrices, if they exist:—

(*a*) (*i*) $\begin{pmatrix} 2 & 4 \\ -1 & 2 \end{pmatrix}$ (*ii*) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ (*b*) (*i*) $\begin{pmatrix} 2 & 2 & -1 \\ -1 & 1 & 1 \\ 1 & -2 & 3 \end{pmatrix}$

(*ii*) $\begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}$ (*iii*) $\begin{pmatrix} 1 & 3 & 4 \\ 1 & 4 & 5 \\ 2 & 5 & 7 \end{pmatrix}$ (*iv*) $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

(*v*) $\begin{pmatrix} 1 & 4 & -2 \\ -2 & -5 & -4 \\ 1 & -2 & 1 \end{pmatrix}$ (*vi*) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(*vii*) $\begin{pmatrix} 3 & -2 & 4 \\ 5 & 3 & -2 \\ 1 & 1 & 1 \end{pmatrix}$ (*viii*) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$ (*ix*) $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 8 & 2 \\ 4 & 9 & -1 \end{pmatrix}$

11. (*i*) If A and B are square matrices of same order, then prove that $|AB|=|A|\,|B|$ and hence show that if A and B are non-singular, so is AB.

(*ii*) If A and B are non-singular square matrices of order 3, prove that $(AB)^{-1}=B^{-1}A^{-1}$.

Answers

4. (*a*) (*i*) $\begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$ (*ii*) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(*b*) (*i*) $\begin{pmatrix} 3 & 0 & 6 \\ 6 & 3 & 0 \\ 9 & 6 & 3 \end{pmatrix}$ (*ii*) $\begin{pmatrix} -1 & 19 & -8 \\ -4 & 14 & -1 \\ 8 & 3 & 2 \end{pmatrix}$ (*iii*) $\begin{pmatrix} -1 & 4 & 3 \\ -1 & 5 & 3 \\ 1 & -6 & -4 \end{pmatrix}$

(*iv*) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (*v*) $\begin{pmatrix} 8 & -5 & -2 \\ -4 & -3 & 1 \\ -7 & 3 & -1 \end{pmatrix}$ (*vi*) $\begin{pmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{pmatrix}$

8. $\begin{pmatrix} 2 & 3 \\ -2 & -3 \end{pmatrix}$ 10. (*a*) (*i*) $\frac{1}{8}\begin{pmatrix} 2 & -4 \\ 1 & 2 \end{pmatrix}$

(*ii*) If $\Delta = ad-bc \neq 0$, $A^{-1}=\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (*b*) (*i*) $\frac{1}{17}\begin{pmatrix} 5 & -4 & 3 \\ 4 & 7 & -1 \\ 1 & 6 & 4 \end{pmatrix}$

(*ii*) $\frac{1}{14}\begin{pmatrix} 3 & -1 & 5 \\ 5 & 3 & -1 \\ -1 & 5 & 3 \end{pmatrix}$ (*iii*) $|A|=0$ $\therefore$ A^{-1} does not exist

(*iv*) $\begin{pmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ (*v*) $\frac{1}{9}\begin{pmatrix} 3 & 0 & 6 \\ 6 & 3 & 0 \\ 9 & 6 & 3 \end{pmatrix}$ (*vi*) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(*vii*) $\frac{1}{37}\begin{pmatrix} 5 & 6 & -8 \\ -7 & -1 & 26 \\ 2 & -5 & 19 \end{pmatrix}$ (*viii*) $\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$ (*ix*) $\begin{pmatrix} -26 & -7 & 12 \\ 11 & 3 & -5 \\ -5 & -1 & 2 \end{pmatrix}$

CHAPTER 4

GROUPS

4.1 Binary operation

1. (*i*) Define a binary operation
   (*ii*) State the two conditions for $*$ to be a binary operation on a non-empty set S.

2. Determine whether or not each of the definitions of * given below does give a binary operation on the given set (*i*) on N, $a * b=a+b$ (*ii*) on N, $a * b=a-b$ (*iii*) on Z, $a * b=a-b$ (*iv*) on C, $a * b=a-b$ (*v*) on N, $a * b=ab$ (*vi*) on C, $a * b=ab$ (*vii*) on Z, $a * b=\frac{a}{b}$ (*viii*) on Q, $a * b=\frac{a}{b}$ (*ix*) on R $-\{0\}$, $a * b=\frac{a}{b}$ (*x*) on Q, $a * b=\frac{a+b}{2}$ (*xi*) on Z, $a * b=\frac{ab}{2}$ (*xii*) on Q, $a * b=\frac{ab}{n}$ where n is a fixed +ve integer (*xiii*) on R, $a * b=\sqrt{ab}$ (*xiv*) on Z, $a * b=a$ (*xv*) on N, $a * b=a^b$ (*xvi*) on R, $a * b=a^b$ (*xvii*) on Z$-\{-1\}$, $a * b=a+b+ab$ (*xviii*) on Q$-\{1\}$, $a * b=a+b-ab$ (*xix*) on Z, $a * b=a+b+5$ (*xx*) on Q, $a * b=ab+1$ (*xxi*) on the set of $m \times n$ matrices, $A * B=A+B$ (*xxii*) on the set of $n \times n$ square matrices, $A * B=AB$ (*xxiii*) on the set of diagonal matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a \neq 0$, $b \neq 0$, $A * B=AB$ (*xxiv*) on the set of singular matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where x is a non-zero real number, $A * B=AB$ (*xxv*) on the set of rational numbers of the form p/q where $p \in Z$ and $q=1$, 2 or 3, $a * b=ab$.

3. Define an Algebraic structure.

4. Define closure law for an algebraic structure with one binary operation (and observe that closure law is true in any algebraic structure!).

5. If (S, $*$) is an algebraic structure and $A \subset S$, will (A, $*$) be an algebraic structure? Illustrate with examples.

6. Define commutative axiom for an algebraic structure (S, $*$); in problem (2) if * is a binary operation, examine whether it is commutative or not.

7. Define associative axiom for an algebraic structure (S, $*$); in problem (2) if * is a binary operation, examine whether it is associative or not.

8. Define identity law for an algebraic structure (S, $*$); in problem (2) if * is a binary operation, examine whether the identity element exists or not.

9. Define inverse law for an algebraic structure (S, *); in problem (2) if * is a binary operation and the identity element exists in S, examine whether inverse exists for each element of S or not.

10. What is meant by "binary operation table"? Explain how the different axioms (defined in this article) except associative axiom can be verified from the binary operation table.

11. Consider the binary operations + and . defined by the two tables:

+	*a*	*b*	*c*	*d*
a	*a*	*b*	*c*	*d*
b	*b*	*a*	*d*	*c*
c	*c*	*d*	*a*	*b*
d	*d*	*c*	*b*	*a*

.	*a*	*b*	*c*	*d*
a	*a*	*a*	*a*	*a*
b	*a*	*b*	*c*	*d*
c	*a*	*c*	*d*	*b*
d	*a*	*d*	*b*	*c*

Show that both the binary operations are commutative and associative. Write down the identities and list the inverse elements.

12. (*i*) Describe Z_4, the set of residue classes of integers modulo 4; generalise.

(*ii*) List the elements of Z_7 and $Z_4 - \{0\}$

(*iii*) Define addition modulo *n* and multiplication modulo *n* in Z_n and show that they are binary in Z_n

(*iv*) Write down the addition table of modulo 7 and multiplication table of modulo 6. Examine whether the operations are associative and commutative or not.

(*v*) Write down the tables for $(Z_5, +)$ and $(Z_5 - \{0\}, \cdot)$ where + is addition modulo 5 and . is multiplication modulo 5. Write down the identities and inverse of each element in each system.

(*vi*) Do the same for $(Z_6, +)$ and $(Z_6 - \{0\}, \cdot)$

(*vii*) In $(Z_n, +)$, find the inverse of an element $k \in Z_n$

13. (*a*) Define a permutation on a non-empty set; illustrate with an example

(*b*) List all the permutations on $\{1, 2, 3\}$ and define the usual binary operation on the set of permutations.

(*c*) Examine whether the set of permutations under this binary operation satisfics all the postulates (discussed in this article) o1 not.

14. (*i*) In $(Q, +)$, what is the inverse of $\frac{8}{3}$?

(*ii*) In $(Q, \cdot)$, what is the inverse of $\frac{8}{5}$?

(*iii*) In $(Z_4, \cdot)$, what are the inverses of 2 and 3?

(*iv*) In $(Z_4, +)$, what is the identity and find the inverse of each element.

(*v*) In $(Q, \cdot)$, what is the identity?

(*vi*) In $(Z, *)$, where $a * b = a+b+1$, $a, b \in Z$, what is the identity and write down the inverse of 5.

(*vii*) In $(Q - \{-1\}, *)$ where $a * b = a + b + ab$, $a, b \in Q - \{-1\}$ what is the identity and the inverse of 'a'.

(*viii*) In $(C, \cdot)$, prove that inverse of non-zero element exists.

(*ix*) In $(S_3, *)$ where S_3 is the set of permutations on $\{1, 2, 3\}$ and $*$ is the binary operation (composition of mapping) defined on it, find the inverses of

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Answers

2. (*i*), (*iii*), (*iv*), (*v*), (*vi*), (*ix*), (*x*), (*xii*), (*xiv*), (*xv*), (*xvii*), (*xviii*), (*xix*), (*xx*), (*xxi*), (*xxii*), (*xxiii*), (*xxiv*)

6. Commutative: (*i*); (*v*), (*vi*), (*x*), (*xii*), (*xvii*), (*xviii*), (*xix*), (*xx*), (*xxi*), (*xxiii*), (*xxiv*)

7. Associative: (*i*), (*v*), (*vi*), (*xii*), (*xiv*), (*xvii*), (*xviii*), (*xix*), (*xxi*), (*xxii*), (*xxiii*), (*xxiv*)

8. Identity: (*v*), (*vi*), (*xii*), (*xvii*), (*xviii*), (*xix*), (*xxi*), (*xxii*), (*xxiii*), (*xxiv*)

9. Inverse: (*xviii*), (*xix*), (*xxi*), (*xxiii*), (*xxiv*)

11. (*i*) identity$=a$; $a'=a$, $b'=b$, $c'=c$, $d'=d$ (*ii*) identity$=b$; a' doesn't exist, $b'=b$, $c'=d$ and $d'=c$.

12. (*i*) $Z_n=\{0, 1, 2, \ldots, (n-1)\}$ (*ii*) $\{0, 1, 2, 3, 4, 5, 6\}$; $\{1, 2, 3\}$ (*v*) identity is 0 and the inverses of 0, 1, 2, 3, 4 are respectively

0, 4, 3, 2, 1; identity is 1 and the inverses of 1, 2, 3, 4 are 1, 3, 2, 4 respectively (*vi*) multiplicative inverse doesn't exist for 2, 3, or 4 (*vii*) $n-k$, $k \neq 0$ and $0' = 0$.

13. (*c*) non-commutative

14. (*i*) $-\frac{8}{3}$] (*ii*) $\frac{5}{3}$ (*iii*) $2'$ doesn't exist; $3' = 3$ (*iv*) $0' = 0$; $1' = 3$, $2' = 2$ and $3' = 1$. (*v*) 1. (*vi*) $e = 1$; -7 (*vii*) 0; $-\frac{a}{1+a}$.

(*ix*) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$; $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$.

4.2 Groups

1. Define a group, an Abelion group and give examples.
2. Give an example of an infinite group and a finite group.
3. (*a*) Show that (Z, +), (Q, +), (R, +) and (C, +) are all abelion groups of infinite order.

   (*b*) Show that $(Z_n, +)$ is an abelion group of finite order.
4. Verify that (*i*) {0, 1, 2, 3, 4, 5} is an abelion group under addition modulo 6 (*ii*) {0, 1, 2, 3} is an abelion group under addition modulo 4
5. (*a*) Why the sets in (3) are not multiplicative groups?

   (*b*) Prove that Q – {0}, R – {0}, C – {0} are all multiplicative groups. What about Z – {0}?

   (*c*) Is $Z_n - \{0\}$ a multiplicative group? Illustrate by taking $n = 6$ and 7
6. Prove that the set {1, 2, 3, 4} is a group under multiplication modulo 5.
7. Construct the multiplication table of integers mod 8 and prove that $(Z_8, \cdot)$ is not a group.
8. (*i*) Prove that the set of odd integers is not a group under addition.

   (*ii*) Prove that the set of even integers (in general n Z) is a group under addition.

   (*iii*) Are the above two sets groups under multiplication? Why?

9. Examine whether the following sets are groups under the given operations or not.
   (*i*) {1, 3, 4, 5, 9} under multiplication mod 11
   (*ii*) {1, 3, 5, 7, 8} under multiplication mod 11
   (*iii*) {1, 2, 4, 5, 7, 8} under multiplication mod 9.

10. Verify that $(\{e\}, *)$ is an abelion group where $e * e = e$.

11. Prove the set of (*i*) the square roots of unity (*ii*) the cube roots of unity (*iii*) the fourth roots of unity form finite abelion groups under multiplication. What are their orders?

12. Prove that the set of square matrices of order $n \times n$ doesn't form a group under matrix multiplication

13. Prove that the set of all $m \times n$ matrices form an abelion additive group.

14. Prove that the set of matrices $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ where α is a real number form an abelion group under multiplication.

15. Prove that the set of matrices
$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $D = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
form a multiplicative group.

16. Prove the set of all 2×2 non-singular matrices form a multiplicative non-abelion group.

17. Prove that the set of all diagonal matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \neq 0$, is a group under matrix multiplication.

18. Prove that the set of complex numbers whose modulii are unity is a multiplicative group.

19. Prove that the set of all rational numbers of the form p/q where $p \in Z$, $q = 1$ or 2 is an additive group.

20. Prove that the set of all integers under the operation defined by $a * b = a + b + 1$ is a group.

21. Prove that the set of all integral powers of 2 form a group under multiplication.

22. If S is the set of all real numbers of the form $a + b\sqrt{2}$ where a and $b \in Q$ such that $a^2 + b^2 \neq 0$, prove that S is a multiplicative group. Is it an additive group?

23. Is the set of all rationals other than 1 a group with the operation $*$ defined by $a * b=a+b-ab$? Give reasons.

24. Is the set of rationals a group under the operation defined by $a * b=ab+1$? Give reasons.

25. Consider the mathematical systems represented by the tables:

$*$	a	b	c
a	a	a	a
b	a	b	c
c	a	c	b

$+$	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

$\cdot$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

Do they from groups? Give reasons.

26. Prove that the set of permutations on $\{1, 2, 3\}$ is a group under the usual composition on it.

27. In the group $(Q-\{0\}, *)$, $a * b=\frac{ab}{2}$. Find the group identity. Find the inverse of 4 and solve $4 * x=5$.

28. Let S be the set of all real numbers except-1. Define $*$ on S by $a * b=a+b+ab$.

    (*i*) Show that $*$ gives a binary operation on S.

    (*ii*) Show that $(S, *)$ is a group.

    (*iii*) Solve $2 * (x * 3)=7$ in S.

29. (*i*) Prove that the identity element of a group is unique.

    (*ii*) Prove that in a group the inverse of an element is unique

    (*iii*) Prove that in a group $a * b=a * c$, $a, b, c \in G$ implies $b=c$ and $a * b=c * b$, $a, b, c \in G$ implies $a=c$. (cancellation laws)

    (*iv*) Prove that in a group the inverse of the inverse of an element is itself.

    (*v*) Show that in a group $(ab)^{-1}=b^{-1}a^{-1}, \forall a, b \in G$. When does $(ab)^{-1}=a^{-1}b^{-1}$ hold good?

    (*vi*) If a and b are any two elements of a group $(G, *)$, then prove that the equations $a * x=b$ and $y * a=b$ have unique solutions in G.

    (*vii*) Prove that a group with 3 elements is abelion.

30. (*i*) If $a^2=a$ for some $a \in G$ where G is a group, then prove that $a=e$.
    (*ii*) If each element of a group G is its own inverse, then prove that G is abelion.
    (*iii*) If $(ab)^2=a^2b^2$, $\forall\ a, b \in G$, then prove that G is abelion and conversely.
    (*iv*) In a group G, for $a, b, c, d \in G$ find $(a\, b^{-1}\, c^{-1}\, d)^{-1}$ and $(a^2\, bc^{-1}\, d^{-2})^{-1}$

31. Define a sub-group of a group and give example.

32. (*i*) Is (N, +) a subgroup of (Z, +)? Why?
    (*ii*) Is (Z, +) a subgroup of (Q, +)? Why?
    (*iii*) Is (Q, +) a subgroup of (R – {0}, ·)? Why?
    (*iv*) Is $(Z_4, +)$ a subgroup of $(Z_5, +)$? Why?

33. Show that H={1, –1} is a subgroup of multiplicative group of non-zero rationals.

34. If H is a subgroup of G, then prove that the identity of H is the same as the identity of G.

35. H is a non-empty subset of a group G. Prove that H is a subgroup of G iff closure and inverse axioms hold in H.

36. Prove that a non-empty subset H of a group G is a subgroup of G iff $\forall\ a, b \in H$, $ab^{-1} \in H$.

37. Show that (*i*) the set of even integers is a subgroup of the set of integers under addition (*ii*) the set of multiples of 5 is a subgroup of the additive group of integers.

38. Let $H=\{3^n : n \in Z\}$ and $G=Q-\{0\}$. Prove that (H, ·) is a subgroup of (G, ·)

Answers

5. (*a*) $0'$ doesn't exist (*c*) when $n=6$, it is not a group because inverse axiom fails; when $n=7$ (prime), it is a group.

8. (*iii*) No; inverse axiom fails 9. (*ii*) is not a group

11. (*i*) 2 (*ii*) 3 (*iii*) 4. 22. Yes 25. I table-No; II table-Yes; III table–Yes; 27. $e=2$; $4'=1$; $x=\frac{5}{2}$. 28. (*iii*) $-\frac{1}{3}$. 30. (*iv*) $d^{-1}c\, ba^{-1}$; $d^2cb^{-1}\, a^{-2}$ 32. (*i*) No. (N, +) is not a group (*ii*) Yes (*iii*) No; Q is not a subset of R–{0} (*iv*) No; Z_4 is not a subset of Z_5.

CHAPTER 5

VECTORS

1. Define *a* 2–dimensional vector; *a* 3–dimensional vector. Give examples. Also define equal vectors.

2. Define the sum $\vec{a} + \vec{b}$ and the difference $\vec{a} - \vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$. Illustrate with examples.

3. Define $k\,\vec{a}$, the product of a number (k) and a vector $\vec{a}$ (scalar multiple of a vector). Illustrate with examples.

4. (*i*) If $\vec{a} = (-1, 0)$, $\vec{b} = (2, -1)$, $\vec{c} = (2, 1)$ and $\vec{d} = (1, 2)$, find $\vec{a} + \vec{b}$, $\vec{c} - \vec{d}$, $3\,\vec{a}$, $4\,\vec{a} + 3\,\vec{d}$, $2\,\vec{c} - \vec{b}$, $2\,\vec{a} + \vec{b} - 3\,\vec{c}$ and $\frac{1}{2}\,(\vec{a} + \vec{b} + \vec{c} + \vec{d})$

   (*ii*) If $\vec{a} = (-1, 0, 2)$, $\vec{b} = (2, -1, 3)$ and $\vec{c} = (2, 3, -5)$ find $\vec{a} + \vec{b}$, $2\,\vec{a} - 3\,\vec{c}$, $2\,(\vec{a} + \vec{b} + \vec{c})$ and $2\,\vec{c} - 3\,\vec{b} + 4\,\vec{a}$

5. (*i*) Find $\vec{a}$ if $3\,\vec{a} + (-2, 1, 5) = 2\,\vec{a} - (1, 0, 1)$

   (*ii*) Find $\vec{a}$ and $\vec{b}$ if $\vec{a} - \vec{b} = (3, 4, 6)$ and $\vec{a} + \vec{b} = (1, 2, 8)$

   (*iii*) Find $\vec{a}$ and $\vec{b}$ if $\vec{a} + \vec{b} = (2, 3, 1)$ and $2\,\vec{a} - 3\,\vec{b} = (1, 2, 3)$

   (*iv*) Solve for x and y: (*a*) $x\,(1, 2) + y\,(2, 1) = (8, 7)$
   (*b*) $2\,(x, 1) + 4\,(0, y) = (6, 10)$.

6. Define the 2–D null (zero) vector and the 3—D null vector.

7. Define negative of a vector. Illustrate with examples.

8. Prove that the set of all 3—dimensional vectors is a group under vector addition.

9. (*i*) Define the 2–dimensional unit vectors $\hat{i}$ and $\hat{j}$ and show that every vector can be written as a (linear) combination of them.

   (*ii*) Define the 3–dimensional unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ and show that every vector (a_1, a_2, a_3) can be written as $a_1\,\hat{i} + a_2\,\hat{j} + a_3\,\hat{k}$.

10. (*i*) Define a directed segment
    (*ii*) Define equality of directed segments
    (*iii*) Explain how a vector can be represented by a directed segment (geometrical interpretation of a vector)

11. Define the length of a vector and obtain expression for the same.

12. (*i*) Define a unit vector and show that $\frac{1}{|\vec{a}|}\vec{a}$ is a unit vector along (or parallel to) $\vec{a}$. Illustrate with examples
    (*ii*) Find a unit vector parallel to $2\vec{a}-3\vec{b}$ where $\vec{a}=(1, 2, 5)$ and $\vec{b}=(2, -1, 2)$
    (*iii*) Find a unit vector along $3\vec{a}+\vec{b}$ where $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$.

13. State the triangular law and parallelogram law of addition of two directed segments (vectors).

14. Describe the geometrical meaning of $k\vec{a}$.

15. Simplify: (*i*) $\vec{AB}+\vec{BA}$ (*ii*) $\vec{BC}+\vec{CA}+\vec{AB}$ (*iii*) $\vec{PQ}+\vec{RP}+\vec{QP}$

16. ABCD is a parallelogram and AC and BD are its diagonals. Express (*i*) $\vec{AC}$ and $\vec{BD}$ in terms of $\vec{AB}$ and $\vec{AD}$
    (*ii*) $\vec{AB}$ and $\vec{AD}$ in terms of $\vec{AC}$ and $\vec{BD}$
    (*iii*) $\vec{AB}$ and $\vec{AC}$ in terms of $\vec{AD}$ and $\vec{BD}$.

17. ABC is a triangle. Simplify: (*i*) $\vec{AC}-\vec{AB}$ (*ii*) $\vec{BA}-\vec{BC}$ (*iii*) $\vec{CA}-\vec{CB}$ (*iv*) $\vec{BC}-\vec{BA}$ (*v*) $\vec{CB}-\vec{CA}$.

18. ABCDEF is a regular hexagon. If $\vec{AB}=\vec{a}$ and $\vec{BC}=\vec{b}$, express $\vec{AC}$, $\vec{CD}$, $\vec{BE}$, $\vec{DE}$, $\vec{EF}$ and $\vec{FA}$ in terms of $\vec{a}$ and $\vec{b}$.

19. Define position vector of a point and prove that $\vec{AB}$=position vector of B – P.V. of A.

20. If A is (2, –1, 5), B is (2, 1, 4) and C is (1, 1, 0), find a unit vector parallel to (*i*) $\overrightarrow{AB}-2\,\overrightarrow{BC}$ (*ii*) $\overrightarrow{AC}-\overrightarrow{BC}$.

21. The position vectors of 4 points P, Q, R, S are $\hat{i}+2\hat{j}+\hat{k}$, $4\hat{i}-3\hat{j}$, $4\hat{i}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ respectively. Express the vectors $\overrightarrow{PR}$, $\overrightarrow{RS}$ and $\overrightarrow{QP}$ in terms of $\hat{i}, \hat{j}$ and $\hat{k}$ and find the length of each one of these vectors.

22. ABCD is a parallelogram. If A is (–1, 2), B is (1, 6) and D is (3, 4), find C.

23. If P is (1, 0), Q is (4, 2) and R is (5, 4) find the coordinates of S if (*i*) PQRS is a parallelogram
(*ii*) PRQS is a parallelogram.

24. Define collinear vectors and obtain a condition for two vectors to be collinear.

25 Show that the points

(*i*) with position vectors $-2\vec{a}+3\vec{b}+5\vec{c}$, $\vec{a}+2\vec{b}+3\vec{c}$ and $7\vec{a}-\vec{c}$ are collinear

(*ii*) with coordinates (1, –2, 3), (2, 3, –4) and (0, –7, 10) are collinear

(*iii*) with position vectors $2\hat{i}+3\hat{j}+4\hat{k}$, $5\hat{i}+6\hat{j}+8\hat{k}$ and $8\hat{i}+9\hat{j}+12\hat{k}$ are collinear

26 (*i*) Define scalar product of two vectors and prove that $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$

(*ii*) Explain the method of finding the angle between two non-zero vectors $\vec{a}$ and $\vec{b}$

(*iii*) Prove that if $\vec{a}$ is orthogonal to $\vec{b}$, then $\vec{a}\cdot\vec{b}=0$ and conversely

(*iv*) Find $\hat{i}\cdot\hat{i}$, $\hat{i}\cdot\hat{j}$, $\hat{i}\cdot\hat{k}$ etc.

(*v*) Give the geometrical meaning of $\vec{a}\cdot\vec{b}$

(*vi*) Prove that $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(*vii*) Find $\vec{a}\cdot\vec{b}$ when $a=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$.

27. (*i*) Find the dot product of the vectors (*a*) $2\hat{i}-4\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$ (*b*) (2, 1, 4) and (–1, 2, 1)

(*ii*) If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}$ find $\vec{a}\cdot\vec{b}$, $\vec{a}\cdot\vec{c}$, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$, $(2\vec{a}+3\vec{c})\cdot(\vec{a}-\vec{b})$ and verify that $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(*iii*) Let $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}-\hat{k}$. Find the dot product of $2\vec{a}+3\vec{b}-2\vec{c}$ and $(\vec{a}\cdot\vec{b})\,\vec{c}$

(*iv*) If $\vec{a}=(1, 1, 1)$ and $\vec{b}=(-1, 1, -1)$, find $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$.

28. Find the angle between the vectors

(*i*) $2\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+2\hat{k}$ (*ii*) $4\hat{i}-3\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$

(*iii*) $2\hat{i}+3\hat{j}+4\hat{k}$ and $4\hat{i}-\hat{j}-\hat{k}$ (*iv*) (3, 2, 0) and (1, 1, 6)

(*v*) $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ if $|\vec{a}|=|\vec{b}|$ (*vi*) $\overrightarrow{AB}$ and $\overrightarrow{AC}$ where A, B, C are (1, 2, –1), (2, 0, 3) and (3, –1, 2) respectively.

29. Prove that the vectors (*i*) $\hat{i}+4\hat{j}+3\hat{k}$ and $4\hat{i}+2\hat{j}-4\hat{k}$ (*ii*) $2\hat{i}+3\hat{j}-4\hat{k}$ and $3\hat{i}+6\hat{j}+6\hat{k}$ are ⊥.

30. Prove that the vectors $2\hat{i}+3\hat{j}+6\hat{k}$, $3\hat{i}-6\hat{j}+2\hat{k}$ and $6\hat{i}+2\hat{j}-3\hat{k}$ are mutually perpendicular.

31. If the vectors (*i*) $-\hat{i}+2\hat{j}+3\hat{k}$ and $m^2\hat{i}+m\hat{j}+\hat{k}$ (*ii*) (2, 6, 5) and (1, m, 2) are perpendicular, find m.

32. Prove that $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2(|\vec{a}|^2+|\vec{b}|^2)$

33. (*i*) Define vector product of two vectors $\vec{a}$ and $\vec{b}$ and prove that $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

(*ii*) Explain the method of finding a unit vector ⊥ both $\vec{a}$ and $\vec{b}$; also sine of the angle between them

(*iii*) Prove that $\vec{a}\times\vec{a}=\vec{0}$

(*iv*) Find $\hat{i}\times\hat{i}$, $\hat{i}\times\hat{j}$, $\hat{i}\times\hat{k}$ etc.

(*v*) Give the geometrical meaning of $\vec{a} \times \vec{b}$

(*vi*) Prove that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

(*vii*) Find $\vec{a} \times \vec{b}$ if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

34. (*i*) Find the cross product of the vectors (*a*) $\hat{j} - 3\hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ (*b*) (2, –1, 1) and (3, 4, –1) (*c*) $2\vec{a} - \vec{b}$ and $2\vec{a} + \vec{b}$ where $\vec{a} = (3, -1, -2)$ and $\vec{b} = (1, 2, -1)$

(*ii*) Let $\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. Find $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$, $|\vec{a} \times \vec{b}|$, $\vec{a} \times (\vec{b} \times \vec{c})$, $(\vec{a} \times \vec{b}) \times \vec{c}$ and verify that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

(*iii*) If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - \hat{k}$, find $|2\vec{a} + 3\vec{b}|$; $\vec{a} \times \vec{b}$; the sine of the angle between $\vec{b}$ and $\vec{c}$; a unit vector ⊥ both $\vec{a}$ and $\vec{c}$; verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

35. Find a unit vector ⊥ each of the vectors given below and also find sine of the angle between them.

(*i*) $2\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - \hat{k}$ (*ii*) $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$

(*iii*) $\hat{i} - 4\hat{j} + 8\hat{k}$ and $2\hat{i} + 2\hat{j} + \hat{k}$ (*iv*) $3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\hat{i} - 2\hat{j} - 2\hat{k}$

36. Prove that (*i*) $\Sigma\, \vec{a} \times (\vec{b} + \vec{c}) = \vec{0}$

(*ii*) $|\vec{a} \times \vec{b}|^2 + |(\vec{a} \cdot \vec{b})|^2 = |\vec{a}|^2\, |\vec{b}|^2$.

37. (*i*) Find the area of △ OAB where A is (0, 2, 4) and B is (3, 0, 4)

(*ii*) Find the area of the parallelogram whose two adjacent sides are represented by $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$

(*iii*) Find the area of the triangle, two of whose sides are represented by the vectors (3, 4, 0) and (5, 7, 1)

(*iv*) Find the area of the triangle whose vertices are (1, 2, 3), (2, –1, 1) and (1, 2, –4).

38. (*i*) Define scalar triple product of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

(*ii*) Give the geometrical meaning of $\vec{a}\cdot(\vec{b}\times\vec{c})$

(*iii*) Show that in the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$, the 'dot' and the 'cross' are interchangable

(*iv*) If $\vec{a}=(a_1, a_2, a_3)$, $\vec{b}=(b_1, b_2, b_3)$ and $\vec{c}=(c_1, c_2, c_3)$

prove that $[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

39. Find the scalar triple product of the following vectors:

(*i*) $\hat{i}+2\hat{j}+3\hat{k}$, $3\hat{j}-\hat{k}$ and $2\hat{i}+4\hat{j}+\hat{k}$

(*ii*) $(3, 0, -1)$, $(-4, 6, 7)$ and $(2, -1, -4)$

(*iii*) $3\hat{i}+8\hat{j}+6\hat{k}$, $9\hat{i}+8\hat{j}+2\hat{k}$ and $6\hat{i}+12\hat{j}+2\hat{k}$

(*iv*) $\hat{i}-\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}-\hat{k}$

(*v*) $\hat{i}+\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+2\hat{k}$

40. Find the volume of the parallelopiped whose co-terminal edges are represented by the vectors

(*i*) $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+2\hat{k}$ and $\hat{i}-3\hat{j}-3\hat{k}$

(*ii*) $7\hat{i}+6\hat{j}-5\hat{k}$, $3\hat{i}-4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-3\hat{k}$

41. Find the volume of the tetrahedron OABC with vertices at origin O and

(*i*) A (2, 3, 1), B (2, 15, 1) and C (10, 3, 1)

(*ii*) A (1, 2, 2), B (3, 1, 1) and C (1, 5, 1)

42. Define coplanar vectors and obtain a condition for three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ to be coplanar.

43. Show that the following vectors are coplanar:

(*i*) $5\hat{i}+6\hat{j}+7\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$ and $7\hat{i}-8\hat{j}+9\hat{k}$

(*ii*) $4\vec{a}-2\vec{b}-2\vec{c}$, $-2\vec{a}+4\vec{b}-2\vec{c}$ and $-2\vec{a}-2\vec{b}+4\vec{c}$

(*iii*) (1, 1, 1), (3, 4, 2) and (3, 1, 5)

44. Show that the following points

(*i*) with position vectors $-6\vec{a}+3\vec{b}+2\vec{c}$, $3\vec{a}-2\vec{b}+4\vec{c}$, $5\vec{a}+7\vec{b}+3\vec{c}$ and $-13\vec{a}+17\vec{b}-\vec{c}$ are coplanar

(*ii*) with coordinates (4, 5, 1), (0, –1, –1), (3, 9, 4) and (–4, 4, 4) are coplanar.

(*iii*) with position vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$, $3\hat{i}+\hat{j}+2\hat{k}$ and $-\hat{i}+\hat{j}$ are coplanar.

45. If the following vectors are coplanar, find m

(*i*) (1, 2, 3), (2, 1, 3) and (4, 7, m)

(*ii*) $\hat{i}+2\hat{j}+5\hat{k}$, $2\hat{i}+m\hat{j}+10\hat{k}$ and $3\hat{i}+9\hat{j}-2\hat{k}$

(*iii*) $m\hat{i}+2\hat{j}+2\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+2\hat{k}$

46. Prove that (*i*) $\vec{a}\cdot(\vec{a}\times\vec{b})=0$ (*ii*) $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$

(*iii*) If $\vec{a}, \vec{b}, \vec{c}$ are orthogonal vectors, $[\vec{a}\ \vec{b}\ \vec{c}]^2=|\vec{a}|^2\,|\vec{b}|^2\,|\vec{c}|^2$

Answers

4. (*i*) (1, –1), (1, –1), (–3, 0), (–1, 6), (2, 3), (–6, –4), (2, 1) (*ii*) (1, –1, 5), (–8, –9, 19), (6, 4, 0), (–6, 9, –11)
5. (*i*) (1, –1, –6) (*ii*) (2, 3, 7), (–1, –1, 1) (*iii*) $(\frac{7}{5}, \frac{11}{5}, \frac{6}{5})$, $(\frac{8}{5}, \frac{4}{5}, -\frac{1}{5})$ (*iv*) (*a*) $x=2,\ y=3$ (*b*) $x=3,\ y=2$

12. (*ii*) $\frac{1}{9}$ (–4, 7, 4) (*iii*) $\frac{1}{\sqrt{51}}(7\hat{i}+\hat{j}-\hat{k})$ 15. (*i*) $\vec{0}$ (*ii*) $\vec{0}$ (*iii*) $\overrightarrow{RP}$ 16. (*i*) $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{AD}$; $\overrightarrow{BD}=\overrightarrow{AD}-\overrightarrow{AB}$ (*ii*) $\overrightarrow{AB}=\frac{1}{2}\overrightarrow{AC}-\frac{1}{2}\overrightarrow{BD}$; $\overrightarrow{AD}=\frac{1}{2}\overrightarrow{AC}+\frac{1}{2}\overrightarrow{BD}$ (*iii*) $\overrightarrow{AB}=\overrightarrow{AD}-\overrightarrow{BD}$; $\overrightarrow{AC}=2\overrightarrow{AD}-\overrightarrow{BD}$ 17. (*i*) $\overrightarrow{BC}$ (*ii*) $\overrightarrow{CA}$ (*iii*) $\overrightarrow{BA}$ (*iv*) $\overrightarrow{AC}$ (*v*) $\overrightarrow{AB}$

18. $\overrightarrow{AC}=\vec{a}+\vec{b}$; $\overrightarrow{CD}=\vec{b}-\vec{a}$; $\overrightarrow{BE}=2(\vec{b}-\vec{a})$; $\overrightarrow{EF}=-\vec{b}$; $\overrightarrow{FA}=\vec{a}-\vec{b}$

20. (*i*) $\frac{2\hat{i}+2\hat{j}+7\hat{k}}{\sqrt{57}}$ (*ii*) $\frac{2\hat{j}-\hat{k}}{\sqrt{5}}$. 21. $3\hat{i}-2\hat{j}$; $-3\hat{i}-\hat{j}$; $-3\hat{i}+5\hat{j}+\hat{k}$; $\sqrt{13}$, $\sqrt{10}$, $\sqrt{35}$. 22. (5, 8) 23. (*i*) (2, 2) (*ii*) (0, –2)

27. (*i*) (*a*) –9 (*b*) 4 (*ii*) 4; 0; 3; 7 (*iii*) 90 (*iv*) 0

28. (*i*) $\cos^{-1}\dfrac{2}{3\sqrt{6}}$ (*ii*) $\cos^{-1}\dfrac{2}{\sqrt{39}}$ (*iii*) $\cos^{-1}\dfrac{1}{\sqrt{29}\ \sqrt{18}}$
(*iv*) $\cos^{-1}\dfrac{5}{\sqrt{13}\ \sqrt{38}}$ (*v*) 90° (*vi*) $\cos^{-1}\dfrac{20}{\sqrt{21}\ \sqrt{22}}$

31. (*i*) 3, –1 (*ii*) –2. 34. (*i*) (*a*) $-\hat{i}-3\hat{j}-\hat{k}$ (*b*) $-3\hat{i}+5\hat{j}+11\hat{k}$
(*c*) $20\hat{i}+4\hat{j}+28\hat{k}$ (*ii*) $-\hat{i}+8\hat{j}+6\hat{k}$; $\hat{j}+\hat{k}$; $\hat{i}-5\hat{j}-4\hat{k}$;
$\sqrt{101}$, $-5\hat{i}-2\hat{j}+2\hat{k}$; $-14\hat{i}+5\hat{j}-9\hat{k}$. (*iii*) $\sqrt{162}$; $3\hat{i}-3\hat{k}$;
$\dfrac{\sqrt{35}}{6}$; $\dfrac{-\hat{i}+6\hat{j}+4\hat{k}}{\sqrt{53}}$. 35. (*i*) $\dfrac{\hat{i}+6\hat{j}+10\hat{k}}{\sqrt{137}}$; $\dfrac{\sqrt{137}}{9\sqrt{2}}$ (*ii*) $\dfrac{-\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$;
$\dfrac{5}{2\sqrt{7}}$ (*iii*) $\dfrac{-4\hat{i}+3\hat{j}+2\hat{k}}{\sqrt{29}}$; $\dfrac{5\sqrt{29}}{27}$ (*iv*) $\dfrac{2\hat{i}+\hat{j}}{\sqrt{5}}$; $\dfrac{8\sqrt{5}}{21}$. 37. in
square units (*i*) $\sqrt{61}$ (*ii*) $2\sqrt{2}$ (*iii*) $\frac{1}{2}\sqrt{26}$ (*iv*) $\frac{7}{2}\sqrt{10}$

39. (*i*) –15 (*ii*) –43 (*iii*) 288 (*iv*) 0 (*v*) 9. 40. in cubic
units (*i*) 42 (*ii*) 80. 41. in cubic units (*i*) 16 (*ii*) $\frac{10}{3}$.
45 (*i*) 11 (*ii*) 4 (*iii*) $\frac{4}{3}$.

SECTION II: TRIGONOMETRY

CHAPTER 1

PROPERTIES OF TRIANGLES

In any $\triangle ABC$, prove the following:—

1. (*i*) *Sine Rule:* $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ where R is the circum-radius

(*ii*) *Projection Rule:* $a=b\cos C+c\cos B$, etc

(*iii*) *Cosine Rule:* $a^2=b^2+c^2-2bc\cos A$, etc

(*iv*) *Napier formula:* $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\operatorname{Cot}\frac{A}{2}$, etc

(*v*) *Half angles formulae:* $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$;

$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$; $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$; etc.

2. (*i*) $\frac{a^2-b^2}{c^2-a^2}=\frac{c}{b}\cdot\frac{\sin(A-B)}{\sin(C-A)}$ (*ii*) $\frac{\sin(B-C)}{\sin(B+C)}=\frac{b^2-c^2}{a^2}$

(*iii*) $\frac{a^2-b^2}{2}\;\frac{\sin A\sin B}{\sin(A-B)}=\frac{1}{2}ab\sin C$ (*iv*) $\frac{a}{b+c}=\frac{\sin\frac{A}{2}}{\cos\frac{B-C}{2}}$

(*v*) $a\sin\left(\frac{A}{2}+C\right)=(b+c)\sin\frac{A}{2}$

(*vi*) $\frac{a+b}{c}=\frac{1+\tan\frac{A}{2}\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$ (*vii*) $\frac{\cos 2A}{a^2}-\frac{\cos 2B}{b^2}=\frac{1}{a^2}-\frac{1}{b^2}$

(*viii*) $\frac{a+b}{a-b}=\tan\frac{A+B}{2}\cot\frac{A-B}{2}$

(*ix*) $a(\cos B+\cos C)=2(b+c)\sin^2\frac{A}{2}$

3. (*i*) $\dfrac{\sin(B-C)}{\sin A}=\dfrac{b\cos C-c\cos B}{b\cos C+c\cos B}$

(*ii*) $a(b\cos C-c\cos B)=b^2-c^2$

(*iii*) $b^2\cos^2 C-c^2\cos^2 B=b^2-c^2$

(*iv*) $b^2\sin 2C+c^2\sin 2B=2bc\sin A$.

4. (*i*) $c^2=(a-b)^2\cos^2\dfrac{C}{2}+(a+b)^2\sin^2\dfrac{C}{2}$

(*ii*) If a, b, c are in H.P., $\sin^2\dfrac{A}{2}, \sin^2\dfrac{B}{2}, \sin^2\dfrac{C}{2}$ are in H.P.

(*iii*) $a^2+b^2+c^2=2(ab\cos C+bc\cos A+ca\cos B)$

(*iv*) If $\cot\dfrac{A}{2}+\cot\dfrac{C}{2}=2\cot\dfrac{B}{2}$, then a, b, c are in A.P.

(*v*) $(a+b+c)\sin\dfrac{A}{2}=2a\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

(*vi*) $b\cos^2\dfrac{C}{2}+c\cos^2\dfrac{B}{2}=s/2$

(*vii*) $a\cos^2\dfrac{C}{2}+c\cos^2\dfrac{A}{2}=\dfrac{3b}{2}$, then a, b, c are in A.P.

(*viii*) If a, b, c are in A.P., then $\tan\dfrac{A}{2}\cdot\tan\dfrac{C}{2}=\frac{1}{3}$.

5. (*i*) If $2\sin^2 B+2\sin^2 C+\cos 2A=1$, then $A=90°$

(*ii*) If $\sin 2B+\sin 2C=\sin 2A$, then the triangle is right angled

(*iii*) If $\cos A=\sin B-\cos C$, then one of the angles is right angled

(*iv*) If $a\cos A=b\cos B$, then the triangle is either isosceles or right angled.

6. (*i*) If $A=60°$, then $(a+b+c)(b+c-a)=3bc$

(*ii*) If $C=60°$, then $\dfrac{1}{a+c}+\dfrac{1}{b+c}=\dfrac{3}{a+b+c}$

(*iii*) If $a=2b$ and $A=3B$, then $C=60°$

(*iv*) If $A=2B$, then $a^2=b(b+c)$

(*v*) If $3a=b+c$, then $\sin\dfrac{A}{2}=\sin\dfrac{B}{2}\sin\dfrac{C}{2}$.

7. (*i*) $\sum(b+c)\cos A=\sum a$ (*ii*) $\sum a(b^2+c^2)\cos A=3\,abc$

(*iii*) $\sum \dfrac{\cos A}{c\cos B+b\cos C}=\dfrac{a^2+b^2+c^2}{2\,abc}$ (*iv*) $\sum \dfrac{\cos^2\frac{A}{2}}{a}=\dfrac{s^2}{abc}$

8. (*i*) $\sum a\sin(B-C)=0$ (*ii*) $\sum a(\sin B-\sin C)=0$

(*iii*) $\sum(b^2-c^2)\sin^2 A=0$ (*iv*) $\sum a^2(\cos^2 B-\cos^2 C)=0$

(*v*) $\sum a\sin\dfrac{A}{2}\sin\dfrac{B-C}{2}=0$ (*vi*) $\sum a^3\sin(B-C)=0$

(*vii*) $\sum \dfrac{b^2-c^2}{a^2}\sin 2A=0$ (*viii*) $\sum \dfrac{b^2-c^2}{a}\cos A=0$

(*ix*) $\sum(b-c)\cot\dfrac{A}{2}=0$ (*x*) $\sum a^2(\cot B-\cot C)=0$

(*xi*) $\sum(b^2-c^2)\cot A=0$ (*xii*) $\sum(b\cos B-c\cos C)\tan A=0$

(*xiii*) $\sum \dfrac{a^2\sin(B-C)}{\sin A}=0$ (*xiv*) $\sum \dfrac{a^2\sin(B-C)}{\sin B+\sin C}=0$

(*xv*) $\sum \dfrac{b^2-c^2}{\cos B+\cos C}=0.$

9. (*i*) $\dfrac{a\sin(B-C)}{b^2-c^2}=\dfrac{b\sin(C-A)}{c^2-a^2}=\dfrac{c\sin(A-B)}{a^2-b^2}$

(*ii*) $\dfrac{a}{bc}+\dfrac{\cos A}{a}=\dfrac{b}{ca}+\dfrac{\cos B}{b}=\dfrac{c}{ab}+\dfrac{\cos C}{c}$

(*iii*) $\dfrac{\cot B-\cot C}{b^2-c^2}=\dfrac{\cot C-\cot A}{c^2-a^2}=\dfrac{\cot A-\cot B}{a^2-b^2}$

(*iv*) $b^2+c^2-2bc\cos(60°+A)=c^2+a^2-2ca\cos(60°+B)$
$=a^2+b^2-2ab\cos(60°+C)$

(*v*) $(s-a)\tan\dfrac{A}{2}=(s-b)\tan\dfrac{B}{2}=(s-c)\tan\dfrac{C}{2}.$

10. In any $\triangle$ ABC, if

(*i*) $a=\sqrt{2}$, $b=2$, $c=\sqrt{3}-1$, find A
(*ii*) $a=1$, $b=\sqrt{6}+\sqrt{2}$, $c=2+\sqrt{3}$, find C
(*iii*) $a=1$, $b=\sqrt{3}$, $c=1$, find B
(*iv*) $b=\sqrt{3}$, $c=1$, $A=30°$, find a
(*v*) $a=75\sqrt{3}$, $c=75$, $B=30°$, find b
(*vi*) $a=2$, $b=\sqrt{3}-1$, $C=60°$, find c

(*vii*) $A=45°$, $a=2$, $c=\sqrt{3}+1$, find C
(*viii*) $B=30°$, $a=2$, $b=\sqrt{2}$, find A
(*ix*) $c=2$, $b=\sqrt{6}$, $C=45°$, find B
(*x*) $B=30°$, $b=2\sqrt{3}$, $A=60°$, find a
(*xi*) $B=30°$, $C=75°$, $c=\sqrt{8}$, find b
(*xii*) $A=45°$, $C=15°$, $b=\sqrt{6}$, find a
(*xiii*) $A=60°$, $B=45°$, $b=5$, find a
(*xiv*) $a=\sqrt{2}$, $c=\sqrt{3}-1$ and $B=135°$, find b.

11. (*i*) Find the greatest angle of the triangle whose sides are 7, 5 and 3
(*ii*) Find the smallest angle of the triangle whose sides are $6+2\sqrt{3}$, $4\sqrt{3}$ and $2\sqrt{6}$
(*iii*) Find the greatest side of the triangle, given that $B=45°$, $C=75°$ and $a=\sqrt{6}$
(*iv*) If $a/b=2+\sqrt{3}$ and $C=60^c$, find A–B.

12. In any $\triangle ABC$, if
(*i*) $a=3$, $b=4$, $c=5$, find $\sin\dfrac{C}{2}$, $\cos\dfrac{B}{2}$ and $\tan\dfrac{A}{2}$
(*ii*) $a=6$, $b=8$, $c=10$, find $\sin\dfrac{A}{2}$, $\cos\dfrac{A}{2}$ and $\tan\dfrac{C}{2}$
(*iii*) $a=5$, $b=13$, $c=12$, find $\sin\dfrac{B}{2}$, $\cos\dfrac{B}{2}$ and $\tan\dfrac{B}{2}$.

Answers

10. (*i*) 30° (*ii*) 75° (*iii*) 120° (*iv*) 1 (*v*) 75 (*vi*) $\sqrt{10-4\sqrt{3}}$
(*vii*) 75° or 105° (*viii*) 45° or 135° (*ix*) 60° or 120° (*x*)
(*xi*) $2(\sqrt{3}-1)$ (*xii*) 2 (*xiii*) $\dfrac{5\sqrt{6}}{2}$ (*xiv*) 2.

11. (*i*) 120° (*ii*) 30° (*iii*) $\sqrt{3}+1$ (*iv*) 90°

12. (*i*) $\dfrac{1}{\sqrt{2}}$, $\dfrac{2}{\sqrt{5}}$, $\dfrac{1}{3}$ (*ii*) $\dfrac{1}{\sqrt{10}}$, $\dfrac{3}{\sqrt{10}}$, 1 (*iii*) $\dfrac{1}{\sqrt{2}}$, $\dfrac{1}{\sqrt{2}}$,

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CHAPTER 2

SOLUTION OF TRIANGLES

1. (*a*) Explain the method of solving a triangle given all the three sides.

(*b*) Solve $\triangle ABC$, given that

(*i*) $a=\sqrt{6}+\sqrt{2}$, $b=2\sqrt{3}$, $c=2\sqrt{2}$
(*ii*) $a=2$, $b=\sqrt{6}$, $c=\sqrt{3}+1$ (*iii*) $a=2\sqrt{3}$, $b=4$, $c=2$
(*iv*) $a=3-\sqrt{3}$, $b=3+\sqrt{3}$, $c=3\sqrt{2}$ (*v*) $a=2$, $b=\sqrt{6}$, $c=\sqrt{3}-1$

2. (*a*) Explain the method of solving a triangle, given two sides and the included angle.

(*b*) Solve $\triangle ABC$, given that

(*i*) $b=\sqrt{3}$, $c=1$, $A=30°$ (*ii*) $a=4$, $b=2$, $C=60°$
(*iii*) $a=2$, $c=\sqrt{6}+\sqrt{2}$, $B=75°$ (*iv*) $a=2$, $c=\sqrt{3}+1$, $B=60°$
(*v*) $a=2$, $c=\sqrt{3}-1$, $B=120°$.

3. (*a*) Explain the method of solving a triangle, given two sides and an angle opposite to one them (ambiguous case)

(*b*) Solve $\triangle ABC$, given that

(*i*) $a=54$, $b=2\sqrt{243}$, $B=30°$ (*ii*) $b=2\sqrt{3}$, $c=3\sqrt{2}$, $C=60°$
(*iii*) $B=30°$, $c=150$, $b=75$ (*iv*) $b=2$, $B=30°$, $c=2\sqrt{3}$
(*v*) $A=120°$, $a=\sqrt{6}$, $b=2$ (*vi*) $A=45°$, $a=2$, $b=\sqrt{6}$.

4. (*a*) Explain the method of solving a triangle, given one side and two angles.

(*b*) Solve $\triangle ABC$, given that

(*i*) $b=4$, $B=15°$, $C=60°$ (*ii*) $A=135°$, $B=30°$, $c=\sqrt{3}-1$
(*iii*) $B=45°$, $C=105°$, $c=\sqrt{2}$ (*iv*) $a=6$, $A=45°$, $B=60°$
(*v*) $A=45°$, $B=60°$, $b=\sqrt{6}$

Answers

1 (*b*). (*i*) $A=75°$. $B=60°$, $C=45°$ (*ii*) $A=45°$, $B=60°$, $C=75°$
(*iii*) $A=60°$, $B=90°$, $C=30°$ (*iv*) $A=15°$, $B=105°$, $C=60°$
(*v*) $A=45°$, $B=120°$, $C=15°$.

2 (*b*). (*i*) $a=1$, $B=120°$, $C=30°$ (*ii*) $c=2\sqrt{3}$, $A=90°$, $B=30°$
(*iii*) $b=\sqrt{6}+\sqrt{2}$, $C=75°$, $A=30°$ (*iv*) $b=\sqrt{6}$, $A=45°$, $C=75°$
(*v*) $b=\sqrt{6}$, $A=45°$, $C=15°$.

3(*b*). (*i*) $A=60°, C=90°, c=36\sqrt{3}$; $A=120°, C=30°, c=18\sqrt{3}$
(*ii*) $B=45°, A=75°, a=3+\sqrt{3}$ (*iii*) $A=60°, C=90°, a=75\sqrt{3}$
(*iv*) $C=60°, A=90°, a=4$; $C=120°, A=30°, a=2$ (*v*) $B=45°$
$C=15°, c=\sqrt{3}-1$ (*vi*) $B=60°, C=75°, c=\sqrt{3}+1$;
$B=120°, C=15°, c=\sqrt{3}-1$.

4(*b*). (*i*) $A=105°, a=4(2+\sqrt{3}), c=2\sqrt{6}(\sqrt{3}+1)$ (*ii*) $C=15°$
$a=2, b=\sqrt{2}$ (*iii*) $A=30°, a=\sqrt{3}-1, b=\sqrt{2}(\sqrt{3}-1$
(*iv*) $C=75°, b=3\sqrt{6}, c=3(\sqrt{3}+1)$ (*v*) $C=75°, a=2$
$c=\sqrt{3}+1$.

CHAPTER 3

INVERSE TRIGONOMETRIC FUNCTIONS

1. Define the six inverse trigonometric functions and stat which of the following are meaningless:—

(*i*) $\sin^{-1}\left(\frac{4}{\sqrt{3}}\right)$ (*ii*) $\cos^{-1}\left(\frac{-5}{\sqrt{11}}\right)$ (*iii*) $\tan^{-1}(100)$ (*iv*) $\sec^{-1}($

2. Find the value of (*i*) $\sin^{-1}(\frac{1}{2})$ (*ii*) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(*iii*) $\tan^{-1}(\sqrt{3})$ (*iv*) $\operatorname{cosec}^{-1}(1)$ (*v*) $\sec^{-1}(2)$ (*vi*) $\cot^{-1}(1$

(*vii*) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (*viii*) $\cos^{-1}(-1)$ (*ix*) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(*x*) $\operatorname{cosec}^{-1}(-2)$ (*xi*) $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ (*xii*) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

3. Find the value of (*i*) $\tan^{-1}(\sin 270°)$ (*ii*) $\sin^{-1}(\tan \pi$

(*iii*) $\sec^{-1}(\sec 20°)$ (*iv*) $\cos^{-1}(\cos 350°)$ (*v*) $\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

(*vi*) $\sin^{-1}(\sin 135°)$ (*vii*) $\cot^{-1}(\cot 790°)$ (*viii*) $\cot^{-1}\left(\tan\frac{3\pi}{4}\right)$

(*ix*) $\tan^{-1}(\sin 180°)$ (*x*) $\cos^{-1}\left(-\frac{1}{2}\tan\frac{\pi}{4}\right)$ (*xi*) $\sin^{-1}(\cos \pi$

(*xii*) $\sin^{-1}[\sin(-600°)]+\cot^{-1}(-\sqrt{3})$ (*xiii*) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$
$-\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ (*xiv*) $\cos^{-1}(-1)-\sin^{-1}(1)$

(*xv*) $\sin^{-1}\left(\cos\dfrac{\pi}{9}\right)-\operatorname{cosec}^{-1}\left(\sec\dfrac{\pi}{9}\right)$.

4. (*a*) Given one inverse trigonometric function, explain how to express other inverse trigonometric functions in terms of it.

(*b*) (*i*) If $y=\sin^{-1}\frac{1}{3}$, find the values of $\cos y$ and $\tan y$

(*ii*) If $\cos^{-1}(-\frac{3}{5})=\sin^{-1}a=\tan^{-1}b$, find a and b, if exist

(*iii*) If $\tan^{-1}(-2)=\sin^{-1}a=\cos^{-1}b$, find a and b, if exist

5. Prove that (*i*) $\sin^{-1}\frac{5}{13}=\tan^{-1}\frac{5}{12}$ (*ii*) $\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{15}{17}$

(*iii*) $\tan^{-1}\frac{3}{4}=\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{4}{5}$ (*iv*) $\cot^{-1}(-\frac{11}{2})=\sin^{-1}\left(\dfrac{-2}{5\sqrt{5}}\right)$

(*v*) $\tan^{-1}x=\cot^{-1}\dfrac{1}{x}$ (*vi*) $\sec^{-1}x=\cos^{-1}\left(\dfrac{1}{x}\right)$

(*vii*) $\operatorname{cosec}^{-1}x=\sin^{-1}\dfrac{1}{x}$ (*viii*) $\tan^{-1}\frac{3}{4}=\cot^{-1}\frac{4}{3}$

(*ix*) $2\tan^{-1}x=\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)=\sin^{-1}\dfrac{2x}{1+x^2}$, $x>0$

(*x*) $2\cos^{-1}x=\cos^{-1}(2x^2-1)$, $x\geqslant 0$

6. Find the value of (*i*) $\sec(\sin^{-1}\frac{3}{5})$ (*ii*) $\sin[\cos^{-1}(-\frac{24}{25})]$

(*iii*) $\cos(\sin^{-1}\frac{4}{5})$ (*iv*) $\tan[\cos^{-1}(-1)]$ (*v*) $\sin[\tan^{-1}(-\sqrt{3})]$

(*vi*) $\sin[2\cos^{-1}(-\frac{3}{5})]$ (*vii*) $\cos[2\sin^{-1}(-\frac{5}{13})]$

(*viii*) $\tan\left[\sin^{-1}(-\frac{1}{2})-\cos^{-1}\dfrac{\sqrt{3}}{2}\right]$

(*ix*) $\sin[\sin^{-1}(\frac{3}{5})+\cos^{-1}(\frac{7}{25})]$ (*x*) $\cos(\tan^{-1}(\frac{8}{15})+\cos^{-1}(\frac{12}{13})]$

(*xi*) $\sin\left(2\cos^{-1}\dfrac{\sqrt{5}}{3}\right)$ (*xii*) $\cos(\tan^{-1}\frac{3}{4})+\tan(\sin^{-1}-\frac{3}{5})$.

7. Prove that

(*i*) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$, $x, y\geqslant 0$, $xy\leqslant 1$

(*ii*) $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$, $x, y\geqslant 0$.

(*iii*) $2\tan^{-1}x=\tan^{-1}\dfrac{2x}{1-x^2}$, $0\leqslant x\leqslant 1$.

8. Prove that (*i*) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\dfrac{\pi}{4}$

(*ii*) $2\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{3}{4}$ (*iii*) $\tan^{-1}\frac{1}{3}-\tan^{-1}\frac{1}{4}=\tan^{-1}\frac{1}{13}$

(*iv*) $\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{2}=\frac{\pi}{4}$

(*v*) $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{4}{7}+\tan^{-1}\frac{9}{7}=\frac{\pi}{2}$

(*vi*) $2\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{5}=\frac{\pi}{4}$

(*vii*) $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}$.

9. Prove that (*i*) $\tan^{-1}\frac{3}{5}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$

(*ii*) $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$ (*iii*) $2\tan^{-1}\frac{1}{2}+\cos^{-1}\frac{4}{5}=\frac{\pi}{2}$

(*iv*) $2\tan^{-1}\frac{1}{5}+\cos^{-1}\frac{63}{65}=\sin^{-1}\frac{3}{5}$

(*v*) $\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{29}$.

10. Prove that (*i*) $\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{56}{65}$

(*ii*) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$

(*iii*) $\sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}=\frac{\pi}{2}$

(*iv*) $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

(*v*) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$

(*vi*) $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$

(*vii*) $\sin^{-1}\frac{3}{5}+\cot^{-1}7=\frac{\pi}{4}$

(*viii*) $\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{2}{\sqrt{5}}=\cot^{-1}\frac{2}{11}$

(*ix*) $\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$

(*x*) $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$.

11. Prove that (*i*) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

(*ii*) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $x>0$

(*iii*) $\sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$.

12. (*i*) If $\tan^{-1} x+\tan^{-1} y=\frac{\pi}{4}$, prove that $x+y+xy=1$

(*ii*) If $\tan^{-1} x+\tan^{-1} y+\tan^{-1} z=\pi$, prove that $x+y+z=xyz$

(*iii*) If $\tan^{-1} x+\tan^{-1} y+\tan^{-1} z=\frac{\pi}{2}$,

prove that $xy+yz+zx=1$

(*iv*) Prove that

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right)+\tan^{-1}\left(\frac{y-z}{1+zy}\right)+\tan^{-1}\left(\frac{z-x}{1+zx}\right)=0,$$

$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$

(*v*) If $\cos^{-1} x+\cos^{-1} y+\cos^{-1} z=\pi$,

prove that $x^2+y^2+z^2+2xyz=1$.

13. Solve for x:—(*i*) $\tan^{-1} x+\tan^{-1} (1-x)=\tan^{-1} \frac{8}{7}$

(*ii*) $\tan^{-1} (1+x)+\tan^{-1} (1-x)=\frac{\pi}{2}$

(*iii*) $\tan^{-1} 3x+\tan^{-1} 2x=\frac{\pi}{4}$

(*iv*) $2\tan^{-1} x=\tan^{-1} 2x$ (*v*) $2\tan^{-1} x+\tan^{-1} 2x=\frac{\pi}{2}$

(*vi*) $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

(*vii*) $\tan^{-1}\left(\frac{x}{x+1}\right)+\cot^{-1}\left(\frac{1-x}{x}\right)=\tan^{-1} 2$

14. Solve: (*i*) $\sin^{-1}x+\sin^{-1} 2x=\frac{\pi}{2}$ (*ii*) $\sin^{-1} x+\cos^{-1} 2x=\frac{\pi}{6}$

(*iii*) $\sin^{-1} x+\cos^{-1} (1-x)=0$

(*iv*) $\sin^{-1} x-\cos^{-1} x=\sin^{-1} (3x-2)$

(*v*) $\sin^{-1} x+\sin^{-1} (1-x)=\cos^{-1} x$

(*vi*) $\cos^{-1} x-\sin^{-1} x=\cos^{-1} (x\sqrt{3})$

(*vii*) $\sin^{-1} x+\sin^{-1} 2x=\frac{\pi}{3}$.

Answers

1. (*ii*) and (*iv*) 2. (*i*) $\frac{\pi}{6}$ (*ii*) $\frac{\pi}{4}$ (*iii*) $\frac{\pi}{3}$ (*iv*) $\frac{\pi}{2}$ (*v*) $\frac{\pi}{3}$ (*vi*) $\frac{\pi}{4}$

(*vii*) $\frac{-\pi}{3}$ (*viii*) π (*ix*) $\frac{-\pi}{6}$ (*x*) $\frac{-\pi}{6}$ (*xi*) $\frac{5\pi}{6}$ (*xii*) $\frac{-\pi}{3}$

3. (*i*) $\frac{-\pi}{4}$ (*ii*) 0 (*iii*) $\frac{\pi}{9}$ (*iv*) $\frac{\pi}{18}$ (*v*) $\frac{\pi}{4}$ (*vi*) $\frac{\pi}{4}$ (*vii*) $\frac{7\pi}{18}$ (*viii*) $\frac{-\pi}{4}$ (*ix*) 0 (*x*) $\frac{2\pi}{3}$ (*xi*) $\frac{-\pi}{2}$ (*xii*) $\frac{\pi}{6}$ (*xiii*) $\frac{\pi}{3}$ (*xiv*) $\frac{\pi}{2}$ (*xv*) 0

4. (*b*) (*i*) $\frac{2\sqrt{2}}{3}$, $\frac{1}{2\sqrt{2}}$ (*ii*) a and b don't exist (*iii*) $a=\frac{-2}{\sqrt{5}}$, b doesn't exist 6. (*i*) $\frac{5}{4}$ (*ii*) $\frac{7}{25}$ (*iii*) $\frac{3}{5}$ (*iv*) 0 (*v*) $-\frac{\sqrt{3}}{2}$ (*vi*) $-\frac{24}{25}$ (*vii*) $\frac{119}{169}$ (*viii*) $-\sqrt{3}$ (*ix*) $\frac{117}{125}$ (*x*) $\frac{140}{221}$ (*xi*) $\frac{4\sqrt{5}}{9}$ (*xii*) $\frac{1}{20}$ 13. (*i*) $\frac{2}{3}$, $\frac{1}{3}$ (*ii*) 0 (*iii*) $\frac{1}{6}$ (*iv*) 0 (*v*) $\frac{1}{\sqrt{5}}$ (*vi*) $\pm\frac{1}{\sqrt{2}}$ (*vii*) $-1, \frac{1}{2}$ 14. (*i*) $\frac{1}{\sqrt{5}}$ (*ii*) $\frac{1}{2}$ (*iii*) 0 (*iv*) $1, \frac{1}{2}$ (*v*) $0, \frac{1}{2}$ (*vi*) $0, \pm\frac{1}{2}$ (*vii*) $-\frac{1}{2}$

CHAPTER 4

GENERAL SOLUTIONS OF TRIGONOMETRIC EQUATIONS

1. Find the general solutions of

(*i*) $2 \sin x=1$ (*ii*) $\cos 2x=\frac{\sqrt{2}}{2}$

(*iii*) $2 \sin^2 2x=1$ (*iv*) $\tan 3x=1$ (*v*) $\sec^2 4x=1$

(*vi*) $3 \cot^2 x=1$ (*vii*) $3 \operatorname{cosec}^2 5x=4$ (*viii*) $\sin x \cos x=\frac{1}{4}$

2. Find the general solutions of

(*i*) $\sin 5x=\sin 4x$ (*ii*) $\cos 3x=\cos x$ (*iii*) $\tan 2\theta=\tan \theta$

(*iv*) $\sin 5\theta+\sin 3\theta=0$ (*v*) $\cos 4x+\cos 2x=0$

(*vi*) $\tan 3x+\tan 2x=0$ (*vii*) $\sin 3\theta=\cos 2\theta$

(*viii*) $\cos 5x=\sin 2x$ (*ix*) $\tan 5x=\cot 4x$

(*x*) $\cos 5x+\sin 3x=0$ (*xi*) $\sin 3x+\cos x=0$

(*xii*) $\tan mx+\cot nx=0$ (*xiii*) $\sin 4x+\cos 2x=0$

(*xiv*) $\cos 2x=\sin \left(x+\frac{\pi}{4}\right)$

3. Find the general solutions of

(*i*) $\cos 9x=\cos 5x-\cos x$ (*ii*) $\sin 7x-\sin x=\sin 3x$

(*iii*) $\cos x-\cos 7x=\sin 4x$ (*iv*) $\sin x+\sin 2x+\sin 3x=0$

(*v*) $\sin x+\sin 3x+\sin 5x+\sin 7x=0$

(*vi*) $\cos x+\cos 3x+\cos 5x+\cos 7x=0$

(*vii*) $\sin 7\theta \sin 5\theta=\sin 3\theta \sin \theta$

(*viii*) $\cos 8\theta \sin 6\theta=\cos 4\theta \sin 2\theta$

(*ix*) $\sin 2x-\cos 3x=\cos 2x-\sin 3x$

(*x*) $\sin \theta+\cos \theta=\sin 2\theta+\cos 2\theta$.

4. Find the general solutions of

(*i*) $\sin 2x=\sqrt{3} \cos x$ (*ii*) $\sin 2\theta+\sin \theta=0$

(*iii*) $\sin 3x-8 \sin^3 x=0$ (*iv*) $\cos 3x+8 \cos^3 x=0$

(*v*) $\tan \theta+\tan 2\theta=\tan 3\theta$

(*vi*) $2 \cos \theta \cot \theta+1=\cot \theta+2 \cos \theta$

(*vii*) $2 \cos^2\theta+\sin \theta=\sin 2\theta+\cos \theta$

(*viii*) $\cos 6x+\cos 4x+\cos 2x+1=0$.

5. Find the general solutions of

(*i*) $1+\sqrt{3} \tan^2 \theta=(\sqrt{3}+1) \tan \theta$

(*ii*) $\tan^2 \theta-\left(\sqrt{3}+\dfrac{1}{\sqrt{3}}\right) \tan \theta+1=0$

(*iii*) $2 \sin^2 x-3 \sin x+1=0$

(*iv*) $2 \cos^2 \theta+\cos \theta-1=0$

(*v*) $\operatorname{cosec}^2 x-3 \operatorname{cosec} x+2=0$.

6. Find the general solutions of

(*i*) $4 \sin^2 x-8 \cos x+1=0$ (*ii*) $6 \cos^2 x+5 \sin x-7=0$

(*iii*) $2 \sin^2 x+\sqrt{3} \cos x+1=0$ (*iv*) $\tan^2 \theta-4 \sec \theta+5=0$

(*v*) $\tan^2 x+3=3 \sec x$ (*vi*) $\sqrt{3} \sec^2\theta+1=$

(*vii*) $3 (\sec^2 x+\tan^2 x)=5$ $(\sqrt{3}+1) \tan \theta+\sqrt{3}$

(*viii*) $2 \cos^2 \theta=3 \sin \theta$ (*ix*) $8 \sin^2 \theta-2 \cos \theta=5$

(*x*) $4 \cos 2\theta=2 \sin \theta+1$ (*xi*) $5 \sin^2 \theta-\cos^2\theta=9 \sin \theta-4$

(*xii*) $\tan \theta-2 \cot \theta=1$ (*xiii*) $\cot x-\tan x=2$

(*xiv*) $3 \cos 2x+7=4 \sin x$ (*xv*) $3 \tan^2 \theta-2 \sin \theta=0$.

7. Find the general solutions of

(i) $\cos x+\sqrt{3}\sin x=1$ (ii) $\sqrt{3}\sin x-\cos x=\sqrt{2}$
(iii) $\sin x+\sqrt{3}\cos x=1$ (iv) $\sin x+\cos x=\sqrt{2}$
(v) $\sin x-\sqrt{3}\cos x=1$ (vi) $\sin x-\cos x=\sqrt{2}$
(vii) $\sqrt{3}\tan x=\sqrt{2}\sec x-1$ (viii) $\tan x+\sec x=\sqrt{3}$
(ix) $6\sin x+8\cos x=7.$

Completed.

Answers (*n* is any integer)

1. (i) $n\pi+(-1)^n\frac{\pi}{6}$ (ii) $n\pi\pm\frac{\pi}{8}$ (iii) $n\frac{\pi}{2}\pm(-1)^n\frac{\pi}{8}$

(iv) $\frac{n\pi}{3}+\frac{\pi}{12}$ (v) $n\frac{\pi}{2}$; $n\frac{\pi}{2}\pm\frac{\pi}{4}$ (vi) $n\pi\pm\frac{\pi}{3}$

(vii) $\frac{n\pi}{5}\pm(-1)^n\frac{\pi}{15}$ (viii) $\frac{n\pi}{2}+(-1)^n\frac{\pi}{12}$.

2. (i) $\frac{n\pi}{5-(-1)^n 4}$ (ii) $\frac{2n\pi}{3\pm1}$ (iii) $n\pi$ (iv) $\frac{n\pi}{5+(-1)^n 3}$

(v) $\frac{(2n+1)\pi}{6}$; $\frac{(2n-1)\pi}{2}$ (vi) $\frac{n\pi}{5}$ ✓ (vii) $\frac{n\pi+(-1)^n\pi/2}{3+(-1)^n 2}$

(viii) $\frac{1}{3}(2n\pi-\frac{\pi}{2})$; $\frac{1}{7}(2n\pi+\frac{\pi}{2})$ (ix) $\frac{n\pi}{9}+\frac{\pi}{18}$ (x) $n\pi+\frac{\pi}{4}$;

$\frac{n\pi}{4}-\frac{\pi}{16}$ (xi) $\frac{n\pi-(-1)^n\frac{\pi}{2}}{3-(-1)^n}$ (xii) $\frac{1}{m-n}(k\pi+\frac{\pi}{2})$ wher

k is any integer. (xiii) $\frac{2n\pi\pm\pi/2}{2\mp4}$ (xiv) $2n\pi-\frac{\pi}{4}$;

$\frac{1}{3}(2n\pi+\frac{\pi}{4})$

3. (i) $\frac{2n\pi}{5}\pm\frac{\pi}{10}$; $\frac{n\pi}{2}\pm\frac{\pi}{12}$ (ii) $\frac{n\pi}{3}$; $\frac{n\pi}{2}\pm\frac{\pi}{12}$ (iii) $\frac{n\pi}{4}$;

$\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$ (iv) $\frac{n\pi}{2}$; $2n\pi\pm\frac{2\pi}{3}$ (v) $\frac{n\pi}{4}$; $n\pi\pm\frac{\pi}{4}$;

$2n\pi\pm\frac{\pi}{2}$ (vi) $\frac{n\pi}{2}\pm\frac{\pi}{8}$; $n\pi+\frac{\pi}{4}$; $2n\pi\pm\frac{\pi}{2}$

(*vii*) $\frac{n\pi}{8}$; $\frac{n\pi}{4}$ (*viii*) $\frac{n\pi}{14-(-1)^n 6}$ (*ix*) $4n\pi \pm \pi$; $\frac{2}{5}[n\pi + \frac{\pi}{4}]$

(*x*) $2n\pi$; $\frac{2n\pi}{3} + \frac{\pi}{6}$

4. (*i*) $2n\pi \pm \frac{\pi}{2}$; $n\pi + (-1)^n \frac{\pi}{3}$ (*ii*) $n\pi$; $2n\pi \pm \frac{2\pi}{3}$ (*iii*) $n\pi$; $n\pi \pm (-1)^n \frac{\pi}{6}$ (*iv*) $2n\pi \pm \frac{\pi}{2}$; $2n\pi \pm \frac{\pi}{3}$; $2n\pi \pm \frac{2\pi}{3}$

(*v*) $n\pi$; $\frac{n\pi}{2}$; $\frac{n\pi}{3}$ (*vi*) $2n\pi \pm \frac{\pi}{3}$; $n\pi + \frac{\pi}{4}$ (*vii*) $2n\pi \pm \frac{\pi}{3}$, $n\pi + \frac{\pi}{4}$ (*viii*) $2n\pi \pm \frac{\pi}{2}$; $\frac{1}{2}(2n\pi \pm \frac{\pi}{2})$; $\frac{1}{3}(2n\pi \pm \frac{\pi}{2})$

5. (*i*) $n\pi + \frac{\pi}{6}$; $n\pi + \frac{\pi}{4}$ (*ii*) $n\pi + \frac{\pi}{3}$; $n\pi + \frac{\pi}{6}$

(*iii*) $n\pi + (-1)^n \frac{\pi}{6}$; $n\pi + (-1)^n \frac{\pi}{2}$ (*iv*) $2n\pi \pm \frac{\pi}{3}$; $2n\pi \pm \pi$.

(*v*) $n\pi + (-1)^n \frac{\pi}{2}$; $n\pi + (-1)^n \frac{\pi}{6}$

$\frac{d}{dx}(\sin x) = \cos x$

6. (*i*) $2n\pi \pm \frac{\pi}{3}$ (*ii*) $n\pi + (-1)^n \frac{\pi}{6}$; $n\pi + (-1)^n \sin^{-1} \frac{1}{3}$

(*iii*) $2n\pi \pm \frac{5\pi}{6}$ (*iv*) $2n\pi \pm \frac{\pi}{3}$ (*v*) $2n\pi$; $2n\pi \pm \frac{\pi}{3}$

(*vi*) $n\pi + \frac{\pi}{4}$; $n\pi + \frac{\pi}{6}$ (*vii*) $n\pi \pm \frac{\pi}{6}$ (*viii*) $n\pi + (-1)^n \frac{\pi}{6}$

(*ix*) $2n\pi \pm \frac{\pi}{3}$; $2n\pi \pm \cos^{-1}(-\frac{3}{4})$ (*x*) $n\pi + (-1)^n \frac{\pi}{6}$; $n\pi + (-1)^n \sin^{-1}(-\frac{3}{4})$ (*xi*) $n\pi + (-1)^n \frac{\pi}{2}$; $n\pi + (-1)^n \frac{\pi}{6}$

(*xii*) $n\pi - \frac{\pi}{4}$; $n\pi + \tan^{-1} 2$ (*xiii*) $\frac{n\pi}{2} + \frac{\pi}{8}$

(*xiv*) $n\pi + (-1)^n \frac{\pi}{2}$ (*xv*) $n\pi$; $n\pi + (-1)^n \frac{\pi}{6}$

7. (*i*) $2n\pi+\frac{\pi}{3}\pm\frac{\pi}{3}$ (*ii*) $2n\pi-\frac{\pi}{3}\pm\frac{3\pi}{4}$ (*iii*) $2n\pi+\frac{\pi}{6}\pm\frac{\pi}{3}$

(*iv*) $2n\pi+\frac{\pi}{4}$ (*v*) $2n\pi-\frac{\pi}{6}\pm\frac{2\pi}{3}$ (*vi*) $2n\pi-\frac{\pi}{4}\pm\pi$

(*vii*) $2n\pi+\frac{\pi}{3}\pm\frac{\pi}{4}$ (*viii*) $2n\pi+\frac{\pi}{6}$

(*ix*) $2n\pi+\cos^{-1}\frac{4}{5}\pm\cos^{-1}\frac{7}{10}$

CHAPTER 5

COMPLEX NUMBERS-DEMOIVRE'S THEOREM

1. Recall the definitions of a complex number, its real and imaginary parts and the conjugate of a complex number.
2. What is Argand plane (complex plane)? Argand diagram?
3. Plot the points representing the complex numbers $-i$, $1+2i$ and $3+8i$; also show that they are collinear
4. Show that the points representing the complex numbers
   (*i*) $7+9i$, $3-7i$ and $-3+3i$ form a right angled isosceles triangle
   (*ii*) $1+i$, $-1-i$ and $-\sqrt{3}+\sqrt{3}\,i$ form an equilateral triangle
   (*iii*) $4+3i$, $-1+3i$, $-1-2i$ and $4-2i$ form a square.
5. ABCD is a square and its centre is at origin. If A represents $2-5i$, find the complex numbers representing the other vertices
6. Express (*i*) $\frac{(1+i)(1+2i)}{1+3i}$ (*ii*) $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$ (*iii*) $\frac{(1+i)^2}{2+i}$ in the form $a+ib$ and plot them on a complex plane
7. Plot the conjugates of (*i*) $\frac{3i}{3+4i}$ (*ii*) $\frac{7-4i}{3+2i}$ (*iii*) $(1+2i)(1-i)$ on an Argand plane.
8. Define the polar form of a complex number and establish the relation between polar form and cartesian form of a complex number.

9. (*a*) Plot the complex numbers (*i*) $\sqrt{2}\,(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$

(*ii*) $2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ (*iii*) $5\,(\cos\pi+i\sin\pi)$

(*iv*) $3\left[\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)\right]$ on a complex plane.

(*b*) Convert the complex numbers in (*a*) into cartesian form and then plot them in a complex plane.

10. Express the following complex numbers in the polar form:

(*i*) $1+i$ (*ii*) $1-i$ (*iii*) $-\sqrt{3}-i$ (*iv*) $-\sqrt{3}+i$ (*v*) $\sqrt{3}-i$

(*vi*) $\frac{1-i\sqrt{3}}{2}$ (*vii*) $-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ (*viii*) -1 (*ix*) $-i$

(*x*) $\frac{i+i^2-i^3+i^4+2}{i^5+i^6+2}$ (*xi*) $-\sqrt{3}-3i$ (*xii*) $-4-4i$ (*xiii*) $4-3i$

(*xiv*) $1+\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}$ (*xv*) $1+\sin\frac{\pi}{15}+i\cos\frac{\pi}{15}$

(*xvi*) $1+\cos\alpha-i\sin\alpha$ (*xvii*) $1-\cos\alpha+i\sin\alpha$

(*xviii*) $\frac{1}{1+\cos\theta+i\sin\theta}$ (*xix*) $\frac{1}{1+\sin\alpha+i\cos\alpha}$

11. Find the modulus and amplitude of each of the following complex numbers:

(*i*) $\frac{1-i}{1+i}$ (*ii*) $\frac{4+3i}{2-i}$ (*iii*) $\frac{2-i}{(1-2i)^2}$ (*iv*) $\frac{(3+2i)^2}{4-3i}$

(*v*) $\frac{1+2i}{1-(1-i)^2}$ (*vi*) $\left(\frac{2+i}{3-i}\right)^2$ (*vii*) $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$

(*viii*) $\frac{z-1}{z+1}$ where $z=2-3i$ (*ix*). $\frac{z+i}{z-4i}$ where $z=1+i$

(*x*) $\frac{z\,(z+2i)}{z+4i}$ where $z=1+i$

12. If z_1 and z_2 are any two non-zero complex numbers, prove that

(*i*) $|z_1 z_2|=|z_1|\,|z_2|$ and $\arg(z_1 z_2)=\arg z_1+\arg z_2$

(*ii*) $\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}$ and $\arg\left(\frac{z_1}{z_2}\right)=\arg z_1-\arg z_2$

13. Explain how will you represent the sum of two complex numbers and the product of two complex numbers in an Argand diagram.

14. (*a*) If $z_1=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ and $z_2=4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$, represent z_1+z_2 and $z_1 z_2$ on a complex plane

    (*b*) Represent the sum and product of $3+i$ and $2-i$ on a complex plane

15. Simplify (*i*) $(\cos\theta+i\sin\theta)(\cos\alpha+i\sin\alpha)$

    (*ii*) $$\frac{\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}}{\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}}$$

    (*iii*) $$\frac{(\cos 2\alpha+i\sin 2\alpha)(\cos 3\beta+i\sin 3\beta)}{\cos 4\gamma+i\sin 4\gamma}$$

16. Find the modulus and amplitude of the complex numbers $z_1=1+i\sqrt{3}$ and $z_2=\sqrt{3}+i$ and hence find the modulus and amplitude of $z_1 z_2$, $z_1\overline{z_2}$, z_1/z_2 and $\overline{z_1}/z_2$

17. If $z_n=\cos\left(\frac{\pi}{2^n}\right)+i\sin\left(\frac{\pi}{2^n}\right)$, prove that $z_1 z_2 z_3 \ldots = -1$

18. Express $z=1+i$ in the polar form and hence plot $\overline{z}$, z^2 and z^3 in an Argand diagram.

19. (*a*) State and prove DeMoivre's theorem

    (*b*) Simplify the following using DeMoivre's theorem.

    (*i*) $\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^{30}$ (*ii*) $\left(\cos\frac{5\theta}{2}+i\sin\frac{5\theta}{2}\right)^4$

    (*iii*) $\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)^5$ (*iv*) $(\sin\theta+i\cos\theta)^3$

    (*v*) $(\sin\theta+i\cos\theta)^n$ (*vi*) $(\cos\theta+i\sin\theta)^5(\cos\theta-i\sin\theta)^4$

    (*vii*) $\dfrac{(\cos\theta+i\sin\theta)^7}{(\cos\theta-i\sin\theta)^4}$ (*viii*) $\dfrac{(\cos\theta+i\sin\theta)^4}{(\sin\alpha+i\cos\alpha)^5}$

    (*ix*) $\dfrac{(\sin\theta+i\cos\theta)^3}{(\cos\alpha+i\sin\alpha)^2}$

20. Simplify:—

(*i*) $\dfrac{(\cos 7\theta - i \sin 7\theta)^{-2} \times (\cos 3\theta + i \sin 3\theta)^4}{(\cos \theta + i \sin \theta)^{-4}}$

(*ii*) $\dfrac{(\cos 2\theta + i \sin 2\theta)^4 \times (\cos 3\theta - i \sin 3\theta)^2}{(\cos 3\theta + i \sin 3\theta)^{-2} \times (\cos \theta - i \sin \theta)^{-3}}$

(*iii*) $\dfrac{(\cos 4\theta + i \sin 4\theta)^3 \times (\cos 5\theta + i \sin 5\theta)^3}{(\cos 6\theta - i \sin 6\theta)^2 \times (\cos \theta - i \sin \theta)^5}$

(*iv*) $\dfrac{(\cos \theta + i \sin \theta)^3 \quad (\cos \theta - i \sin \theta)^{-8}}{(\cos 4\theta + i \sin 4\theta)^5 \quad (\cos 3\theta + i \sin 3\theta)^2}$

(*v*) $\dfrac{(\cos 5\alpha - i \sin 5\alpha)^4 \; (\cos 4\beta + i \sin 4\beta)^{-2}}{(\cos 3\alpha + i \sin 3\alpha)^{-7} \; (\cos 3\beta - i \sin 3\beta)^5}$

21. (*i*) Simplify $(\sqrt{3}+i)^4 - (\sqrt{3}-i)^4$

(*ii*) Simplify $(1+i\sqrt{3})^5 + (1-i\sqrt{3})^5$

(*iii*) Simplify $(1+i)^6 + (1-i)^6$

(*iv*) Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos \dfrac{n\pi}{4}$

(*v*) Prove that $(-1+i\sqrt{3})^{3n} + (-1-i\sqrt{3})^{3n} = 2^{3n+1}$

(*vi*) Prove that $(1+\cos\theta + i \sin\theta)^n + (1+\cos\theta - i\sin\theta)^n$

$$= 2^{n+1} \cos^n \frac{\theta}{2} \cos \frac{n\theta}{2}$$

(*vii*) $\left(1+\sin\dfrac{\pi}{5} + i\cos\dfrac{\pi}{5}\right)^{\frac{10}{8}} + \left(1+\sin\dfrac{\pi}{5} - i\cos\dfrac{\pi}{5}\right)^{\frac{10}{8}} = 0.$

22. If $z = \cos\theta + i\sin\theta$, prove that $\dfrac{z^{2n}-1}{z^{2n}+1} = i \tan n\theta$

23. (*i*) Prove that $\left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos\left(\dfrac{n\pi}{2} - n\theta\right)$

$$+ i \sin\left(\frac{n\pi}{2} - n\theta\right)$$

(*ii*) Simplify $\left(\dfrac{1+\sin\dfrac{\pi}{8} + i\cos\dfrac{\pi}{8}}{1+\sin\dfrac{\pi}{8} - i\cos\dfrac{\pi}{8}}\right)^{\frac{1}{2}}$

(*iii*) Simplify $\left(\dfrac{1+\cos\dfrac{\pi}{n}+i\sin\dfrac{\pi}{n}}{1+\cos\dfrac{\pi}{n}-i\sin\dfrac{\pi}{n}}\right)^{n}$.

(*iv*) Simplify $\left(\dfrac{1+\cos\alpha+i\sin\alpha}{1+\cos\alpha-i\sin\alpha}\right)^{10}$

(*v*) Prove that $\left(\dfrac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^{n}=\cos n\theta+i\sin n\theta$

(*vi*) Prove that $\left(\dfrac{1-\cos\dfrac{\pi}{10}+i\sin\dfrac{\pi}{10}}{1+\cos\dfrac{\pi}{10}-i\sin\dfrac{\pi}{10}}\right)^{10}=-1$

24. (*i*) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ prove that $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$ and $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$

(*ii*) Using DeMoivre's theorem prove that $\sin 3\theta=3\sin\theta-4\sin^3\theta$ and $\cos 3\theta=4\cos^3\theta-3\cos\theta$

25. (*i*) If $x=\cos\theta+i\sin\theta$, show that

$$x^2+\frac{1}{x^2}=2\cos 2\theta \text{ and } x^3-\frac{1}{x^3}=2i\sin 3\theta$$

(*ii*) If $x=\cos\alpha+i\sin\alpha$ and $y=\cos\beta+\sin\beta$, prove that

$$\frac{x^3}{y^4}+\frac{y^4}{x^3}=2\cos(3\alpha-4\beta) \text{ and } \frac{x^3}{y^4}-\frac{y^4}{x^3}=2i\sin(3\alpha-4\beta)$$

(*iii*) If $x=\cos\theta+i\sin\theta$ and $y=\cos\phi+i\sin\phi$, prove that

$$\frac{x}{y}+\frac{y}{x}=2\cos(\theta-\phi) \text{ and } \frac{x}{y}-\frac{y}{x}=2i\sin(\theta-\phi)$$

(*iv*) If $x=\cos\alpha+i\sin\alpha$ and $y=\cos\beta+i\sin\beta$,

prove that $xy+\dfrac{1}{xy}=2\cos(\alpha+\beta)$, $xy-\dfrac{1}{xy}$ $=2i\sin(\alpha+\beta)$ and $x^2y^3+\dfrac{1}{x^2y^3}=2\cos(2\alpha+3\beta)$

(*v*) If $x=\cos\alpha+i\sin\alpha$, $y=\cos\beta+i\sin\beta$ and

$z=\cos\gamma+i\sin\gamma$, prove that $\dfrac{x^4y^3}{z^2}+\dfrac{z^2}{x^4y^3}=$

$2\cos(4\alpha+3\beta-2\gamma)$ and $\dfrac{x^5y^6}{z^4}-\dfrac{z^4}{x^5y^6}=$
$2i\sin(5\alpha+6\beta-4\gamma)$

26. (*i*) If $x+\dfrac{1}{x}=2\cos\theta$, prove that $x=\cos\theta\pm i\sin\theta$

(*ii*) If $x+\dfrac{1}{x}=2\cos\alpha$, prove that $x^n+\dfrac{1}{x^n}=2\cos n\alpha$

and (one of the values of) $x^n-\dfrac{1}{x^n}=2i\sin n\alpha$

(*iii*) If $x+\dfrac{1}{x}=2\cos\alpha$ and $y+\dfrac{1}{y}=2\cos\beta$, prove that

$\dfrac{x^m}{y^n}+\dfrac{y^n}{x^m}=2\cos(m\alpha-n\beta)$ and (one of the values of)

$\dfrac{x^n}{y^m}-\dfrac{y^m}{x^n}=2i\sin(n\alpha-m\beta)$.

27. Recall the cube roots and fourth roots of unity and plot them on a complex plane.

28. Explain the method of finding n^{th} root of a complex number; in particular cube roots of a complex number

29. Find the cube roots of the following complex numbers and represent them on a complex plane

(*i*) $\sqrt{3}+i$ (*ii*) $\sqrt{3}-i$ (*iii*) $-1-i$ (*iv*) $-2+2i$ (*v*) $\dfrac{1+i\sqrt{3}}{2}$

(*vi*) $1-i\sqrt{3}$ (*vii*) $1+\sqrt{3}i$ (*viii*) 125 (*ix*) $8i$.

30. Solve: (*i*) $x^5=1$ (*ii*) $x^3+27=0$ (*iii*) $x^5-x^3+x^2-1=0$
(*iv*) $x^4=81-81i$

31. If ω is an imaginary cube root of unity, prove that

(*i*) $(1+\omega-\omega^2)^3-(1-\omega+\omega^2)^3=0$

(*ii*) $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5=32$

(*iii*) $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})=16$

(*iv*) $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})=49$

(*v*) $(a-b)(a-\omega b)(a-\omega^2 b)=a^3-b^3$

(*vi*) $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)=a^3+b^3$

(*vii*) $(a+b)^2+(a\omega+b\omega^2)^2+(a\omega^2+b\omega)^2=6ab$

Answers

5. $(2, 5), (-2, 5)$ and $(-2, -5)$ 6. (*i*) $(\frac{4}{5}, \frac{3}{5})$ (*ii*) $(\frac{-3}{5}, 0)$ (*iii*) $(\frac{3}{5}, \frac{4}{5})$ 7. (*i*) $(\frac{12}{25}, \frac{-9}{25})$, (*ii*) $(1, 2)$ (*iii*) $(4, 2)$

9. (*b*) (*i*) $\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{3}}{2}\right)$ (*ii*) $(-\sqrt{2}, \sqrt{2})$ (*iii*) $(-5, 0)$

(*iv*) $\left(\frac{3}{2}, -3\frac{\sqrt{3}}{2}\right)$

10. (*i*) $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (*ii*) $\sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right]$

(*iii*) $2\left[\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(\frac{-5\pi}{6}\right)\right]$

(*iv*) $2\left[\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)\right]$ (*v*) $2\left[\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right]$

(*vi*) $\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)$ (*vii*) $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

(*viii*) $\cos\pi+i\sin\pi$ (*ix*) $\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)$

(*x*) $2(\cos 0+i\sin 0)$ (*xi*) $2\sqrt{3}\left[\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)\right]$

(*xii*) $4\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right]$

(*xiii*) $5(\cos\alpha+i\sin\alpha)$ where $\alpha=\sin^{-1}(\frac{-3}{5})$ (*xiv*) $2\cos\frac{\pi}{20}$ $\left(\cos\frac{\pi}{20}+i\sin\frac{\pi}{20}\right)$ (*xv*) $2\cos\frac{13\pi}{60}\left(\cos\frac{13\pi}{60}+i\sin\frac{13\pi}{60}\right)$

(*xvi*) $2\cos\frac{\alpha}{2}\left[\cos\left(-\frac{\alpha}{2}\right)+i\sin\left(\frac{-\alpha}{2}\right)\right]$

(*xvii*) $2\sin\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)\right]$

(*xviii*) $\frac{1}{2}\sec\frac{\theta}{2}\left[\cos\left(\frac{-\theta}{2}\right)+i\sin\left(\frac{-\theta}{2}\right)\right]$

(*xix*) $\frac{1}{2}\sec\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)+i\sin\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)\right.$

11. (*i*) $-1; -\frac{\pi}{2}$ (*ii*) $\sqrt{5}; \sin^{-1}\frac{2}{\sqrt{5}}$ (*iii*) $\frac{1}{\sqrt{5}}; \pi - \sin^{-1}\frac{11\sqrt{5}}{25}$
(*iv*) $\frac{13}{5}; \pi - \sin^{-1}\frac{63}{65}$ (*v*) $1;\ 0$ (*vi*) $\frac{1}{2}; \frac{\pi}{2}$ (*vii*) $2; -\frac{\pi}{2}$
(*viii*) $\frac{\sqrt{5}}{3}; -\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (*ix*) $\frac{1}{\sqrt{2}}; \frac{3\pi}{4}$ (*x*) $\sqrt{\frac{10}{13}};$
$\sin^{-1}\frac{7}{\sqrt{130}}$

14. (*a*) $z_1+z_2=(1+2\sqrt{3},\ 2+\sqrt{3});\ z_1 z_2=8\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$
$=(0,\ 8)$ (*b*) $(5,\ 0);\ (7,\ -1)$

15. (*i*) $\cos(\theta+\alpha)+i\sin(\theta+\alpha)$ (*ii*) $\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}$
(*iii*) $\cos(2\alpha+3\beta-4\gamma)+i\sin(2\alpha+3\beta-4\gamma)$.

16. $z_1;\ 2,\ \frac{\pi}{3};\ z_2 : 2,\ \frac{\pi}{6};\ z_1 z_2 : 4,\ \frac{\pi}{2};\ z_1\overline{z_2} : 4,\ \frac{\pi}{6};$
$\frac{z_1}{z_2} : 1,\ \frac{\pi}{6};\ \frac{\overline{z_1}}{z_2} : 1,\ \frac{-\pi}{6}$

18. $\overline{z}=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right);\ z^2=2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right);$
$z^3=2\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

19. (*b*) (*i*) 1 (*ii*) $\cos 10\theta+i\sin 10\theta$ (*iii*) -1 (*iv*) $-(\cos 3\theta+i\sin 3\theta)$
(*v*) $\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right)$ (*vi*) $\cos\theta+i\sin\theta$
(*vii*) $\cos 11\theta+i\sin 11\theta$ (*viii*) $\sin(4\theta+5\alpha)-i\cos(4\theta+5\alpha)$
(*ix*) $-[\sin(3\theta+2\alpha)+i\cos(3\theta+2\alpha)]$.

20. (*i*) $\cos 30\theta+i\sin 30\theta$ (*ii*) $\cos 5\theta+i\sin 5\theta$
(*iii*) $\cos 44\theta+i\sin 44\theta$ (*iv*) $\cos 15\theta-i\sin 15\theta$
(*v*) $\cos(\alpha+7\beta)+i\sin(\alpha+7\beta)$

21. (*i*) $16\sqrt{3}\,i$ (*ii*) 32 (*iii*) 0

23. (*ii*) $\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}$ (*iii*) -1 (*iv*) $\cos 10\alpha+i\sin 10\alpha$

29. The three cube roots are given by the following, where $k=0,1,2$

(*i*) $2^{\frac{1}{3}}\left[\cos\dfrac{12k+1}{18}\pi+i\sin\dfrac{12k+1}{18}\pi\right]$ (*ii*) $2^{\frac{1}{3}}\left[\cos\dfrac{12k-1}{18}\pi+i\sin\dfrac{12k-1}{18}\pi\right]$ (*iii*) $\sqrt{2}^{\frac{1}{3}}\left[\cos\dfrac{8k-3}{12}\pi+i\sin\dfrac{8k-3}{12}\pi\right]$

(*iv*) $(2\sqrt{2})^{\frac{1}{3}}\left[\cos\dfrac{8k+3}{12}\pi+i\sin\dfrac{8k+3}{12}\pi\right]$ (*v*) $\cos\dfrac{6k+1}{9}\pi+i\sin\dfrac{6k+1}{9}\pi$ (*vi*) $2^{\frac{1}{3}}\left[\cos\dfrac{6k-1}{9}\pi+i\sin\dfrac{6k-1}{9}\pi\right]$

(*vii*) $2^{\frac{1}{3}}\left[\cos\dfrac{6k+1}{9}\pi+i\sin\dfrac{6k+1}{9}\pi\right]$ (*viii*) $5\left[\cos\dfrac{2k\pi}{3}+i\sin\dfrac{2k\pi}{3}\right]$ (*ix*) $2\left[\cos\dfrac{4k+1}{6}\pi+i\sin\dfrac{4k+1}{6}\pi\right]$

30. (*i*) $\cos\dfrac{2k\pi}{5}+i\sin\dfrac{2k\pi}{5}$, $k=0, 1, 2, 3, 4$ (*ii*) $-3,\ \frac{3}{2}+i\dfrac{3\sqrt{3}}{2},\ -\frac{3}{2}-i\dfrac{3\sqrt{3}}{2}$ (*iii*) $1, -1, -1,\ \frac{1}{2}+i\dfrac{\sqrt{3}}{2},\ \frac{1}{2}-i\dfrac{\sqrt{3}}{2}$

(*iv*) $3\left[\cos\dfrac{8k-1}{16}\pi+i\sin\dfrac{8k-1}{16}\pi\right]$, $k=0, 1, 2, 3.$

SECTION III: ANALYTICAL GEOMETRY

CHAPTER 1

PAIR OF STRAIGHT LINES

1. (*i*) Prove that a homogeneous equation in x and y of degree 2 i.e. $ax^2+2hxy+by^2=0$ represents a pair of lines through origin.

   (*ii*) If m_1 and m_2 are the slopes of the straight lines represented by $ax^2+2hxy+by^2=0$, prove that $m_1+m_2=-\frac{2h}{b}$ and $m_1\ m_2=\frac{a}{b}$.

2. Find the separate equations of the pair of lines (*i*) $x^2-5xy+4y^2=0$ (*ii*) $y^2+xy-2x^2=0$ (*iii*) $2x^2-5xy+2y^2=0$

3. What is the point of intersection of the pair of lines (*i*) $12x^2-10xy+y^2=0$. (*ii*) $4x^2+5xy+y^2=0$.

4. Prove that the angle (acute) between the pair of lines $ax^2+2hxy+by^2=0$ is $\tan^{-1}\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$. Also deduce the condition for the lines to be

   (*i*) coincident (*ii*) perpendicular.

5. Find the angle between the pair of lines

   (*i*) $x^2-7xy+12y^2=0$ (*ii*) $3x^2+10xy+8y^2=0$

   (*iii*) $2x^2+xy-3y^2=0$ (*iv*) $x^2-xy-6y^2=0$ (*v*) $4x^2-4xy+y^2=0$

   (*vi*) $2x^2+3xy-2y^2=0$

6. Show that (*i*) $5x^2-4xy-5y^2=0$ represents a pair of perpendicular lines (*ii*) $9x^2-6xy+y^2=0$ represents a pair of coincident lines.

7. Prove that the pair of lines through origin and perpendicular to the lines $ax^2+2hxy+by^2=0$ is $bx^2-2hxy+ay^2=0$.

8. Prove that the lines $x^2-4xy+y^2=0$ and $x+y=1$ form an equilateral triangle.

9. Prove that the condition for the general quadratic equation in x and y i.e. $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of lines is $abc+2fgh-af^2-bg^2-ch^2=0$.

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10. Show that the following equation represents a pair of lines.
    (*i*) $12x^2+7xy-12y^2-x+7y-1=0$.
    (*ii*) $3x^2-8xy-3y^2+2x+14y-8=0$.
    (*iii*) $(x-1)^2-2(x-1)y-y^2=0$ (*iv*) $2x^2+xy-y^2-3x+6y-9=0$.
    (*v*) $6x^2-7xy-5y^2-17x+37y-14=0$.
    (*vi*) $3x^2-10xy+3y^2+14x-10y+8=0$.

11. Find the value of k if the following equation represents a pair of lines:
    (*i*) $kx^2-2xy-3y^2+12y-9=0$. (*ii*) $x^2-5xy+4y^2+x+2y+k=0$.
    (*iii*) $9x^2-6xy+ky^2+18x-6y+8=0$.
    (*iv*) $2x^2+xy-y^2-kx+6y-9=0$.
    (*v*) $x^2+6xy+ky^2+4x+12y-5=0$.

12. Find the values of k for which the following equation represents a pair of lines:
    (*i*) $x^2-kxy-2y^2-3y-1=0$. (*ii*) $2x^2+2kxy+3y^2-x+2y-1=0$.

13. Find the separate equations of the pair of lines and their point of intersection:
    (*i*) $4x^2+xy-3y^2-9x+5y+2=0$.
    (*ii*) $2x^2-5xy+2y^2+7x-5y+3=0$.
    (*iii*) $x^2-5xy+4y^2+x+2y-2=0$. (*iv*) $x^2-y^2+x-3y-2=0$.

14. Find the value of k if the following equation represents a pair of lines and also find their point of intersection:
    (*i*) $3x^2-11xy+10y^2-7x+13y+k=0$
    (*ii*) $3x^2-14xy-5y^2+10x-18y+k=0$
    (*iii*) $x^2-5xy+ky^2+x+2y-2=0$
    (*iv*) $kx^2-5xy+2y^2+7x-11y+5=0$.

15. (*i*) Show that the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ is parallel to the pair of lines $ax^2+2hxy+by^2=0$. (*ii*) Deduce that the acute angle between the lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\tan^{-1}\left|\dfrac{2\sqrt{h^2-ab}}{a+b}\right|$. (*iii*) Also find the conditions for them to be parallel and perpendicular.

16. Find the (*acute*) angle between the following pair of lines and their point of intersection:
    (*i*) $2x^2+7xy+3y^2+18x+19y+28=0$. (*ii*) $x^2-2xy-3y^2+12y-9=0$ (*iii*) $x^2+9y^2+6xy+4x+12y-5=0$. (*iv*) $2x^2+5xy+3y^2+6x+7y+4=0$. (*v*) $3x^2-10xy+3y^2+14x-10y+8=0$.

17. Show that the following lines are parallel:
(*i*) $9x^2-6xy+y^2+18x-6y+8=0$. (*ii*) $x^2+2\sqrt{3}xy+3y^2-3x-3\sqrt{3}y-4=0$.

18. Show that the following lines are perpendicular:
(*i*) $3x^2-8xy-3y^2+2x+4y-1=0$. (*ii*) $x^2-y^2+x-3y-2=0$.

19. Show that the equation (*i*) $x^2+y^2+2xy-8x-8y-9=0$ represents a pair of parallel straight lines. (*ii*) $3x^2+8xy-3y^2+29x-3y+18=0$ represents a pair of perpendicular lines.

20. Show that the following equation represents a pair of parallel straight lines. Find the distance between them.
(*i*) $4x^2+12xy+9y^2+14x+21y+12=0$. (*ii*) $9x^2-6xy+y^2+18x-6y+8=0$ (*iii*) $8x^2+8xy+2y^2+2x+y-15=0$. (*iv*) $9x^2-24xy+16y^2-9x+12y-4=0$.

21. Find a and b if the equation
(*i*) $12x^2+7xy-ay^2-18x+by+6=0$ represents a pair of perpendicular lines. (*ii*) $16x^2-8xy+ay^2-16x+4y+b=0$ represents a pair of parallel lines.

22. Prove that the pairs of lines $x^2-y^2+x-3y-2=0$ and $x^2-y^2=0$ form a rectangle.

Answers

2. (*i*) $x-4y=0$; $x-y=0$ (*ii*) $x-y=0$; $2x+y=0$ (*iii*) $x-2y=0$; $2x-y=0$. 3. (*i*) and (*ii*) (0, 0) 5. (*i*) $\tan^{-1}\dfrac{1}{13}$ (*ii*) $\tan^{-1}\dfrac{2}{11}$ (*iii*) $\tan^{-1} 5$ (*iv*) 45° (*v*) 0° (*vi*) 90°

11. (*i*) 1 (*ii*) -2 (*iii*) 1 (*iv*) 3 (*v*) 9. 12. (*i*) ± 1 (*ii*) $\frac{7}{2}, -\frac{5}{2}$.

13. (*i*) $4x-3y-1=0$, $x+y-2=0$; (1, 1) (*ii*) $2x-y+1=0$; $x-2y+3=0$; $(\frac{1}{3}, \frac{5}{3})$ (*iii*) $x-y-1=0$, $x-4y+2=0$; (2, 1) (*iv*) $x+y+2=0$; $x-y-1=0$; $(-\frac{1}{2}, -\frac{3}{2})$

14. (*i*) 4; (3, 1) (*ii*) 8; $\left(-\dfrac{11}{8}, \dfrac{1}{8}\right)$ (*iii*) 4; (2, 1) (*iv*) 2; $(-3, -1)$

16. (*i*) 45°; $(-1, -2)$ (*ii*) $\tan^{-1} 2$; $(\frac{3}{2}, \frac{3}{2})$ (*iii*) 0°; the lines don't intersect (*iv*) $\tan^{-1} \frac{1}{5}$; $(-1, 0)$ (*v*) $\tan^{-1} \frac{4}{3}$; $(-\frac{1}{4}, \frac{5}{4})$

20. (*i*) $\dfrac{1}{\sqrt{13}}$ (*ii*) $\dfrac{2}{\sqrt{10}}$ (*iii*) $\dfrac{11}{2\sqrt{5}}$ (*iv*) 1.

21. (*i*) $a=12$, $b=\dfrac{-23}{2}$, 1 (*ii*) $a=1$, b is any real number.

CHAPTER 2

CIRCLE

2.1 General Equation

1. Define a circle and derive the equation of the circle with centre at (h, k) and radius r.
2. Write down the equation of the circle with
   (*i*) centre at $(0, 0)$ and radius 5 units. (*ii*) centre at $(1, -2)$ and radius $\sqrt{3}$ units. (*iii*) centre at $(-2, -3)$ and radius 1 unit.
3. Find the centre and radius of the circle $x^2+y^2+2gx+2fy+c=0$.
4. (*i*) What is the condition for the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a circle.
   (*ii*) If $3x^2-ky^2-5x+2y-3=0$ represents a circle, find k.
5. Find the centre and radius of the circle:
   (*i*) $x^2+y^2+8x+4y-5=0$ (*ii*) $x^2+y^2-2x-6y+7=0$
   (*iii*) $x^2+y^2-3x-7y+4=0$. (*iv*) $2x^2+2y^2-4x+4y-5=0$.
   (*v*) $3x^2+3y^2-6x+27y-2=0$.
6. Find the equation of the circle with centre $(2, 1)$ and passing through origin.
7. Show that the line $x+y-3=0$ is a diameter of the circle $x^2+y^2-2x-4y-20=0$.
8. Find the centre of the circle, two of whose diameters are (*i*) $x+y=2$ and $2x+3y=5$ (*ii*) $x+y=2$ and $x-y=0$.
9. If one end of a diameter of a circle $x^2+y^2+x-6y-27=0$ is $(-5, -1)$, find its other end.
10. Find the equation of the circle (*i*) two of whose diameters are $x+y=6$ and $x+2y=4$ and whose radius is 10 units (*ii*) two of whose diameters are $x-y=2$ and $2x+y=7$ and which passes through the centre of the circle $x^2+y^2+4x-2y-1=0$.
11. Find the equation of the circle whose centre is same as the centre of the circle $x^2+y^2-6x-2y+4=0$ and which passes through the centre of the circle $x^2+y^2-14x-8y+5=0$.

12. Find the equation of the chord of the circle $x^2+y^2-4x-2y-20=0$ which is bisected at the point (2, 3).

13. Find the equation of the circle passing through the points
    (*i*) (5, 5), (–3, –1), (4, 6). (*ii*) (3, –4), (–2, 1), (–3, –1).
    (*iii*) (1, 5), (5, 7), (8, 6). (*iv*) (1, 1), (–2, 2), (–6, 0)
    (*v*) (3, 0), (1, –6), (4, –1).

14. Find the circumcentre and circumradius of the triangle formed by the vertices (5, 3), (1, 5), (3, –1).

15. Show that the following points are concyclic.
    (*i*) (1, 1), (–2, 2), (–2, –8), (–6, 0).
    (*ii*) (5, 5), (6, 4), (–1, –3),(–2, –2).

16. Find the equation of the circle with its centre on the line
    (*i*) $3x+4y=7$ and passing through the points (0, 4) and (4, 2)
    (*ii*) $4x+y=16$ and passing through the points (4, 1) and (6, 5).

17. A circle has its centre on the x–axis and passes through the points (5, 1) and (3, 4). Find its equation.

18. Find the equation of the circle passing through the points (2, –1) and (3, 2) and having (*i*) x-axis (*ii*) y-axis as a diameter

19. Show that the following circles touch each other and also find their point of contact.
    (*i*) $x^2+y^2-4x-10y+25=0$; $x^2+y^2+2x-2y-7=0$
    (*ii*) $x^2+y^2-5x+6y+15=0$; $x^2+y^2-2x+6y+6=0$
    (*iii*) $x^2+y^2+2x-4y-4=0$; $x^2+y^2-4x+4y+4=0$.

20. Find the equation of the circle having (x_1, y_1) and (x_2, y_2) as the extremities of a diameter.

21. Find the equation of the circle having (*i*) (–4, 5) and (3, 7) (*ii*) (5, 0) and (0, 5) as end points of a diameter.

22. Find the equation of a circle drawn on the line joining the centres of the circles $x^2+y^2+2x+6y-2=0$ and $x^2+y^2-4x-8y+5=0$ as the end points of a diameter

23. Find the points of intersection of the following circle with the given line:
    (*i*) $x^2+y^2-6x-2y+5=0$; $x-y-1=0$
    (*ii*) $x^2+y^2-x+3y-10=0$; $x+y+1=0$.

Answers

2. (*i*) $x^2+y^2-25=0$ (*ii*) $x^2+y^2-2x+4y+2=0$
(*iii*) $x^2+y^2+4x+6y+12=0$. 4. (*i*) $h=0$; $a=b$ (*ii*) -3.

5. (*i*) $(-4, -2)$; 5 (*ii*) $(1, 3)$; $\sqrt{3}$ (*iii*) $(\frac{3}{2}, \frac{7}{2})$; $\sqrt{\frac{21}{2}}$
(*iv*) $(1, -1)$; $\frac{3}{\sqrt{2}}$ (*v*) $(1, -\frac{9}{8})$; $\sqrt{\frac{263}{12}}$ 6. $x^2+y^2-4x-2y=0$

8. $(1, 1)$. 9. $(4, 7)$. 10. (*i*) $x^2+y^2-16x+4y-32=0$
(*ii*) $x^2+y^2-6x-2y-15=0$. 11. $x^2+y^2-6x-2y-15=0$.

12. $y-3=0$. 13. (*i*) $x^2+y^2-2x-4y-20=0$ (*ii*) $x^2+y^2-x+3y-10=0$
(*iii*) $x^2+y^2-10x-4y+4=0$ (*iv*) $x^2+y^2+4x+6y-12=0$
(*v*) $2x^2+2y^2-5x+11y-3=0$. 14. $(2, 2)$; $\sqrt{10}$

16. (*i*) $x^2+y^2-2x-2y-8=0$ (*ii*) $x^2+y^2-6x-8y+15=0$.

17. $2x^2+2y^2-x-47=0$. 18. (*i*) $x^2+y^2-8x+11=0$
(*ii*) $3x^2+3y^2-8y-23=0$. 19. (*i*) $(\frac{4}{5}, \frac{17}{8})$ (*ii*) $(3, -3)$ (*iii*) $(\frac{4}{5}, -\frac{3}{8})$

21. (*i*) $x^2+y^2+x-12y+23=0$ (*ii*) $x^2+y^2-5x-5y=0$.

22. $x^2+y^2-x-y-14=0$. 23. (*i*) $(4, 3)$; $(1, 0)$ (*ii*) $(3, -4)$, $(-2, 1)$

2.2 Equation of tangent—Length of tangent

1. Find the equation of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at any point (x_1, y_1) on it.
2. Find the equation of the tangent to the circle
   (*i*) $x^2+y^2=100$ at $(-6, 8)$. (*ii*) $x^2+y^2+4x+y+1=0$ at $(-3, 1)$
   (*iii*) $5(x^2+y^2)+4x+2y-60=0$ at $(1, 3)$.
   (*iv*) $x^2+y^2+8x+10y-105=0$ at $(1, 6)$.
3. Show that the line
   (*i*) $4x+5y=39$ touches the circle $x^2+y^2+6x-4y-28=0$.
   (*ii*) $x+3y-3=0$ touches the circle $x^2+y^2-4x+6y+3=0$.
4. Show that the line
   (*i*) $3x+y-15=0$ touches the circle $x^2+y^2-2x-4y-5=0$ and also find the coordinates of the point of contact.
   (*ii*) $4x-y-5=0$ is a tangent to the circle $x^2+y^2+4x-8y+3=0$ and also find the coordinates of the point of contact.
5. Find the value of k if
   (*i*) $x+ky-5=0$ touches the circle $x^2+y^2-2x+6y-6=0$.
   (*ii*) $x-2y+k=0$ touches the circle $x^2+y^2+3x-2y+2=0$.

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6. Find the equations of the tangents to the circle
   (*i*) $x^2+y^2+10x-2y+6=0$ which are parallel to the line $2x+y=7$.
   (*ii*) $x^2+y^2-8x+12=0$ which are parallel to the line $3x-4y+25=0$.
   (*iii*) $x^2+y^2-2x-4y-4=0$ which are perpendicular to the line $3x-4y+5=0$
   (*iv*) $x^2+y^2-4x+6y=23$ which are perpendicular to the line $8x+6y+251=0$.

7. Find the condition for the line
   (*i*) $y=mx+c$ to be a tangent to the circle $x^2+y^2=a^2$.
   (*ii*) $lx+my+n=0$ to be a tangent to the circle $(x-a)^2+(y-b)^2=r^2$.

8. Find the equation of the circle
   (*i*) with centre at $(-2, 4)$ and touching the line $4x-y=5$.
   (*ii*) with centre at (h, k) and touching the line $ax+by+c=0$.
   (*iii*) with its centre at the point $(0, -4)$ and touching the x-axis.
   (*iv*) passing through the point $(2, 5)$ and touching the y-axis at $(0, 1)$.

9. Find the length of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ from a point (x_1, y_1).

10. Find the length of the tangent drawn from the given point to the given circle:

(*i*) $x^2+y^2-2x+4y+1=0$;	$(1, 1)$
(*ii*) $2(x^2+y^2)+x-y+5=0$;	$(0, 0)$
(*iii*) $2x^2+2y^2+5x-3y+7=0$;	$(-1, 4)$
(*iv*) $2(x^2+y^2)-3x+4y+6=0$;	$(4, 1)$

11. Show that the lengths of the tangents from the point $(1, 5)$ to the circles $x^2+y^2+4x+2y-3=0$ and $x^2+y^2-6x+4y-3=0$ are equal.

12. Show that the lengths of the tangents from the point $(3, 2)$ to the circles $x^2+y^2+4x+2y-5=0$, $x^2+y^2+6x+y-9=0$ and $2(x^2+y^2)+7x-y+3=0$ are equal.

13. Find the equation of the locus of a point which moves such that the length of the tangent from it to the circle $x^2+y^2=16$ is 5 times its distance from the line $3x-4y+5=0$.

14. Show that the locus of a point, which moves such that it lengths of the tangents to the circles $x^2+y^2-7x+3y+1=$ and $2(x^2+y^2)-5x+6y+12=0$ are in the ratio 1:2, is a circl

15. A point moves such that the lengths of the tangents from i to the circles $x^2+y^2-8x+5=0$ and $x^2+y^2-4y+3=0$ an in the ratio 2:3. Show that the locus is a circle and find it centre and radius.

16. Find the equation of the circle such that the lengths of th tangents from the points $(-1, 0)$, $(0, 2)$ and $(-2, -1)$ are 1 $\sqrt{10}$, $3\sqrt{3}$ respectively.

Answers

2. (*i*) $3x-4y+50=0$ (*ii*) $2x-3y+9=0$ (*iii*) $7x+16y-55=$ (*iv*) $5x+11y-71=0$. 4. (*i*) $(4, 3)$ (*ii*) $(2, 3)$. 5. (*i*) $0, \frac{24}{7}$ (*ii*) $1,$
6. (*i*) $2x+y-1=0$; $2x+y+19=0$ (*ii*) $3x-4y-22=0$; $3x-4y-2=$ (*iii*) $4x+3y+5=0$; $4x+3y-25=0$ (*iv*) $3x-4y+12=0$; $3x-4y-48=0$. 7. (*i*) $c^2=a^2(1+m^2)$ (*ii*) $(la+mb+n)^2=r^2(l^2+m^2$
8. (*i*) $x^2+y^2+4x-8y+3=0$ (*ii*) $(a^2+b^2)\,[(x-h)^2+(y-k)^2]=$ $(ah+bk+c)^2$ (*iii*) $x^2+y^2+8y=0$ (*iv*) $x^2+y^2-10x-2y+1=$
10. (*i*) $\sqrt{5}$ (*ii*) $\sqrt{\frac{5}{2}}$ (*iii*) $2\sqrt{3}$ (*iv*) 4. 13. $8x^2-24xy+15y^2+30$ $-40y+41=0$. 15. $\left(\frac{36}{5}, \frac{-8}{5}\right)$; $\frac{\sqrt{1195}}{5}$.
16. $x^2+y^2-26x+12y-18=0$.

2,3 Radical Axis—Radical Centre

1. Define power of a point and obtain the power of the poin (x_1, y_1) w.r.t. the circle $x^2+y^2+2gx+2fy+c=0$.
2. State whether each of the points $(2. 2)$, $(-2, 4)$ and $(5, 6$ lies inside, on or outside the circle $x^2+y^2-4x-6y-3=0$
3. Define radical axis of two circles and find its equation. Als show that the radical axis is perpendicular to the line of centre
4. Find the radical axis of the circles:
   (*i*) $x^2+y^2+2x-2y+1=0$; $x^2+y^2-2x+y+1=0$
   (*ii*) $x^2+y^2-x+4y+1=0$; $2x^2+2y^2-x+2y=0$
   (*iii*) $5(x^2+y^2)-x+y-7=0$; $x^2+y^2+x=0$
   (*iv*) $2x^2+2y^2+3x+5y+9=0$; $3x^2+3y^2-3x-2y-1=0$
   (*v*) $(x+3)^2+(y+1)^2=9$; $(x+1)^2+(y+2)^2=10$

5. (*i*) If two circles intersect at P and Q, prove that the common chord PQ is their radical axis.
   (*ii*) Find the points of intersection of the circles $x^2+y^2-6x-2y+5=0$ and $x^2+y^2-8y-1=0$
6. If the circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x+6y+15=0$ touch each other, write down the equation of the common tangent.
7. Prove that the radical axes of three circles taken in pairs are concurrent.
8. Define radical centre of three circles.
9. Find the radical centre of the following three circles:
   (*i*) $x^2+y^2-6x-6y+4=0$; $x^2+y^2-2x-4y+3=0$; $x^2+y^2+2x+2y+1=0$.
   (*ii*) $x^2+y^2+4x+2y+1=0$; $2x^2+2y^2+8x+6y-3=0$; $x^2+y^2+6x-2y-3=0$
   (*iii*) $2(x^2+y^2)+7x-y+3=0$; $x^2+y^2+6x+y-9=0$; $x^2+y^2+4x+2y-5=0$. Also examine whether the radical centre lies inside or outside the given circles.
   (*iv*) $x^2+y^2+x+y-3=0$; $3(x^2+y^2)-1=0$; $2(x^2+y^2)+x-2=0$.

Answers

2. inside; outside; outside. 4. (*i*) $4x-3y=0$. (*ii*) $x-6y-2=0$. (*iii*) $6x-y+7=0$ (*iv*) $15x+19y+29=0$. (*v*) $2x-y+3=0$.
5. (4, 3); (1, 0) 6. $x-3=0$. 9. (*i*) $(\frac{1}{8}, \frac{1}{4})$ (*ii*) $(7, \frac{5}{2})$ (*iii*) (3, 2); outside. (*iv*) $(\frac{4}{3}, \frac{4}{3})$.

2.4 Orthogonal Circles

1. Define orthogonality of two circles and obtain the condition for the circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ to cut orthogonally.
2. Show that the following circles are orthogonal:
   (*i*) $x^2+y^2-2x-2y-7=0$; $x^2+y^2+4x+2y+1=0$
   (*ii*) $x^2+y^2-6x+8y+7=0$; $x^2+y^2+12x+14y+13=0$
   (*iii*) $2x^2+2y^2+6x+8y-3=0$; $x^2+y^2-5x-14y-34=0$
   (*iv*) $x^2+y^2+4x+1=0$; $5(x^2+y^2)+3x+14y+1=0$.
3. Find the value of k if the following circles are orthogonal:
   (*i*) $x^2+y^2+y=0$; $x^2+y^2+4x+2y+k=0$
   (*ii*) $x^2+y^2-4x-6y+11=0$; $x^2+y^2+kx-2y+3=0$.

4. Prove that (*i*) a circle with any point as centre and the length of the tangent from it to a given circle as radius cuts the given circle orthogonally (*ii*) a circle with any point on the radical axis of two circles as centre and the length of the tangent from the point to either of the two circles as radius cuts both the circles orthogonally (*iii*) the circle with the radical centre of three circles as centre and the length of the tangent from it to any one of the three circles as radius cuts the three circles orthogonally.

5. Find the equation of the circle with centre at
   (*i*) (0, 0) and cutting orthogonally the circle $x^2+y^2+12x+4y+10=0$
   (*ii*) (1, 1) and cutting orthogonally the circle $x^2+y^2=1$
   (*iii*) (1, 2) and cutting orthogonally the circle $x^2+y^2-6x+6y-2=0$.

6. Find the equation of the circle passing through origin and cutting orthogonally the circles (*i*) $x^2+y^2+8x+4y-6=0$ and $x^2+y^2+2x+6y+1=0$. (*ii*) $x^2+y^2-8y+12=0$ and $x^2+y^2-4x-6y-3=0$.

7. Find the equation of the circle passing through the point (1, –1) and cutting orthogonally the circles $x^2+y^2-2x+3y-7=0$ and $x^2+y^2+5x-5y+9=0$.

8. Find the equation of the circle which cuts orthogonally the circles (*i*) $x^2+y^2-4x-6y+11=0$ and $x^2+y^2-10x-4y+21=0$ and has its centre on $2x+3y-7=0$ (*ii*) $x^2+y^2+2x-2y-2=0$ and $x^2+y^2+6x-4y+4=0$ and has its centre on the line $4x-3y+3=0$. (*iii*) $x^2+y^2+7x+6y+11=0$ and $x^2+y^2-x+22y+3=0$ and has its centre on the line $x+y-5=0$.

9. Find the equation of the circle (*i*) passing through origin, having its centre on $x+y=4$ and cutting orthogonally the circle $x^2+y^2-4x+2y+4=0$. (*ii*) passing through (1, 2), having its centre on $2x+y-1=0$ and cutting orthogonally the circle $x^2+y^2-2x+1=0$. (*iii*) passing through the point (1, 1), having its centre on the line $x+2y+1=0$ and cutting orthogonally the circle $x^2+y^2+2x+2y+2=0$.

10. Find the equation of the circle passing through the point (–6, 0), having the length of the tangent from the point (1, 1) as $\sqrt{5}$ units and cutting orthogonally the circle $x^2+y^2-4x-6y-3=0$.

11. Find the equation of the circle cutting orthogonally the three circles (*i*) $x^2+y^2+7x-9y+29=0$; $x^2+y^2-2x+3y-7=0$; $x^2+y^2+5x-5y+9=0$; (*ii*) $x^2+y^2+y=0$; $x^2+y^2+4x+7=0$; $2(x^2+y^2)+3x+5y+9=0$. (*iii*) $x^2+y^2+2x+4y+1=0$; $x^2+y^2-2x+6y-3=0$; $2(x^2+y^2)+6x+8y-3=0$

Answers

3. (*i*) 1 (*ii*) -4. 5. (*i*) $x^2+y^2=10$ (*ii*) $x^2+y^2-2x-2y+1=0$. (*iii*) $x^2+y^2-2x-4y-4=0$ 6. (*i*) $x^2+y^2-2x+y=0$ (*ii*) $x^2+y^2+6x-3y=0$. 7. $x^2+y^2-16x-18y-4=0$. 8. (*i*) $x^2+y^2-4x-2y+3=0$ (*ii*) $x^2+y^2+6x+6y+2=0$ (*iii*) $x^2+y^2-6x-4y-44=0$

9. (*i*) $x^2+y^2-4x-4y=0$ (*ii*) $x^2+y^2-2y-1=0$
(*iii*) $x^2+y^2-2x+2y-2=0$

10. $x^2+y^2+6x-3y=0$ 11. (*i*) $x^2+y^2-16x-18y-4=0$ (*ii*) $x^2+y^2+4x+2y+1=0$. (*iii*) $x^2+y^2-5x-14y-34=0$.

CHAPTER 3

CONIC SECTION

3.1 Definition of conic

1. Define a conic, a parabola, an ellipse and a hyperbola.
2. Name the conics whose eccentricities are $\frac{1}{3}$, $\frac{\sqrt{5}}{2}$ and 1.
3. Find the equation of the parabola (*i*) with focus (1, 2) and directrix $x+y=4$ (*ii*) with focus (3, -4) and directrix $x-y+5=0$.
4. Find the equation of the ellipse (*i*) whose focus is (-2, 3), the corresponding directrix is $x+y=0$ and the eccentricity is $\frac{1}{2}$ (*ii*) whose focus is (1, -1), the corresponding directrix is $x-y+3=0$ and $e=\frac{1}{2}$.
5. Find the equation of the hyperbola
   (*i*) whose focus is (0, 3), the corresponding directrix is $x+y-1=0$ and $e=2$
   (*ii*) whose focus is (1, -1), the directrix is $x-y+1=0$ and $e=\sqrt{2}$.

Answers

2. Ellipse, hyperbola and parabola.
3. (*i*) $x^2-2xy+y^2+4x-6=0$ (*ii*) $x^2+2xy+y^2-22x+26y+25=0$
4. (*i*) $7x^2-2xy+7y^2+32x-48y+104=0$
   (*ii*) $7x^2+2xy+7y^2-22x+22y+7=0$.
5. (*i*) $x^2+4xy+y^2-4x+2y-7=0$ (*ii*) $2xy-4x+4y+1=0$.

3.2 Standard forms of parabola, ellipse and hyperbola

1. (*a*) Derive the equation of the *parabola* in the form $y^2=4a$
   (*b*) Write down the focus, directrix, vertex and axis of th parabola $y^2=4ax$.
   (*c*) Define latus rectum of the parabola $y^2=4ax$ and obtai its length and end points.
   (*d*) Find the focus, directrix, vertex, axis, the end points o latus rectum and the length of latus rectum of th following parabolas:
   (*i*) $y^2=12x$ (*ii*) $y^2=16x$ (*iii*) $y^2+9x=0$ (*iv*) $y^2+x=$
2. (*a*) Derive the equation of the *ellipse* in the form
   $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$$
   (*b*) Prove that the sum of the focal distances of any poi of an ellipse is equal to the length of its major axis.
   (*c*) Write down the centre, vertices, lengths of major an minor axes, foci and directrices of the ellipse
   $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b.$$
   (*d*) Define latus rectum; find the coordinates of the en points of letera recta and the length of a latus rectu of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$.
   (*e*) Find the centre, vertices, lengths of major and mino axes, the length of LR, eccentricity, foci, directrices an the end points of LR of the following ellipses:
   (*i*) $\frac{x^2}{9}+\frac{y^2}{4}=1$ (*ii*) $\frac{x^2}{100}+\frac{y^2}{36}=1$ (*iii*) $\frac{x^2}{16}+\frac{y^2}{9}=1$
   (*iv*) $x^2+64y^2=64$

3. (*a*) Derive the equation of the *hyperbola* in the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(*b*) Prove that the difference between the focal distances of any point on a hyperbola is equal to the length of transverse axis.

(*c*) Write down the centre, vertices, lengths of transverse and conjugate axes, foci and directrices of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(*d*) Define latus rectum; find the coordinates of the end points of latera recta and the length of a latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(*e*) Find the centre, vertices, lengths of transverse and conjugate axes, the length of LR, eccentricity, foci, directrices and the end points of the latera recta of the following hyperbolas:

(*i*) $\frac{x^2}{625}-\frac{y^2}{144}=1$ (*ii*) $\frac{x^2}{9}-\frac{y^2}{4}=1$ (*iii*) $\frac{x^2}{16}-\frac{y^2}{9}=1$

(*iv*) $x^2-y^2=4$ (*v*) $36x^2-25y^2=900$ (*vi*) $\frac{9x^2}{4}-\frac{3y^2}{32}=1$.

4. (*a*) The length of major axis of an *ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 32 and one of its foci is (8, 0). Find its eccentricity.

(*b*) Find the '*e*' of an ellipse whose length of LR is equal to half of its length of major axis.

(*c*) Find the distance between foci, the distance between directrices and the length of LR of the ellipse

(*i*) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (*ii*) $4x^2+9y^2=36$.

(*d*) The distance between the foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 10. If $e=1/2$ find the length of its major axis.

(*e*) The distance between foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 50. If the length of semi-major axis is 150, find *e*.

(*f*) Find the '*e*' of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if the distance between directrices is 3 times the distance between foci.

5. (*a*) If one of the foci of a *hyperbola* $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $(8\sqrt{2}, 0)$ and $e=2\sqrt{2}$, find the length of transverse axis.

(*b*) The distance between the directrices of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 25. If $e=2$, find the lengths of the transverse and conjugate axes.

(*c*) The distance between foci of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 120. If the transverse axis is 40, find *e*.

(*d*) Find '*e*' of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if (*i*) the distance between foci is 4 times the distance between the directrices. (*ii*) the length of LR is 1/3 of its transverse axis.

(*e*) Find the distance between foci, distance between directrices and length of LR of the hyperbola:

(*i*) $25x-144y^2=900$ (*ii*) $x^2-8y^2=32$

(*iii*) $\frac{9x^2}{4}-\frac{3y^2}{32}=1$ (*iv*) $x^2-16y^2=1$.

Answers

1. (*d*)

	Focus	Directrix	Vertex	Axis	End points of L.R.	Length of LR
(*i*)	(3, 0)	$x=-3$	(0, 0)	x-axis	$(3, \pm 6)$	12
(*ii*)	(4, 0)	$x=-4$	(0, 0)	*x*-axis	$(4, \pm 8)$	16
(*iii*)	$(-\frac{9}{4}, 0)$	$x=\frac{9}{4}$	(0, 0)	*x*-axis	$(-\frac{9}{4}, \mp \frac{9}{2})$	9
(*iv*)	$(-\frac{1}{4}, 0)$	$x=\frac{1}{4}$	(0, 0)	*x*-axis	$(-\frac{1}{4}, \mp \frac{1}{2})$	1

The answers of 2 (*e*) and 3 (*e*) are given in the order: centre, vertices, lengths of axes, length of LR, *e*, foci, directrices and the end points of LR.

2. (*e*) (*i*) (0, 0); $(\pm 3, 0), (0, \pm 2)$. 6; 4; $\frac{8}{3}$; $\frac{\sqrt{5}}{3}$; $(\pm\sqrt{5}, 0)$; $x=\pm\frac{9}{\sqrt{5}}$; $(\pm\sqrt{5}, \pm\frac{4}{3})$ (*ii*) (0, 0); $(\pm 10, 0), (0, \pm 6)$; 20; 12; $\frac{36}{5}$; $\frac{4}{5}$; $(\pm 8, 0)$; $x=\pm\frac{25}{2}$; $(\pm 8, \pm\frac{18}{5})$

(*iii*) $(0, 0)$; $(\pm 4, 0), (0, \pm 3)$; 8; 6; $\frac{9}{2}$; $\frac{\sqrt{7}}{4}$; $(\pm\sqrt{7}, 0)$; $x=\pm\frac{16}{\sqrt{7}}$; $(\pm\sqrt{7}, \pm\frac{9}{4})$ (*iv*) $(0, 0)$; $(\pm 8, 0), (0, \pm 1)$; 16; 2; $\frac{1}{4}$; $\frac{\sqrt{63}}{8}$; $(\pm\sqrt{63}, 0)$; $x=\pm\frac{64}{\sqrt{63}}$; $(\pm\sqrt{63}, \pm\frac{1}{8})$.

3. (*e*) (*i*) $(0, 0)$; $(\pm 25, 0)$; 50; 24; $\frac{288}{25}$; $\frac{\sqrt{769}}{25}$; $(\pm\sqrt{769}, 0)$; $x=\pm\frac{625}{\sqrt{769}}$; $(\pm\sqrt{769}, \pm\frac{144}{25})$. (*ii*) $(0, 0)$; $(\pm 3, 0)$; 6; 4; $\frac{8}{3}$; $\frac{\sqrt{13}}{3}$; $(\pm\sqrt{13}, 0)$; $x=\pm\frac{9}{\sqrt{13}}$; $(\pm\sqrt{13}, \pm\frac{4}{3})$.

(*iii*) $(0, 0)$; $(\pm 4, 0)$; 8; 6; $\frac{9}{2}$; $\frac{5}{4}$; $(\pm 5, 0)$; $x=\pm\frac{16}{5}$; $(\pm 5, \pm\frac{9}{4})$ (*iv*) $(0, 0)$; $(\pm 2, 0)$; 4; 4; 4; $\sqrt{2}$; $(\pm 2\sqrt{2}, 0)$; $x=\pm\sqrt{2}$, $(\pm 2\sqrt{2}, \pm 2)$ (*v*) $(0, 0)$; $(\pm 5, 0)$; 10; 12; $\frac{72}{5}$; $\frac{\sqrt{61}}{5}$; $(\pm\sqrt{61}, 0)$; $x=\pm\frac{25}{\sqrt{61}}$; $(\pm\sqrt{61}, \pm\frac{36}{5})$ (*vi*) $(0, 0)$; $(\pm\frac{8}{3}, 0)$; $\frac{4}{3}$; $\frac{2\sqrt{32}}{\sqrt{3}}$; 32; 5; $(\pm\frac{10}{3}, 0)$; $x=\pm\frac{2}{15}$; $(\pm\frac{10}{3}, \pm 16)$.

4. (*a*) $\frac{1}{2}$. (*b*) $\frac{1}{\sqrt{2}}$ (*c*) (*i*) 6; $\frac{50}{3}$; $\frac{32}{5}$ (*ii*) $2\sqrt{5}$; $\frac{18}{\sqrt{5}}$; $\frac{8}{3}$. (*d*) 20. (*e*) $\frac{1}{6}$. (*f*) $\frac{1}{\sqrt{3}}$. 5. (*a*) 8. (*b*) 50; $50\sqrt{3}$. (*c*) 3. (*d*) (*i*) 2. (*ii*) $\frac{2}{\sqrt{3}}$. (*e*) (*i*) 13; $\frac{144}{13}$; $\frac{25}{12}$ (*ii*) 12; $\frac{32}{3}$; $\sqrt{2}$ (*iii*) $\frac{20}{3}$; $\frac{4}{15}$; 32 (*iv*) $\frac{\sqrt{17}}{2}$; $\frac{8}{\sqrt{17}}$; $\frac{1}{8}$.

3.3 Other forms of parabola, ellipse and hyperbola

1. (*a*) Write down the vertex, axis, focus, directrix, the end points and the length of latus rectum of the following (*other forms of*) *parabolas:*

   (*i*) $x^2=4ay$ (*ii*) $(y-k)^2=4a(x-h)$ (*iii*) $(x-h)^2=4a(y-k)$.

(*b*) Find the focus. directrix, vertex, axis, the end points of latus rectum and the length of latus rectum of the followin parabolas

(*i*) $8x^2=y$ (*ii*) $x^2=8y$ (*iii*) $x^2+6y=0$
(*iv*) $(y+3)^2=-4(x+2)$ (*v*) $(y-2)^2=16(x+1)$
(*vi*) $(x-2)^2=32(y+1)$ (*vii*) $(x+1)^2+4(y-3)=0$
(*viii*) $y^2=-8(x+2)$ (*ix*) $(y-2)^2=40x$
(*x*) $x^2=4(y-4)$ (*xi*) $(x+1)^2=-16y$.

(*c*) Find the focus, directrix, vertex, axis, the end points of latus rectum and the length of latus rectum of the follow-ing parabolas:

(*i*) $x^2-2x+4y+8=0$ (*ii*) $x^2-2x-8y-23=0$
(*iii*) $y^2+x-y+2=0$ (*iv*) $x^2+4x+2y-1=0$
(*v*) $x^2+8x+12y+4=0$ (*vi*) $y^2+8x-4y-4=0$
(*vii*) $y^2-6x+18=0$ (*viii*) $x^2+4y+4=0$

2. (*a*) Write down the centre, vertices, lengths of major and minor axes, the length of latus rectum, eccentricity, foci, directrices and the end points of latera recta of the (*othe* *forms of*) *ellipses:*

(*i*) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1,\ a<b$ (*ii*) $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1,\ a>b$

(*iii*) $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1,\ a<b.$

(*b*) (*i*) $\dfrac{x^2}{25}+\dfrac{y^2}{256}=1$ (*ii*) $\dfrac{x^2}{36}+\dfrac{y^2}{64}=1$ (*iii*) $\dfrac{x^2}{36}+\dfrac{y^2}{49}=1$

(*iv*) $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$ (*v*) $9x^2+4y^2=20.$

(*c*) (*i*) $\dfrac{(x+1)^2}{64}+\dfrac{(y+3)^2}{36}=1$ (*ii*) $\dfrac{(x+3)^2}{25}+\dfrac{(y-1)^2}{16}=1$

(*iii*) $\dfrac{(x-2)^2}{16}+\dfrac{(y-3)^2}{9}=1$ (*iv*) $\dfrac{(x-2)^2}{16}+\dfrac{(y-1)^2}{25}=1$

(*v*) $4x^2+9y^2+16x-18y-11=0$
(*vi*) $3x^2+4y^2+6x-8y-5=0$
(*vii*) $9x^2+16y^2+18x-64y-71=0$
(*viii*) $25x^2+16y^2+150x-128y-1119=0.$

3. (*a*) **Write down the centre, vertices, lengths of transverse and conjugate axes, the length of latus rectum, eccentricity, foci, directrices and the end points of latera recta of the (*other forms of*) *hyperbolas*:**

(*i*) $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ (*ii*) $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

(*iii*) $\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$

(*b*) (*i*) $9y^2-4x^2=36$ (*ii*) $y^2-x^2=1$ (*iii*) $\frac{y^2}{16}-\frac{x^2}{25}=1.$

(*c*) (*i*) $\frac{(x-1)^2}{9}-\frac{(y-2)^2}{16}=1$ (*ii*) $(x+2)^2-4(y+2)^2=4$

(*iii*) $\frac{(y-4)^2}{100}-\frac{(x+3)^2}{64}=1$ (*iv*) $9x^2-16y^2+72x-32y-16=0$

(*v*) $9x^2-18y^2+54x-36y+79=0$

(*vi*) $x^2-4y^2-2x+16y-40=0$

(*vii*) $3y^2-4x^2-8x-24y-40=0$

(*viii*) $9x^2-4y^2-18x-16y+29=0.$

Answers

1.		Focus	Directrix	Vertex	Axis	End points of L.R.	Length of L.R.
(*b*)	(*i*)	$(0, \frac{1}{32})$	$y=-\frac{1}{32}$	$(0, 0)$	y-axis	$(\pm\frac{1}{16}, \frac{1}{32})$	$\frac{1}{8}$
	(*ii*)	$(0, 2)$	$y=-2$	$(0, 0)$	y-axis	$(\pm 4, 2)$	8
	(*iii*)	$(0, -\frac{3}{2})$	$y=\frac{3}{2}$	$(0, 0)$	y-axis	$(\mp 3, -\frac{3}{2})$	6
	(*iv*)	$(-3, -3)$	$x=-1$	$(-2, -3)$	$y=-3$	$(-3, -5)$; $(-3, -1)$	4
	(*v*)	$(3, 2)$	$x=-5$	$(-1, 2)$	$y=2$	$(3, 10)$; $(3, -6)$	16
	(*vi*)	$(2, 7)$	$y=-9$	$(2, -1)$	$x=2$	$(18, 7)$; $(-14, 7)$	32
	(*vii*)	$(-1, 2)$	$y=4$	$(-1, 3)$	$x=-1$	$(1, 2)$; $(-3, 2)$	4
	(*viii*)	$(-4, 0)$	$x=0$	$(-2, 0)$	$y=0$	$(-4, \mp 4)$	8
	(*ix*)	$(10, 2)$	$x=-10$	$(0, 2)$	$y=2$	$(10, 22)$; $(10, -18)$	40
	(*x*)	$(0, 5)$	$y=3$	$(0, 4)$	$x=0$	$(\pm 2, 5)$	4
	(*xi*)	$(-1, -4)$	$y=4$	$(-1, 0)$	$x=1$	$(-9, -4)$; $(7, -4)$	16
(*c*)	(*i*)	$(1, -\frac{11}{4})$	$y=-\frac{3}{4}$	$(1, -\frac{7}{4})$	$x=-1$	$(-1, -\frac{11}{4})$; $(3, -\frac{11}{4})$	4
	(*ii*)	$(1, -1)$	$y=-5$	$(1, -3)$	$x=1$	$(5, -1)$; $(-3, -1)$	8

(*iii*)	$(-2, \frac{1}{2})$	$x=-\frac{3}{2}$	$(-\frac{7}{4}, \frac{1}{2})$	$y=\frac{1}{2}$	$(-2, 0), (-2, 1)$	1
(*iv*)	$(-2, 2)$	$y=3$	$(-2, \frac{5}{2})$	$x=-2$	$(-3, 2); (-1, 2)$	2
(*v*)	$(-4, -2)$	$y=4$	$(-4, 1)$	$x=-4$	$(-10, -2); (2, -2)$	12
(*vi*)	$(-1, 2)$	$x=3$	$(1, 2)$	$y=2$	$(-1, 6); (-1, -2)$	8
(*vii*)	$(\frac{9}{2}, 0)$	$x=\frac{3}{2}$	$(3, 0)$	$y=0$	$(\frac{9}{2}, \pm 3)$	6
(*viii*)	$(0, -2)$	$y=0$	$(0, -1)$	$x=0$	$(\mp 2, -2)$	4

The answers of (2) (*b*); (2) (*c*); (3) *b*) and (3) (*c*) are given in the order: centre; vertices; lengths of axes, length of LR, foci, directrices and the end points of LR

2. (*b*) (*i*) $(0, 0)$; $(\pm 5, 0), (0, \pm 16)$; 32; 10; $\frac{25}{8}$; $\frac{\sqrt{231}}{16}$;

$(0, \pm \sqrt{231})$; $y=\pm \frac{256}{\sqrt{231}}$; $(\pm \frac{25}{4}, \pm \sqrt{231})$

(*ii*) $(0, 0)$; $(\pm 6, 0), (0, \pm 8)$; 16; 12; 9; $\frac{\sqrt{7}}{4}$; $(0, \pm 2\sqrt{7})$

$y=\pm \frac{32}{\sqrt{7}}$; $(\pm \frac{9}{2}, \pm 2\sqrt{7})$

(*iii*) $(0, 0)$; $(\pm 6, 0), (0, \pm 7)$; 14; 12; $\frac{72}{7}$; $\frac{\sqrt{13}}{7}$;

$(0, \pm \sqrt{13})$; $y=\pm \frac{49}{\sqrt{13}}$; $(\pm \frac{36}{7}, \pm \sqrt{13})$

(*iv*) $(0, 0)$; $(\pm 4, 0), (0, \pm 5)$; 10; 8; $\frac{32}{5}$; $\frac{3}{5}$; $(0, \pm 3)$; $y=\pm \frac{25}{3}$; $(\pm \frac{16}{5}, \pm 3)$

(*v*) $(0, 0)$; $(\pm \frac{2\sqrt{5}}{3}, 0), (0, \pm \sqrt{5})$; $2\sqrt{5}$; $\frac{4\sqrt{5}}{3}$; $\frac{40}{9\sqrt{5}}$

$\frac{\sqrt{5}}{3}$; $(0, \pm \frac{5}{3})$; $y=\pm 3$; $(\pm \frac{20}{9\sqrt{5}}, \pm \frac{5}{3})$.

(*c*) (*i*) $(-1, -3)$; $(-1, 3), (-1, -9), (7, -3), (-9, -3)$; 16; 12; $\frac{\sqrt{7}}{4}$; $(-1 \pm 2\sqrt{7}, -3)$; $x=-1 \pm \frac{32}{\sqrt{7}}$; $(\pm 2\sqrt{7} -1,$
$-3 \pm$

(*ii*) $(-3, 1)$; $(-8, 1), (2, 1), (-3, 5), (-3, -3)$; 10; 8; $\frac{3}{5}$; $(0, 1), (-6, 1)$; $x=-3 \pm \frac{25}{3}$; $(-3 \pm 3, 1 \pm \frac{16}{5})$

(*iii*) $(2, 3)$; $(6, 3), (-2, 3), (2, 6), (2, 0)$; 8; 6; $\frac{9}{2}$; $\frac{\sqrt{7}}{4}$;

$(2 \pm \sqrt{7}, 3)$; $x=2 \pm \frac{\sqrt{7}}{16}$; $(2 \pm \sqrt{7}, 3 \pm \frac{9}{4})$.

(*iv*) $(2, 1)$; $(2, 6), (2, -4), (6, 1), (-2, 1)$; 10; 8; $\frac{32}{5}$; $\frac{3}{5}$; $(2, 4), (2, -2)$; $y=\frac{28}{3}$. $y=-\frac{22}{3}$; $(2 \pm \frac{16}{5}, 1 \pm 3)$

(*v*) $(-2, 1)$; $(1, 1), (-5, 1), (-2, 3), (-2, -1)$; 6; 4; $\frac{8}{3}$; $\frac{\sqrt{5}}{3}$; $(-2\pm\sqrt{5}, 1)$; $x=-2 \pm \frac{9}{\sqrt{5}}$; $(-2 \pm \sqrt{5}, 1 \pm \frac{4}{3})$.

(*vi*) $(-1, 1)$; $(-1 \pm 2, 1)$ $(-1, 1 \pm \sqrt{3})$; 4; $2\sqrt{3}$; 3; $\frac{1}{2}$; $(0, 1), (-2, 1)$; $x=3$, $x=-5$; $(0, \frac{5}{2}), (0, -\frac{1}{2}), (-2, \frac{5}{2}), (-2, -\frac{1}{2})$.

(*vii*) $(-1, 2)$; $(3, 2), (-5, 2), (-1, 5), (-1, -1)$; 8; 6; $\frac{9}{2}$; $\frac{\sqrt{7}}{4}$; $(-1 \pm \sqrt{7}, 2)$; $x=-1 \pm \frac{16}{\sqrt{7}}$; $(-1 \pm \sqrt{7}, 2 \pm \frac{9}{4})$.

(*viii*) $(-3, 4)$; $(-3, -6), (-3, 14), (-11, 4), (5, 4)$; 20; 16; $\frac{64}{5}$; $\frac{3}{5}$; $(-3, -2), (-3, 10)$; $y=-\frac{38}{3}$, $y=\frac{62}{3}$; $(-3 \pm \frac{32}{5}, 4 \pm 6)$.

3. (*b*) (*i*) $(0, 0)$; $(0, \pm 2)$; 4; 6; 9; $\frac{\sqrt{13}}{2}$; $(0, \pm \sqrt{13})$; $y=\pm \frac{4}{\sqrt{13}}$; $(\pm \frac{9}{2}, \pm \sqrt{13})$.

(*ii*) $(0, 0)$; $(0, \pm 1)$; 2; 2; 2; $\sqrt{2}$; $(0, \pm \sqrt{2})$; $y=\pm \frac{1}{\sqrt{2}}$; $(\pm 1, \pm \sqrt{2})$.

(*iii*) $(0, 0)$; $(0, \pm 4)$; 8; 10, $\frac{25}{2}$; $\frac{\sqrt{41}}{4}$; $(0, \pm \sqrt{41})$; $y=\pm \frac{16}{\sqrt{41}}$; $(\pm \frac{25}{4}, \pm \sqrt{41})$.

(*c*) (*i*) $(1, 2)$; $(4, 2), (-2, 2)$; 6; 8; $\frac{32}{3}$; $\frac{5}{3}$; $(6, 2), (-4, 2)$; $x=1 \pm \frac{9}{5}$; $(1 \pm 5, 2 \pm \frac{16}{3})$.

(*ii*) $(-2, 2)$; $(0, -2), (-4, -2)$; 4; 2; 1; $\frac{\sqrt{5}}{2}$; $(-2 \pm \sqrt{5}, -2)$; $x=-2 \pm \frac{4}{\sqrt{5}}$; $(-2 \pm \sqrt{5}, -2 \pm \frac{1}{2})$.

(*iii*) $(-3, 4)$; $(-3, 14), (-3, -6)$; 20; 16; $\frac{64}{5}$; $\frac{\sqrt{41}}{5}$; $(-3, 4 \pm 2\sqrt{41})$, $y=4 \pm \frac{50}{\sqrt{41}}$; $(-3 \pm \frac{32}{5}, 4 \pm 2\sqrt{41})$.

(*iv*) $(-4, -1)$; $(-8, -1), (0, -1)$; 8; 6; $\frac{9}{2}$; $\frac{5}{4}$; $(1, -1), (-9, -1)$; $x=\pm \frac{16}{5} -4$; $(-4 \pm 5, -1 \pm \frac{9}{4})$.

(*v*) $(-3, -1)$; $\left(-3, -1 \pm \frac{2\sqrt{2}}{3}\right)$; $\frac{4\sqrt{2}}{3}$; $\frac{8}{3}$; $\frac{8\sqrt{2}}{3}$; $\sqrt{3}$;

$\left(-3, -1 \pm \frac{2\sqrt{6}}{3}\right)$; $y=-1 \pm \frac{2\sqrt{6}}{9}$; $\left(-3 \pm \frac{4\sqrt{2}}{3}, -1 \pm \frac{2\sqrt{6}}{3}\right)$.

(*vi*) $(1, 2)$; $(6, 2)$, $(-4, 2)$; 10; 5; $\frac{5}{2}$; $\frac{\sqrt{5}}{2}$; $\left(1 \pm \frac{5\sqrt{5}}{2}, 2\right)$;

$x=1 \pm 2\sqrt{5}$; $\left(1 \pm \frac{5\sqrt{5}}{2}, 2 \pm \frac{5}{4}\right)$.

(*vii*) $(-1, 4)$; $(-1, 4 \pm 2\sqrt{7})$; $4\sqrt{7}$, $2\sqrt{21}$; $3\sqrt{7}$; $\frac{\sqrt{7}}{2}$;

$(-1, 4 \pm 7)$; $y=0$, $y=8$; $\left(-1 \pm \frac{3\sqrt{7}}{2}, 4 \pm 7\right)$.

(*viii*) $(1, -2)$; $(1, 1)$, $(1, -5)$; 6; 4: $\frac{8}{3}$; $\frac{\sqrt{13}}{3}$; $(1, -2 \pm \sqrt{13})$;

$y=-2 \pm \frac{9}{\sqrt{13}}$; $\left(1 \pm \frac{4}{3}, -2 \pm \sqrt{13}\right)$.

3.4 Determination of parabola, ellipse and hyperbola

1. (*a*) Find the equation of the parabola given that:
   (*i*) its focus is $(-4, 0)$ and directrix is $x=4$
   (*ii*) its focus is $(0, 3)$ and directrix is $y=-3$
   (*iii*) its vertex is $(0, 0)$ and focus is $(-2, 0)$
   (*iv*) its vertex is $(0, 0)$ and focus is $(0, 8)$

   (*b*) Find the equation of the parabola given that:
   (*i*) directrix is $x=3$ and focus is $(4, 0)$
   (*ii*) directrix is $x=1$ and focus is $(2, 0)$
   (*iii*) directrix is $y=1$ and focus is $(0, 0)$
   (*iv*) directrix is $y=-2$ and focus is $(0, 1)$
   (*v*) vertex is $(0, 0)$, axis is y-axis and it passes through $(\frac{1}{2}, 2)$
   (*vi*) vertex is $(0, 0)$, axis is x-axis and it passes through $(1, -4)$
   (*vii*) vertex is $(-2, 1)$ and directrix is $x=1$
   (*viii*) vertex is $(2, 5)$ and directrix is $x=3$
   (*ix*) vertex is $(-3, 2)$ and directrix is $y=4$.

   (*c*) Find the equation of the parabola if
   (*i*) its focus is $(2, 3)$ and vertex is $(-2, 3)$

(*ii*) its focus is $(-3, 2)$ and vertex is $(-2, 2)$
(*iii*) its focus is $(4, 1)$ and vertex is $(4, -5)$
(*iv*) its focus is $(-1, 1)$ and vertex is $(-1, 4)$
(*v*) its focus is $(2, -3)$ and vertex is $(2, -1)$

Also find the directrix and axis.

(*d*) Find the equations of parabolas having the latus rectum joining the points

(*i*) $(-3, 1)$ and $(1, 1)$
(*ii*) $(2, 2)$ and $(3, 2)$
(*iii*) $(2, 1)$ and $(2, 3)$
(*iv*) $(-1, 1)$ and $(-1, 0)$

(*e*) Find the equations of the parabolas with

(*i*) directrix $x+2=0$, axis $y=3$ and length of latus rectum$=6$
(*ii*) directrix $x=4$, axis $y+2=0$ and length of latus rectum$=4$
(*iii*) directrix $y+1=0$, axis $x=2$ and length of latus rectum$=8$
(*iv*) directrix $y=3$, axis $x+1=0$ and length of latus rectum$=16$.

2. (*a*) Find the equation of the ellipse in the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b, \text{ if}$$

(*i*) a focus is $(3, 0)$ and $e=\frac{3}{5}$
(*ii*) a directrix is $x=4$ and $e=\frac{1}{4}$
(*iii*) distance between foci$=3$ and $e=\frac{1}{2}$
(*iv*) distance between foci$=4$ and $e=\frac{1}{4}$
(*v*) distance between directrices$=10\sqrt{2}$ and $e=1/\sqrt{2}$
(*vi*) passing through $(2, 1)$ and $e=\frac{1}{2}$
(*vii*) the foci are $(\pm\sqrt{2}, 0)$ and $e=\frac{1}{4}$.

(*b*) Find the equation of the ellipse referred to its axes as coordinate axes (*x*-axis as major axis) if

(*i*) the distance between foci$=8$ and distance between directrices$=32$
(*ii*) the distance between foci$=4$ and distance between directrices$=16$
(*iii*) length of LR$=8$ and distance between foci$=2\sqrt{21}$
(*iv*) length of LR$=\frac{8}{3}$ and distance between foci$=2\sqrt{5}$
(*v*) length of LR$=8$, e$=1/\sqrt{2}$
(*vi*) length of LR$=\frac{8}{3}$, $e=\sqrt{5}/3$

(*vii*) length of LR=10 and the length of minor axis i
equal to the distance between foci.

(*viii*) length of minor axis=10 and the distance betwee
foci=$5\sqrt{2}$

(*c*) Find the equation of the ellipse whose centre is the origin, whose axes are coordinate axes and which passes through (*i*) (–2, 2) and (3, –1) (*ii*) (1, 4) and (–6, 1).

3. (*a*) Find the equation of the hyperbola in the form
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if

(*i*) a focus is (4, 0) and $e=2$

(*ii*) a directrix is $x=6$ and $e=\sqrt{2}$

(*iii*) distance between foci=12 and $e=8$

(*iv*) distance between foci=12 and $e=2$

(*v*) distance between directrices=20 and $e=2$

(*vi*) distance between directrices=12 and $e=3$

(*vii*) conjugate axis is 7 and it passes through (3, –2)

(*viii*) vertices are (2, 0) and (–2, 0) and foci are at (3, 0
and (–3, 0).

(*b*) Find the equation of the hyperbola referred to its axe as axes of coordinates (*x*-axis as transverse axis), give that

(*i*) the distance between foci=24 and distance betwee
directrices=6

(*ii*) the distance between foci=6 and distance betwee
directrices=$\frac{8}{3}$

(*iii*) the distance between foci=10, length of LR=$\frac{9}{2}$

(*iv*) the distance between foci=$2\sqrt{5}$, length of LR=$\frac{4\sqrt{}}{3}$

(*v*) $e=\sqrt{2}$ and length of LR=$8\sqrt{2}$

(*vi*) $e=5$ and length of LR=32

(*vii*) length of conjugate axis=5 and distance betwee
foci=26

(*viii*) length of conjugate axis=6 and distance betwe
foci=8

(*c*) Find the equation of the hyperbola with axes as th coordinate axes and passing through
(*i*) (3, –2) and (2, –1) (*ii*) (2, –2) and $e=2$.

4. **What are the parametric equations of standard forms of parabola, ellipse and hyperbola.**

Answers

1. (*a*) (*i*) $y^2=-16x$ (*ii*) $x^2=12y$ (*iii*) $y^2=-8x$ (*iv*) $x^2=32y$
   (*b*) (*i*) $y^2=2x-7$ (*ii*) $y^2=2x-3$ (*iii*) $x^2=-2y+1$ (*iv*) $x^2=6y+3$ (*v*) $8x^2=y$ (*vi*) $y^2=16x$ (*vii*) $(y-1)^2=-12x-24$ (*viii*) $(y-5)^2=-4(x-2)$ (*ix*) $(x+3)^2=-8(y-2)$.
   (*c*) (*i*) $(y-3)^2=16(x+2)$; $x=-6$; $y=3$ (*ii*) $(y-2)^2=-4(x+2)$; $x=-1$, $y=2$ (*iii*) $(x-4)^2=24(y+5)$; $y=-11$; $x=4$ (*iv*) $(x+1)^2=-12(y-4)$; $y=7$; $x=-1$ (*v*) $(x-2)^2=-8(y+1)$; $y=1$; $x=2$.
   (*d*) (*i*) $(x+1)^2=-4(y-2)$; $(x+1)^2=4y$ (*ii*) $(x-\frac{5}{2})^2=y-\frac{7}{4}$; $(x-\frac{5}{2})^2=-y+\frac{9}{4}$ (*iii*) $(y-2)^2=2(x-\frac{3}{2})$; $(y-2)^2=-2(x-\frac{5}{2})$ (*iv*) $(y-\frac{1}{2})^2=-(x+\frac{3}{4})$; $(y-\frac{1}{2})^2=(x+\frac{5}{4})$.
   (*e*) (*i*) $(y-3)^2=6(x+\frac{1}{2})$; $(y-3)^2=-6(x+\frac{7}{2})$ (*ii*) $(y+2)^2=4(x-5)$; $(y+2)^2=-4(x-3)$ (*iii*) $(x-2)^2=8(y-1)$; $(x-2)^2=-8(y+3)$ (*iv*) $(x+1)^2=16(y-7)$; $(x+1)^2=-16(y+1)$.

2. (*a*) (*i*) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (*ii*) $x^2+\frac{y^2}{15/16}=1$ (*iii*) $\frac{x^2}{9}+\frac{y^2}{27/4}=1$
   (*iv*) $\frac{x^2}{64}+\frac{y^2}{60}=1$ (*v*) $\frac{x^2}{25}+\frac{y^2}{25/2}=1$ (*vi*) $\frac{x^2}{16/3}+\frac{y^2}{4}=1$
   (*vii*) $\frac{x^2}{32}+\frac{y^2}{30}=1$.
   (*b*) (*i*) $\frac{x^2}{64}+\frac{y^2}{48}=1$ (*ii*) $\frac{x^2}{16}+\frac{y^2}{12}=1$ (*iii*) $\frac{x^2}{49}+\frac{y^2}{28}=1$
   (*iv*) $\frac{x^2}{9}+\frac{y^2}{4}=1$ (*v*) $\frac{x^2}{64}+\frac{y^2}{32}=1$ (*vi*) $x^2+\frac{y^2}{4/9}=1$
   (*vii*) $\frac{x^2}{100}+\frac{y^2}{50}=1$ (*viii*) $\frac{x^2}{75/2}+\frac{y^2}{25}=1$.
   (*c*) (*i*) $\frac{x^2}{32/3}+\frac{y^2}{32/5}=1$ (*ii*) $\frac{x^2}{115/3}+\frac{y^2}{115/7}=1$.

3. (*a*) (*i*) $\frac{x^2}{4}-\frac{y^2}{12}=1$ (*ii*) $\frac{x^2}{72}-\frac{y^2}{72}=1$ (*iii*) $\frac{x^2}{9/16}-\frac{y^2}{567/16}=1$
   (*iv*) $\frac{x^2}{9}-\frac{y^2}{27}=1$ (*v*) $\frac{x^2}{400}-\frac{y^2}{1200}=1$ (*vi*) $\frac{x^2}{324}-\frac{y^2}{2592}=1$

(*vii*) $\dfrac{x^2}{441/65}-\dfrac{y^2}{49/4}=1$ (*viii*) $\dfrac{x^2}{4}-\dfrac{y^2}{5}=1.$

(*b*) (*i*) $\dfrac{x^2}{36}-\dfrac{y^2}{108}=1$ (*ii*) $\dfrac{x^2}{4}-\dfrac{y^2}{5}=1$ (*iii*) $\dfrac{x^2}{16}-\dfrac{y^2}{9}=1$

(*iv*) $\dfrac{x^2}{3}-\dfrac{y^2}{2}=1$ (*v*) $\dfrac{x^2}{32}-\dfrac{y^2}{32}=1$ (*vi*) $\dfrac{x^2}{4/9}-\dfrac{y^2}{32/3}=1$

(*vii*) $\dfrac{x^2}{651/4}-\dfrac{y^2}{25/4}=1$ (*viii*) $\dfrac{x^2}{7}-\dfrac{y^2}{9}=1.$

(*c*) (*i*) $\dfrac{x^2}{7/3}-\dfrac{y^2}{7/5}=1$ (*ii*) $\dfrac{x^2}{11/3}-\dfrac{y^2}{11}=1.$

3.5 Tangent and Normal

1. Find the equations of tangent and normal to the given conic at the given point:

   (*i*) $y^2=4ax$ at (x_1, y_1); at $(at^2, 2at)$

   (*ii*) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at (x_1, y_1); at $(a\cos\theta, b\sin\theta)$

   (*iii*) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ at (x_1, y_1); at $(a\sec\theta, b\tan\theta)$.

2. Find the equations of tangent and normal

   (*a*) to the parabola (*i*) $y^2=8x$ at $(2, 4)$

   (*ii*) $y^2+4x=0$ at $(-1, -2)$

   (*iii*) $2y^2=3x$ at $(6, 3)$

   (*iv*) $2x^2+3y=0$ at $(3, -6)$

   (*b*) to the ellipse (*i*) $\dfrac{x^2}{36}+\dfrac{y^2}{9}=1$ at $(6, 0)$

   (*ii*) $4x^2+14y^2=70$ at $(0, 5)$

   (*iii*) $9x^2+4y^2=20$ at $(\frac{4}{3}, 1)$

   (*c*) to the hyperbola (*i*) $\dfrac{x^2}{9}-\dfrac{y^2}{16}=1$ at $(3, 0)$

   (*ii*) $\dfrac{x^2}{16}-\dfrac{y^2}{25}=1$ at $(4, 0)$

   (*iii*) $16x^2-9y^2=144$ at $(6, 4\sqrt{3})$

3. Find the equations of tangent and normal to the parabola
   (*i*) $x^2+2x-3y+3=0$ at $(1, 2)$
   (*ii*) $y^2+3x-3y+8=0$ at $(-4, 4)$
   (*iii*) $y^2-x+y+1=0$ at $(3, 1)$
   (*iv*) $y^2+x-y+2=0$ at $(-2, 1)$
   (*v*) $y^2=12x$ at the end points of LR

4. Find the equations of tangent and normal to the ellipse
   (*i*) $3x^2+5y^2=13$ at the points whose abscissa is 1.
   (*ii*) $4x^2+9y^2=20$ at the points whose ordinate is $\frac{4}{3}$.
   (*iii*) $9x^2+25y^2=225$ at the end points of LR (tangents only).
   (*iv*) $x^2+2y^2-4x+4y-6=0$ at $(4, 1)$.
   (*v*) $4x^2+9y^2+16x-18y-11=0$ at $(1, 1)$.

5. Find the equations of tangent and normal to the hyperbola
   (*i*) $\frac{x^2}{36}-\frac{y^2}{9}=1$ at the points whose abscissa is 6.
   (*ii*) $\frac{y^2}{16}-\frac{x^2}{25}=1$ at the points whose ordinate is -4.
   (*iii*) $4x^2-2y^2-3y+23=0$ at $(1, 3)$.
   (*iv*) $4x^2-9y^2=36$ at the end points of LR (tangents only).
   (*v*) $2x^2-y^2-8x-6=0$ at $(-1, 2)$.

6. Obtaion the condition for the line $y=mx+c$ to be a tangent to
   (*i*) the parabola $y^2=4ax$ (*ii*) the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
   (*iii*) the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

7. Find the equations of the tangents to the given conic which are parallel to the given line:
   (*i*) $y^2=8x$; $2x+3y+5=0$ (*ii*) $\frac{x^2}{16}+\frac{y^2}{4}=1$; $3x+y+5=0$
   (*iii*) $\frac{x^2}{36}-\frac{y^2}{9}=1$; $x+y+10=0$.

8. Find the equations of the tangents to the given conic which are perpendicular to the given line:
   (*i*) $y^2=16x$; $x+y=5$ (*ii*) $\frac{x^2}{4}+\frac{y^2}{8}=1$; $2x+y+1=0$
   (*iii*) $\frac{x^2}{20}-\frac{y^2}{5}=1$; $4x+3y-5=0$.

Funding: Tattva Heritage Foundation,Kolkata. Digitization: eGangotri.

Answers

2. (*a*) (*i*) $x-y+2=0$; $x+y-6=0$ (*ii*) $x-y-1=0$; $x+y+3=0$.
(*iii*) $x-4y+6=0$; $4x+y-27=0$ (*iv*) $4x+y-6=0$; $x-4y-27=0$
(*b*) (*i*) $x=6, y=0$ (*ii*) $y=5, x=0$ (*iii*) $3x+y-5=0$; $3x-9y+5=0$
(*c*) (*i*) $x-3=0$; $y=0$ (*ii*) $x-4=0$; $y=0$
(*iii*) $8x-3\sqrt{3}y-12=0$; $3\sqrt{3}x+8y-50\sqrt{3}=0$.

3. (*i*) $4x-3y+2=0$; $3x+4y-11=0$
(*ii*) $3x+5y-8=0$; $5x-3y+32=0$
(*iii*) $x-3y=0$; $3x+y-10=0$ (*iv*) $x+y+1=0$; $x-y+3=0$
(*v*) $x-y+3=0$; $x+y-9=0$ and $x+y+3=0$; $x-y-9=0$.

4. (*i*) $3x+5\sqrt{2}\,y-13=0$; $5\sqrt{2}x-3y-2\sqrt{2}=0$ and
$3x-5\sqrt{2}y-13=0$; $5\sqrt{2}x+3y-2\sqrt{2}=0$
(*ii*) $x+3y-5=0$; $9x-3y-5=0$ and $x-3y+5=0$; $9x+3y+5=0$
(*iii*) $4x+5y \pm 25=0$; $4x-5y \pm 25=0$
(*iv*) $x+2y-6=0$; $2x-y-7=0$ (*v*) $x=1$, $y=1$.

5. (*i*) $x=6$; $y=0$ (*ii*) $x=0$; $y+4=0$ (*iii*) $8x-15y+37=0$;
$15x+8y-39=0$ (*iv*) $\sqrt{13}x-3y\pm9=0$ and $\sqrt{13}x+3y\pm9=0$
(*v*) $3x+y+1=0$; $x-3y+7=0$

6. (*i*) $c=\dfrac{a}{m}$ (*ii*) $c^2=a^2m^2+b^2$ (*iii*) $c^2=a^2m^2-b^2$

7. (*i*) $2x+3y+9=0$ (*ii*) $3x+y=\pm2\sqrt{37}$ (*iii*) $x+y\pm3\sqrt{3}=0$.

8. (*i*) $x-y+4=0$ (*ii*) $x-2y\pm3=0$ (*iii*) $3x-4y\pm10=0$.

SECTION IV: CALCULUS

CHAPTER 1

LIMITS AND CONTINUITY

1.1 LIMITS

1. Define a function of a variable x and its value at $x=a$.

2. Let $f(x)=2x^2+3x+1$; $g(x)=2\cos x-\cos 2x$; $h(x)=\log_e(x+1)+e^x$. Find (*i*) $f(0)$ (*ii*) $f(-2)$ (*iii*) $f(\sin x)$ (*iv*) $g(0)$ (*v*) $g(\pi)$ (*vi*) $g(x+h)$ (*vii*) $h(0)$ (*viii*) $h(x^2)$.

3. Find the value of the following function, if exists, at $x=0$

(*i*) $\dfrac{\sin x}{x}$ (*ii*) $\operatorname{cosec} x-\cot x$ (*iii*) $\dfrac{a^x-1}{x}$ (*iv*) $(1+x)^{\frac{1}{x}}$

4. Give the meaning of "$\underset{x\to a}{\text{Lt}}\, f(x)=l$".

5. Find (*i*) $\underset{x\to 0}{\text{Lt}}\,(x^2-3x+5)$ (*ii*) $\underset{x\to 0}{\text{Lt}}\,\dfrac{4x^4+3}{x^2-4x+5}$

(*iii*) $\lim\limits_{x\to 1}\dfrac{x^2+4x+3}{x^2+5x+4}$ (*iv*) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin x}{x}$ (*v*) $\lim\limits_{x\to 0}\dfrac{x^3-2^3}{x-2}$

(*vi*) $\lim\limits_{x\to 0}(\sec x-\cos x)$ (*vii*) $\underset{x\to \pi}{\text{Lt}}\,\dfrac{1+\cos 2x}{1-\sin x}$

6. Evaluate (*i*) $\underset{x\to -1}{\text{Lt}}\,\dfrac{(x+1)\,(x+2)}{(x+1)\,(2x+1)}$ (*ii*) $\underset{x\to 2}{\text{Lt}}\,\dfrac{x^2-x-2}{x^2-5x+6}$

(*iii*) $\underset{x\to 1}{\text{Lt}}\,\dfrac{x^2-7x+6}{x^2+2x-3}$ (*iv*) $\underset{x\to 1}{\text{Lt}}\,\dfrac{x^3-4x^2+3x}{x^3-1}$.

(*v*) $\underset{x\to -3}{\text{Lt}}\,\dfrac{2x^2+9x+9}{4x^2+11x-3}$ (*vi*) $\underset{x\to -2}{\text{Lt}}\,\dfrac{x^2+4x+4}{2x^2+5x+2}$

(*vii*) $\underset{x\to 2}{\text{Lt}}\,\dfrac{x^4-16}{x^3-8}$ (*viii*) $\underset{x\to 1}{\text{Lt}}\left(\dfrac{1}{x-1}-\dfrac{3}{x^3-1}\right)$. (*ix*) $\underset{x\to \pi}{\text{Lt}}\,\dfrac{1+\cos^3 x}{\tan^2 x}$

7. Evaluate (*i*) $\underset{x\to 0}{\text{Lt}}\,\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$ (*ii*) $\underset{x\to 0}{\text{Lt}}\,\dfrac{x}{\sqrt{1-x}-1}$

(*iii*) $\underset{x\to 0}{\text{Lt}}\,\dfrac{\sqrt{1+x+x^2}-1}{x}$ (*iv*) $\underset{x\to 0}{\text{Lt}}\,\dfrac{x^2}{\sqrt{4+x^2}-\sqrt{4-x^2}}$.

(v) $\underset{x\to 1}{\text{Lt}}\ \dfrac{x-1}{\sqrt{5x-1}-\sqrt{3x+1}}$ (vi) $\underset{x\to 2}{\text{Lt}}\ \dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$

(vii) $\underset{x\to 3}{\text{Lt}}\ \dfrac{x^3-27}{\sqrt{2x+3}-\sqrt{3x}}$ (viii) $\underset{x\to 2}{\text{Lt}}\ \dfrac{x^2-x-2}{\sqrt{3x+4}-\sqrt{x+8}}$

8. **Evaluate:** (i) $\underset{x\to \infty}{\text{Lt}}\ \dfrac{13x^3-14x^2+17x+8}{26x^3+18x^2+12x+1}$.

(ii) $\underset{x\to \infty}{\text{Lt}}\ \dfrac{3x^2-8x+1}{10x^2-x+20}$ (iii) $\underset{n\to \infty}{\text{Lt}}\ \dfrac{\sqrt{5n^2+7n+5}}{n-3}$

(iv) $\underset{n\to \infty}{\text{Lt}}\ (\sqrt{n^2+1}-n)$ (v) $\underset{n\to \infty}{\text{Lt}}\ (\sqrt{n^3+n}-\sqrt{n^3+1})$

(vi) $\underset{n\to \infty}{\text{Lt}}\ n(\sqrt{4+n^2}-n)$ (vii) $\underset{n\to \infty}{\text{Lt}}\ \dfrac{1+2+3+\dots+n}{n+5}$

(viii) $\underset{n\to \infty}{\text{Lt}}\ \dfrac{1^2+2^2+3^2+\dots+n^2}{n^3}$

(ix) $\underset{n\to \infty}{\text{Lt}}\ \dfrac{1^3+2^3+3^3\dots+n^3}{n^4}$

(x) $\underset{x\to \infty}{\text{Lt}}\ \dfrac{1^3+2^3+3^3+\dots+x^3}{(x+2)^5}$

(xi) $\underset{n\to \infty}{\text{Lt}}\left(\dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+\dots+\dfrac{1}{3^n}\right)$

(xii) $\underset{n\to \infty}{\text{Lt}}\left(\dfrac{3}{2}+\dfrac{3}{4}+\dfrac{3}{8}+\dots\text{to } n \text{ terms}\right)$

(xiii) $\underset{n\to \infty}{\text{Lt}}\left(\dfrac{1+3+5+\dots\text{to } n \text{ terms}}{1+2+3+\dots\text{to } n \text{ terms}}\right)$

9. **Prove that** $\underset{x\to a}{\text{Lt}}\ \dfrac{x^n-a^n}{x-a}=na^{n-1}$ **for all rational values of** n

10. **Evaluate:**

(i) $\underset{x\to a}{\text{Lt}}\ \dfrac{x^6-a^6}{x-a}$ (ii) $\underset{x\to 2}{\text{Lt}}\ \dfrac{x^3-8}{x-2}$ (iii) $\underset{x\to 3}{\text{Lt}}\ \dfrac{x^4-81}{x-3}$

(iv) $\underset{x\to 2}{\text{Lt}}\ \dfrac{\sqrt{x}-\sqrt{2}}{x-2}$ (v) $\underset{x\to a}{\text{Lt}}\ \dfrac{x^{8/5}-a^{8/5}}{x-a}$ (vi) $\underset{x\to 3}{\text{Lt}}\ \dfrac{x-3}{x^5-3^5}$

(vii) $\underset{x\to a}{\text{Lt}}\ \dfrac{x-a}{\sqrt[3]{x^2}-\sqrt[3]{a^2}}$ (viii) $\underset{x\to -2}{\text{Lt}}\ \dfrac{x^5+32}{x+2}$ (ix) $\underset{x\to b}{\text{Lt}}\ \dfrac{x^5-b^5}{x^4-b^4}$

Funding: Tattva Heritage Foundation,Kolkata. Digitization: eGangotri.

(*x*) $\underset{x\to -a}{\text{Lt}}\ \frac{x^7+a^7}{x^5+a^5}$ (*xi*) $\underset{x\to 2}{\text{Lt}}\ \frac{x^2-4}{x\sqrt{x}-2\sqrt{2}}$ (*xii*) $\underset{h\to 0}{\text{Lt}}\ \frac{h}{(4+h)^4-4^4}$

11. Prove that

(*i*) $\underset{\theta\to 0}{\text{Lt}}\ \frac{\sin\theta}{\theta}=1$

(*ii*) $\underset{\theta\to 0}{\text{Lt}}\ \frac{\tan\theta}{\theta}=1$ where θ is measured in radian.

12. Evaluate:

(*i*) $\underset{\theta\to 0}{\text{Lt}}\ \frac{\sin 3\theta}{3\theta}$ (*ii*) $\underset{x\to 0}{\text{Lt}}\ \frac{\sin 5x}{x}$ (*iii*) $\underset{x\to 0}{\text{Lt}}\ \frac{x}{\sin\frac{x}{2}}$

(*iv*) $\underset{\theta\to 0}{\text{Lt}}\ \frac{\sin 3\theta}{\sin 2\theta}$ (*v*) $\underset{x\to 0}{\text{Lt}}\ \frac{\sin mx}{\sin nx}$ (*vi*) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan 15x}{x}$

(*vii*) $\underset{x\to 0}{\text{Lt}}\ \frac{\sin 5x}{\tan 4x}$ (*viii*) $\underset{x\to 0}{\lim}\ \frac{\sin 2x\tan 4x}{x^2}$

(*ix*) $\underset{\theta\to 0}{\text{Lt}}\ \frac{\sin^2\theta}{\tan^2 2\theta}$ (*x*) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan(2x^4)\sin^2(4x)}{x^6}$

(ɔ princi) $\frac{\sin x}{x}$ when x is measured in degrees.

(*i*) x

13. e:

(*ii*) s

(*i*) ') s $\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos x}{x}$ (*ii*) $\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos x}{x^2}$ (*iii*) $\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos x}{x\sin x}$

(*iv*) $\underset{x\to 0}{\text{Lt}}\ (\operatorname{cosec} x-\cot x)$ (*v*) $\underset{x\to 0}{\text{Lt}}\ \frac{x\tan x}{1-\cos x}$ (*vi*) $\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos 2x}{1-\cos 4x}$

(*vii*) $\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos px}{1-\cos qx}$ (*viii*) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan 3x-\sin 2x}{x}$ (*ix*) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan 3x-x}{3x-\sin x}$

(*x*) $\underset{x\to 0}{\text{Lt}}\ \frac{2\sin x-\sin 2x}{x^2}$ (*xi*) $\underset{x\to 0}{\text{Lt}}\ \frac{\sin 3x-3\sin x}{x^3}$

(*xii*) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan x-\sin x}{x^3}$ (*xiii*) $\underset{x\to 0}{\text{Lt}}\ \frac{\cos ax-\cos bx}{x^2}$

(*xiv*) $\underset{x\ \sqrt{}}{\text{Lt}}\ \frac{\sin(a+x)-\sin(a-x)}{x}.$

14. **Evaluate:**

(*i*) $\underset{\theta\to\frac{\pi}{2}}{\text{Lt}} \dfrac{\cos\theta}{\frac{\pi}{2}-\theta}$ (*ii*) $\underset{x\to 1}{\text{Lt}} \dfrac{1-x}{\cos\frac{\pi}{2}x}$ (*iii*) $\underset{\theta\to\frac{\pi}{2}}{\text{Lt}} \dfrac{1-\sin\theta}{\left(\frac{\pi}{2}-\theta\right)^2}$

(*iv*) $\underset{\theta\to\frac{\pi}{2}}{\text{Lt}} (\sec\theta-\tan\theta)$ (*v*) $\underset{\theta\to\frac{\pi}{2}}{\text{Lt}} \dfrac{\cos 3\theta+3\cos\theta}{\left(\frac{\pi}{2}-\theta\right)^3}$

(*vi*) $\underset{x\to 0}{\text{Lt}} \dfrac{\sin^{-1}x}{x}$ (*vii*) $\underset{x\to a}{\text{Lt}} \dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$

Answers

2. (*i*) 1 (*ii*) 3 (*iii*) $2\sin^2x+3\sin x+1$ (*iv*) 1 (*v*) -3
(*vi*) $2\cos(x+h)-\cos(2x+2h)$ (*vii*) 1 (*viii*) $\log_e(x^2+1)+e^{x^2}$

3. doesn't exist

5. (*i*) 5 (*ii*) $\frac{8}{5}$ (*iii*) $\frac{4}{5}$ (*iv*) $\frac{2}{\pi}$ (*v*) 4 (*vi*) 0 (*vii*) 2

6. (*i*) -1 (*ii*) -3 (*iii*) $-\frac{5}{4}$ (*iv*) $-\frac{3}{5}$ (*v*) $\frac{8}{13}$ (*vi*) 0 (*vii*) $\frac{8}{3}$ (*viii*) 1 (*ix*)

7. (*i*) 1 (*ii*) -2 (*iii*) $\frac{1}{2}$ (*iv*) 2 (*v*) 2 (*vi*) -8 (*vii*) -162 (*viii*) $3\sqrt{10}$

8. (*i*) $\frac{1}{2}$ (*ii*) $\frac{3}{10}$ (*iii*) $\sqrt{5}$ (*iv*) 0 (*v*) 0 (*vi*) 2 (*vii*) [illegible] (*ix*)
(*x*) 0 (*xi*) $\frac{1}{2}$ (*xii*) 3 (*xiii*) 2

10. (*i*) $6a^5$ (*ii*) 12 (*iii*) 108 (*iv*) $\frac{1}{2\sqrt{2}}$ (*v*) $\frac{8}{5}a^{\frac{8}{5}}$ (*vi*)
(*viii*) 80 (*ix*) $\frac{5b}{4}$ (*x*) $\frac{7}{5}a^2$ (*xi*) $\frac{4\sqrt{2}}{3}$ (*xii*) $\frac{1}{256}$

Lt
$x\to($
luat
Lt

12. (*i*) 1 (*ii*) 5 (*iii*) 2 (*iv*) $\frac{3}{2}$ (*v*) $\frac{m}{n}$ (*vi*) 15 (*vii*) $\frac{5}{4}$ (*viii*) 8 (*ix*)
(*x*) 32 (*xi*) $\frac{\pi}{180}$

13. (*i*) 0 (*ii*) $\frac{1}{2}$ (*iii*) $\frac{1}{2}$ (*iv*) 0 (*v*) 2 (*vi*) $\frac{1}{4}$ (*vii*) p^2/q^2 (*viii*) 1 (*ix*)
(*x*) 0 (*xi*) -4 (*xii*) $\frac{1}{2}$ (*xiii*) $\frac{b^2-a^2}{2}$ (*xiv*) $2\cos a$

14. (*i*) 1 (*ii*) $\frac{2}{\pi}$ (*iii*) $\frac{1}{2}$ (*iv*) 0 (*v*) 4 (*vi*) 1 (*vii*) $2\sqrt{a}\cos a$

1.2 Continuity

1. Define continuity of a function $f(x)$ at $x=a$

2. Discuss the continuity of the function

(*i*) $\frac{\sin x}{x}$ at $x=0$ and $x=\pi$ (*ii*) $\tan x$ at $x=\frac{\pi}{4}$ and $\frac{\pi}{2}$

(*iii*) x^2 at any point (*iv*) *a* polynomial in x at any point

(*v*) $\frac{x^2-4}{x-2}$ at $x=1$ and $x=2$ (*v*) $\sqrt{1-x^2}$ at $x=0$ and $x=2$

(*vi*) $\sin^{-1} x$ at $x=0$ (*vii*) e^{5x} at $x=1$

Answers

(*i*) no. yes; (*ii*) yes; no (*iii*) yes (*iv*) yes (*v*) yes; no (*vi*) yes (*vii*) yes

CHAPTER 2

DIFFERENTIATION

2.1 Standard forms and rules of differentiation

1. Define differential coefficient of $f(x)$ w. r. t. x
2. Differentiate the following functions w. r. t. x from first principles.

   (*i*) x^n; $\sqrt{x}$; $\frac{1}{x^5}$; $\frac{1}{\sqrt[3]{x^2}}$ and *k*, *a* constant.

   (*ii*) $\sin x$; $\cos x$; $\tan x$; $\operatorname{cosec} x$; $\sec x$ and $\cot x$.

   (*iii*) $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\operatorname{cosec}^{-1} x$, $\sec^{-1} x$ and $\cot^{-1} x$

   (*iv*) a^x; e^x and $\log x$

   (*v*) $\sin 3x$; $\cos 4x$; $\tan 5x$; ax^2+bx+c; $\frac{1}{2x+3}$ and $\frac{1+x}{1-x}$.

3. Prove the rule of derivative of a scalar multiple of a function and differentiate the following w. r. t. x

   (*i*) $5x^2$ (*ii*) $-2 \sec x$ (*iii*) $2 \sin^{-1} x$ (*iv*) $\frac{\log x}{5}$ (*v*) $\frac{5}{x}$ (*vi*) $\frac{e^x}{2}$

   (*vii*) $\log_{10} x$

4. Prove the rule of derivative of sum of two functions and that of difference of two functions and find the derivatives of the following w. r. t. x

   (*i*) $\sqrt{x}+\cot x$ (*ii*) $3 \operatorname{cosce} x-4^x$ (*iii*) $5 \cos x-3 \sin x$ (*iv*) x^e+e^x

(*v*) $2\sec^{-1}x+8x$ (*vi*) $3\log x+\log_5 x$ (*vii*) $8^x-\frac{x^2}{4}$ (*viii*) $4x^2+3x$

(*ix*) $5e^x-4\log x$ (*x*) $3\tan x-4\sec^{-1}x$ (*xi*) $x^{-4}+\sqrt{x}+5$

(*xii*) $3e^x-2\log x+8x$ (*xiii*) $3\log x-4\sin x+4\cot x-3\sin^{-1}x$

(*xiv*) $\frac{4}{x^2}-\frac{2}{x}+4+8x+9x^2$ (*xv*) $5^x+x^5+5^5$ (*xvi*) $\left(x-\frac{1}{x}\right)^2$

(*xvii*) $2(\sin^{-1}x+\cos^{-1}x)+\tan^{-1}x+\cot^{-1}x$ (*xviii*) $\tan x \cot x$

5. (*i*) If $f(x)=x^2+2x+3$ find $f(0)$ and $f'(0)$

(*ii*) If $f(t)=3\tan t-4\cos t+2$ find $f'(0)$

(*iii*) If $f(s)=2\cos s-3\sin s+s^2$ find $f'(\pi)$

6. **Prove the rule of derivative of product of two functions and differentiate the following w. r. t. x**

(*i*) $x^2\,3^x$ (*ii*) $x\log x-x$ (*iii*) $x^5\operatorname{cosec} x$ (*iv*) $(3x^2-x)e^x$

(*v*) $(x+1)\operatorname{cosec}^{-1}x$ (*vi*) $(2x-1)(x^2-6x+3)$ (*vii*) $(\sec x+4)(2x-5)$

(*viii*) $e^x\log x$ (*ix*) $\sqrt{x}\sin x$ (*x*) $\sin x\cos x$

7. **Prove the rule of derivative of quotient of two functions and find the differential coefficient of the following w.r.t. x**

(*i*) $\frac{x^2+4}{3x-7}$ (*ii*) $\frac{\tan x}{x}$ (*iii*) $\frac{\sin x}{1+\tan x}$ (*iv*) $\frac{x}{1-x}$ (*v*) $\frac{\tan^{-1}x}{1+x^2}$

(*vi*) $\frac{x+1}{\log x}$ (*vii*) $\frac{e^x-1}{x-1}$ (*viii*) $\frac{3+2\cos x}{2-3\sin x}$ (*ix*) $\frac{x^5-3x+2}{5x^2-7x+1}$

(*x*) $\frac{\sqrt{x}+\sqrt{5}}{\sqrt{5}-\sqrt{x}}$

8. **Differentiate the following w.r.t. x:**

(*i*) $(1-x^2)\tan x$ (*ii*) $x\tan^{-1}x$ (*iii*) $(ax^2+bx+c)(px+q)$

(*iv*) $(\sin x+\cos x)e^x$ (*v*) $\frac{1+\tan x}{1-\tan x}$ (*vi*) $\frac{e^x}{x^2}$

(*vii*) $\frac{1+x+x^2}{\sqrt{x}}$ (*viii*) $\frac{x+e^x}{1-\cos x}$ (*ix*) $\frac{x^3-1}{\log x}$

(*x*) $(x+a)(x+b)(x+c)$ (*xi*) $(x^2+1)(e^x-1)(\log x+2)$

(*xii*) $(3\cos x-5)(1-4x^2)(4+\sin^{-1}x)$ (*xiii*) $\frac{(x+2)\sin x}{x^2+1}$

(*xiv*) $\frac{x+2}{x+1}\operatorname{cosec} x$ (*xv*) $\frac{(\sec x-x^2)(\cos^{-1}x+2x)}{(3x^2-4x+1)}$

(*xvi*) $\frac{xe^x}{(x+4)\operatorname{cosec}^{-1}x}$

9. Find the differential coefficient of the following w.r.t. x

(*i*) $\dfrac{x^2 \cos x+1}{x \sin x+x^2}$ (*ii*) $(x^3+1)(x^2-1)(x^4+4)$ (*iii*) $e^x \sin x \log x$

(*iv*) $\dfrac{e^x+\sin x}{\sec x-x^5}$ (*v*) $x \tan x - x^2 \sec x$ (*vi*) $\dfrac{x^3 \tan x}{1+x}$

(*vii*) $\dfrac{(1+\sqrt{x}) \sin x}{x \log x}$ (*viii*) $\dfrac{x^2}{x^3+1}-\dfrac{x^2+1}{x^2-1}$ (*ix*) $x^2 \sin^{-1} x+\dfrac{x}{\cos x}$

(*x*) $\dfrac{3^x\, x^{15}}{x \tan^{-1} x}$

Answers

3. (*i*) $10x$ (*ii*) $-2 \sec x \tan x$ (*iii*) $2/\sqrt{1-x^2}$ (*iv*) $\dfrac{1}{5x}$ (*v*) $-5/x^2$
   (*vi*) $\frac{1}{2} e^x$ (*vii*) $\frac{1}{x} \log_{10} e$
4. (*i*) $\dfrac{1}{2\sqrt{x}}-\text{cosec}^2 x$ (*ii*) $-3 \text{ cosec } x \cot x-4^x \log 4$ (*iii*) $-5 \sin x$
   $-3 \cos x$ (*iv*) $ex^{e-1}+e^x$ (*v*) $\dfrac{2}{x\sqrt{x^2-1}}+8$ (*vi*) $\dfrac{3}{x}+\dfrac{1}{x}\log_5 e$
   (*vii*) $8^x \log 8-\frac{1}{2}x$ (*viii*) $8x+3$ (*ix*) $5e^x-\dfrac{4}{x}$ (*x*) $3 \sec^2 x-\dfrac{4}{x\sqrt{x^2-1}}$
   (*xi*) $-4x^{-5}+\dfrac{1}{2\sqrt{x}}$ (*xii*) $3e^x-\dfrac{2}{x}+8$
   (*xiii*) $\dfrac{3}{x}-4\cos x-4 \text{ cosec}^2 x-\dfrac{3}{\sqrt{1-x^2}}$ (*xiv*) $-\dfrac{8}{x^3}+\dfrac{2}{x^2}+8+18x$
   (*xv*) $5^x \log 5+5x^4$ (*xvi*) $2x-\dfrac{2}{x^3}$ (*xvii*) 0 (*xviii*) 0.
5. (*i*) 3;2 (*ii*) 3 (*iii*) $3+2\pi$
6. (*i*) $x\, 3^x (x \log 3+2)$ (*ii*) $\log x$ (*iii*) $x^4 \text{ cosec } x\,(5-x \cot x)$
   (*iv*) $e^x (3x^2+5x-1)$ (*v*) $-\dfrac{x+1}{x\sqrt{x^2-1}}-\text{cosec}^{-1} x$ (*vi*) $6x^2-26x+12$
   (*vii*) $(2x-5) \sec x \tan x+2(\sec x+4)$ (*viii*) $e^x(\log x+\frac{1}{x})$
   (*ix*) $\dfrac{1}{2\sqrt{x}} \sin x+\sqrt{x} \cos x$ (*x*) $\cos 2x$
7. (*i*) $\dfrac{3x^2-14x-12}{(3x-7)^2}$ (*ii*) $\dfrac{x \sec^2 x-\tan x}{x^2}$

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(iii) $\dfrac{(1+\tan x)\cos x-\sin x \sec^2 x}{(1+\tan x)^2}$ (iv) $\dfrac{1}{(1-x)^2}$

(v) $\dfrac{1-2x\tan^{-1}x}{(1+x^2)^2}$ (vi) $\dfrac{x\log x-x-1}{x(\log x)^2}$

(vii) $\dfrac{(x-1)e^x-(e^x-1)}{(x-1)^2}$ (viii) $\dfrac{6-4\sin x+9\cos x}{(2-3\sin x)^2}$

(ix) $\dfrac{(5x^2-7x+1)(5x^4-3)-(x^5-3x+2)(10x-7)}{(5x^2-7x+1)^2}$

(x) $\dfrac{\sqrt{5}}{\sqrt{x}(\sqrt{5}-\sqrt{x})^2}$

8. (i) $(1-x^2)\sec^2 x-2x\tan x$ (ii) $\dfrac{x}{1+x^2}+\tan^{-1}x$

(iii) $(px+q)(2ax+b)+(ax^2+bx+c)p$ (iv) $2e^x\cos x$

(v) $\dfrac{2\sec^2 x}{(1-\tan x)^2}$ (vi) $\dfrac{e^x(x-2)}{x^3}$

(vii) $\dfrac{3x^2+x-1}{2x^{3/2}}$ (viii) $\dfrac{(1-\cos x)(1+e^x)-(x+e^x)\sin x}{(1-\cos x)^2}$

(ix) $\dfrac{3x^3\log x-(x^3-1)}{x(\log x)^2}$ (x) $(x+b)(x+c)+(x+c)(x+a)$
$+(x+a)(x+b)$

(xi) $(x^2+1)(e^x-1).\dfrac{1}{x}+(\log x+2)[(e^x-1).\,2x+(x^2+1)e^x)$

(xii) $(3\cos x-5)(1-4x^2).\dfrac{1}{\sqrt{1-x^2}}+(4+\sin^{-1}x)[(1-4x^2)(-3\sin x)$
$+(3\cos x-4)(-8x)]$

(xiii) $\dfrac{(x^2+1)[\sin x+(x+2)\cos x]-(x+2)\sin x.\,2x}{(x^2+1)^2}$

(xiv) $\dfrac{(x+1)[\operatorname{cosec} x-(x+2)\operatorname{cosec} x\cot x]-(x+2)\operatorname{cosec} x}{(x+1)^2}$

(xv) $(3x^2-4x+1)\ [(\cos^{-1}x+2x)(\sec x\tan x-2x)+(\sec x-x^2)$
$\dfrac{\left(-\dfrac{1}{\sqrt{1-x^2}}+2)\right]-(\sec x-x^2)(\cos^{-1}x+2x)(6x-4}{(3x^2-4x+1)^2}$

(xvi) $$\frac{(x+4)\operatorname{cosec}^{-1}x\,(e^x+xe^x)-xe^x\left[\operatorname{cosec}^{-1}x+(x+4).\frac{-1}{x\sqrt{x^2-1}}\right]}{[(x+4)\ \operatorname{cosec}^{-1}x]^2}$$

9. (i) $$\frac{(x^2+x\sin x)(2x\cos x-x^2\sin x)-(x^2\cos x+1)\ (2x+\sin x+x\cos x)}{(x\ \sin\ x+x^2)^2}$$

(ii) $(x^2-1)(x^4+4)(3x^2)+(x^3+1)(x^4+4)(2x)+(x^3+1)(x^2-1)(4x^3)$

(iii) $e^x\sin x\log x+e^x\cos x\log x+\dfrac{e^x\sin x}{x}$

(iv) $$\frac{(\sec x-x^5)\ (e^x+\cos x)-(e^x+\sin x)\ (\sec x\ \tan x-5x^4)}{(\sec x-x^5)^2}$$

(v) $\tan x+x\sec^2x-x^2\sec x\tan x-2x\sec x$

(vi) $$\frac{(1+x)\ [\ 3x^2\tan x+x^3\sec^2 x]-x^3\tan x}{(1+x)^2}$$

(vii) $$\frac{x\log x[\sin x\frac{1}{2\sqrt{x}}+(1+\sqrt{x})\cos x]-(1+\sqrt{x})\sin x(\log x+1)}{(x\log\ x)^2}$$

(viii) $\dfrac{2x-x^4}{(x^3+1)^2}+\dfrac{4x}{(x^2-1)^2}$ (ix) $2x\sin^{-1}x+\dfrac{x^2}{\sqrt{1-x^2}}+\dfrac{\cos x+x\sin x}{\cos^2 x}$

(x) $$\frac{x\tan^{-1}x\,[x^{15}\ 3^x\log 3+3^x 15\,x^{14}]-3^x\,x^{15}\left[\tan^{-1}x+\frac{x}{1+x^2}\right]}{(x\ \tan^{-1}x)^2}$$

Rules of Differentiation (contd.)

10. State and prove the chain rule of differentiation.

11. Differentiate the following w. r. t. x:

(i) $(1-x)^5$ (ii) $(x^2-3x+1)^4$ (iii) $\dfrac{1}{4-x^2}$ (iv) $\sqrt{x^2+x-1}$

(v) $\sqrt{\sin^{-1}x}$ (vi) $\dfrac{1}{\sqrt[3]{1-e^x}}$ (vii) $\dfrac{1}{(4x^2-3x+7)^3}$

(viii) $(\sec^{-1}x)^3$ (ix) $\sqrt[13]{x^2-\log x}$

(x) $(3x-4e^x+3\ \log\ x+\operatorname{cosec}\ x)^2$ (xi) $\log_x e$ (xii) $\sin^{15}x$

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12. Differentiate the following w. r. t. x:
(*i*) $\sin\sqrt{x}$ (*ii*) $\cos(\log x)$ (*iii*) $\tan(3x^2+1)$
(*iv*) $\cot 5x$ (*v*) $\operatorname{cosec}(2\sin x)$ (*vi*) $\sec(e^x+1)$
(*vii*) $\sin(\sin x)$ (*viii*) $\cos(\tan^{-1}x)$ (*ix*) $\tan(\sec x)$
(*x*) $\operatorname{cosec}(x^2-x+1)$ (*xi*) $\cot(5^x)$ (*xii*) $\sec(2\sec^{-1}x$
(*xiii*) $\sin x$ when x is measured in degrees.

13. Differentiate the following w.r.t. x:
(*i*) $e^{\sqrt{x}}$ (*ii*) $e^{5\cos^{-1}x}$ (*iii*) $\log(4x^2+9)$ (*iv*) $\log(4-x)$ (*v*) 10^{x}
(*vi*) 3^{4x^2} (*vii*) e^{1/x^2} (*viii*) $e^{x-x^2+\cos x}$ (*ix*) $\log(\sec x+\tan x$
(*x*) $\log\dfrac{a-x}{a+x}$ (*xi*) $\log_{10}(\log x)$ (*xii*) $\log\sqrt{\sin x}$

14. Differentiate the following w.r.t. x:
(*i*) $\sin^{-1}(1-x)$ (*ii*) $\cos^{-1}(\sqrt{x})$ (*iii*) $\tan^{-1}(x^2+1)$ (*iv*) $\sec^{-1}(x$
(*v*) $\operatorname{cosec}^{-1}\left(\dfrac{2\log x}{3}\right)$ (*vi*) $\cot^{-1}\left(\dfrac{x}{3}\right)$ (*vii*) $\sin^{-1}(\tan x)$
(*viii*) $\cos^{-1}(2e^x-3)$ (*ix*) $\tan^{-1}\left(\dfrac{2}{x^2}\right)$ (*x*) $\sec^{-1}(\sqrt{x}+2)$
(*xi*) $\operatorname{cosec}^{-1}(4x)$ (*xii*) $\cot^{-1}(\sqrt[3]{x})$.

15. (*a*) Define the six hyperbolic functions and obtain the derivatives

(*b*) Differentiate the following w.r.t. x:
(*i*) $\sinh 2x$ (*ii*) $\cosh(1+x^2)$ (*iii*) $\tanh 3x$ (*iv*) $\coth\left(\dfrac{1}{x}\right)$
(*v*) $\operatorname{cosech}(\sin x)$ (*vi*) $\operatorname{sech}^3 x$ (*vii*) $\log(\cosh x)$
(*viii*) $x\operatorname{cosech}(x^2)$ (*ix*) $\tan^{-1}(\sinh x)$
(*x*) $e^x(\sinh 2x-x)$ (*xi*) $\dfrac{\cosh x-\cos x}{\sinh x+\sin x}$

16. (*a*) Define the six inverse hyperbolic functions and obtai their derivatives.

(*b*) Differentiate the following w.r.t. x:
(*i*) $\sinh^{-1}(2x)$ (*ii*) $\cosh^{-1}\dfrac{2x}{3}$ (*iii*) $\tanh^{-1}(1-x)$
(*iv*) $\sinh^{-1}(\tan x)$ (*v*) $\coth^{-1}(2x^2)$ (*vi*) $\operatorname{sech}^{-1}(x^2)$
(*vii*) $\operatorname{cosech}^{-1}(e^x)$ (*viii*) $\tanh^{-1}\left(\dfrac{\tan x}{2}\right)$ (*ix*) $\operatorname{sech}^{-1}(\cos x$

17. Differentiate the following w.r.t. x:

(*i*) $\sin(m\sin^{-1}\sqrt{x})$ (*ii*) $\cos(e^{x^2})$ (*iii*) $\tan(\log\sin x)$

(*iv*) $\sqrt{\sec\sqrt{x}}$ (*v*) $e^{e^{x^2}}$ (*vi*) $\operatorname{cosec}(e^{4\tan^{-1}x})$

(*vii*) $\cot[(1-x^2)^{10}]$ (*viii*) $\log(x+\sqrt{1+x^2})$

(*ix*) $\log(\log\sin x)$ (*x*) $e^{(\sin^{-1}x)^2}$ (*xi*) $[\sin^{-1}(2\cos x)]^{\frac{5}{2}}$

(*xii*) $\dfrac{1}{x-\sqrt{x^2-1}}$ (*xiii*) $\cos^{-1}(\sin^4 x)$ (*xiv*) $(\operatorname{cosec}^{-1}\sqrt{x})^{10}$

(*xv*) $(e^{2x+3}+3x^2)^7$ (*xvi*) $\dfrac{1}{(e^{x^2}-1)^5}$ (*xvii*) $[\tan^{-1}(e^x)]^2$

(*xviii*) $\sin^3(4x^2)$ (*xix*) $\cosh^4 2x$ (*xx*) $\operatorname{sech}^2(4x^2)$ (*xxi*) $\sec^3(\sec x)$

(*xxii*) $\tan(\tanh\sqrt{x})$ (*xxiii*) $[\sinh^{-1}3x]^3$ (*xxiv*) $\coth^{-1}(\sec^2 x)$

18. Differentiate the following w.r.t. x:

(*i*) $x\operatorname{cosec}^{-1}\dfrac{1}{x}$ (*ii*) $\cot^{-1}\dfrac{2}{x}+\tan^{-1}\dfrac{x}{2}$

(*iii*) $\sqrt{x^2-5}\,\sqrt[3]{x^2+3}$ (*iv*) $(x^2+x+1)^4\tan(x^3)$

(*v*) $e^{-ax}\tan^2(x^2+1)$ (*vi*) $\cos 5x\sin^{-1}(\tan x)$

(*vii*) $\dfrac{e^{\sin x}}{\sqrt{\log x}}$ (*viii*) $\sqrt{\dfrac{1-x^2}{1+x^2}}$ (*ix*) $\sqrt{\dfrac{1-\cos 4x}{1+\cos 4x}}$

(*x*) $\dfrac{(2x^2-1)^2}{\sqrt{4-x}}$ (*xi*) $\dfrac{\sin^3 2x}{1+e^{-x}}$ (*xii*) $\tan\left(\dfrac{x+1}{x+2}\right)$

(*xiii*) $\dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}+\dfrac{x\sqrt{a^2-x^2}}{2}$

(*xiv*) $x\log(x+\sqrt{1+x^2})-\sqrt{1+x^2}$ (*xv*) $\sin^7 x\cos^{10}3x$

(*xvi*) $\tan^3(x^2)\sqrt{\sin(x^3)}$ (*xvii*) $e^{\frac{1}{x}}\sin^2 4x$

(*xviii*) $\sqrt{\cot^{-1}(4\sqrt{x})}\log(2-x^2)$ (*xix*) $\cosh 3x\sinh 2x$

(*xx*) $\cosh^2 x\cos^3 x$ (*xxi*) $\sin^3 3x\cosh 2x$ (*xxii*) $e^{\sqrt{x}}\log(\sinh x)$

(*xxiii*) $\sinh^{-1}\left(\dfrac{1-x}{1+x}\right)$ (*xxiv*) $\log\dfrac{a+b\cos x}{b+a\cos x}$ (*xxv*) $\tan^{-1}\sqrt{\dfrac{1-x^2}{1+x^2}}$

(*xxvi*) $\tan^2 x\tanh^{-1}x$ (*xxvii*) $\dfrac{\operatorname{sech}^{-1}x}{\sqrt{1-x^2}}$ (*xxviii*) $\log\dfrac{x-\sqrt{1-x^2}}{x+\sqrt{1-x^2}}$

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19. Differentiate the following w.r.t. x:

(*i*) $\dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}$ (*ii*) $\dfrac{\log(1+x^2)e^{-x^2}}{\sin 3x}$

(*iii*) $\dfrac{\tan(\log x)\log(\cos^{-1}x)}{2x-1}$ (*iv*) $\dfrac{(4x-1)^3(x^2+2)^4}{(3x^2+5)^2}$

(*v*) $(1+\sin^3 x)\sqrt{\cos x}\,e^{3x}$ (*vi*) $\dfrac{\tan^{-1}(\log x)\log(1-\sqrt{x})}{\sqrt[3]{1-x^5}}$

(*vii*) $\dfrac{\sec^{-1}(2x)\,.\,\sin^2 x}{\sqrt{\log x}}$ (*viii*) $\dfrac{e^{\operatorname{cosec}^{-1}x}\cdot\sqrt{\cot x}}{(1-3x)^2}$

(*ix*) $\dfrac{\log(1+\tan x)}{(1+x^2)\sqrt{1+x}}$ (*x*) $\dfrac{\operatorname{cosec}^3\sqrt{x}\,.\,\cos 2x}{\tan^3 x}$

(*xi*) $\dfrac{\sqrt{1-x^2}\cdot\sqrt[4]{\sin x}}{\cos(\log x)}$ (*xii*) $\dfrac{e^{x^2}\tan^{-1}x}{\sqrt{1+x^2}}$

(*xiii*) $e^{3x}\sin 3x\cos 4x$ (*xiv*) $\dfrac{\operatorname{cosec} 2x\,.\,\sec^2 4x}{\tan(3x^2)}$

(*xv*) $\dfrac{\sqrt{x}\cosh 5x}{\sin^4 x}$ (*xvi*) $\dfrac{\log\tanh 5x}{xe^x}$

20. (*i*) If $y=\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$, prove that $\dfrac{dy}{dx}=\frac{1}{2}$

(*ii*) If $y=\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$, prove that $\dfrac{dy}{dx}=-\frac{1}{2}$

(*iii*) If $y=\cos^{-1}\sqrt{\dfrac{1+\cos x}{2}}$, prove that $\dfrac{dy}{dx}=\frac{1}{2}$

(*iv*) If $y=\tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}}$, prove that $\dfrac{dy}{dx}=\frac{1}{2}$

21. Differentiate the following w.r.t. x:

(*i*) $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ (*ii*) $\sin^{-1}(3x-4x^3)$ (*iii*) $\sin^{-1}\left(\dfrac{x}{\sqrt{1+}}\right.$

(*iv*) $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ (*v*) $\cos^{-1}(1-2x^2)$ (*vi*) $\cos^{-1}(2x^2-$

(*vii*) $\cos^{-1}(4x^3-3x)$ (*viii*) $\sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$ (*ix*) $\sec^{-1}\left(\dfrac{x}{\sqrt{x^2}}\right.$

(*x*) $\tan^{-1}\dfrac{2x}{1-x^2}$ (*xi*) $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$ (*xii*) $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

(*xiii*) $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)$ (*xiv*) $\tan^{-1}\left(\dfrac{x+a}{1-ax}\right)$ (*xv*) $\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}$

(*xvi*) $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$.

Answers

11. (*i*) $-5(1-x)^4$ (*ii*) $4(x^2-3x+1)^3(2x-3)$ (*iii*) $\dfrac{2x}{(4-x^2)^2}$

(*iv*) $\dfrac{2x+1}{2\sqrt{x^2+x-1}}$ (*v*) $\dfrac{1}{2\sqrt{\sin^{-1}x}\sqrt{1-x^2}}$ (*vi*) $\frac{1}{3}(1-e^x)^{-\frac{4}{3}}e^x$

(*vii*) $-3(4x^2-3x+7)^{-4}.(8x-3)$ (*viii*) $\dfrac{3(\sec^{-1}x)^2}{x\sqrt{x^2-1}}$

(*ix*) $\frac{1}{13}(x^2-\log x)^{-\frac{12}{13}}.\left(2x-\dfrac{1}{x}\right)$

(*x*). $2(3x-4e^x+3\log x+\text{cosec}\,x)\left(3-4e^x+\dfrac{3}{x}-\text{cosec}\,x\cot x\right)$

(*xi*) $-\dfrac{1}{x(\log x)^2}$ (*xii*) $15\sin^{14}x\cos x$.

12. (*i*) $\dfrac{\cos\sqrt{x}}{2\sqrt{x}}$ (*ii*) $-\dfrac{\sin(\log x)}{x}$ (*iii*) $6x\sec^2(3x^2+1)$

(*iv*) $-5\,\text{cosec}^2\,5x$ (*v*) $-2\cos x.\,\text{cosec}(2\sin x)\cot(2\sin x)$

(*vi*) $e^x\sec(e^x+1)\tan(e^x+1)$ (*vii*) $\cos x.\cos(\sin x)$

(*viii*) $-\dfrac{\sin(\tan^{-1}x)}{1+x^2}$ (*ix*) $\sec^2(\sec x).\sec x\tan x$

(*x*) $-(2x-1)\,\text{cosec}(x^2-x+1)\cot(x^2-x+1)$

(*xi*) $-5^x\log 5\,\text{cosec}^2(5^x)$ (*xii*) $\dfrac{2\sec(2\sec^{-1}x)\tan(2\sec^{-1}x)}{x\sqrt{x^2-1}}$

(*xiii*) $\dfrac{\pi}{180}\cos\left(\dfrac{\pi}{180}x\right)$.

13. (*i*) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$ (*ii*) $\dfrac{-5e^{5\cos^{-1}x}}{\sqrt{1-x^2}}$ (*iii*) $\dfrac{8x}{4x^2+9}$ (*iv*) $\dfrac{-1}{4-x}$

(*v*) $10e^x (\log 10)\, e^x$ (*vi*) $3^{4x^2} (\log 3)\, 8x$ (*vii*) $-\frac{2}{x^3} \cdot e^{\frac{1}{x^2}}$

(*viii*) $e^{x-x^2+\cos x} . (1-2x-\sin x)$ (*ix*) $\sec x$ (*x*) $\frac{-1}{a+x} - \frac{1}{a-x}$

(*xi*) $\frac{\log_{10} e}{x \log x}$ (*xii*) $\frac{1}{2} \cot x$.

14. (*i*) $-\frac{1}{\sqrt{2x-x^2}}$ (*ii*) $-\frac{1}{2\sqrt{x-x^2}}$ (*iii*) $\frac{2x}{2+2x^2+x^4}$

(*iv*) $\frac{3}{x\sqrt{x^6-1}}$ (*v*) $\frac{-3}{x \log x \sqrt{4 (\log x)^2-9}}$ (*vi*) $\frac{-3}{9+x^2}$

(*vii*) $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ (*viii*) $\frac{-2e^x}{\sqrt{1-(2e^x-3)^2}}$ (*ix*) $\frac{-4x}{x^4+4}$

(*x*) $\frac{1}{2\sqrt{x}(\sqrt{x}+2)\sqrt{(\sqrt{x}+2)^2-1}}$ (*xi*) $\frac{-1}{x\sqrt{16x^2-1}}$

(*xii*) $\frac{-1}{3x^{2/3}(1+x^{2/3})}$.

15. (*i*) $2 \cosh 2x$ (*ii*) $2x \sinh (1+x^2)$ (*iii*) $3\, \mathrm{sech}^2\, 3x$

(*iv*) $\frac{1}{x^2} \mathrm{cosech}^2 \frac{1}{x}$ (*v*) $-\cos x\, \mathrm{cosech} (\sin x) \coth (\sin x)$

(*vi*) $-3\, \mathrm{sech}^3 x \tanh x$ (*vii*) $\tanh x$

(*viii*) $-2x^2\, \mathrm{cosech}\, (x^2) \coth (x^2) + \mathrm{cosech}\, (x^2)$ (*ix*) $\frac{\cosh x}{1+\sinh^2 x}$

(*x*) $(\sinh 2x - x + 2 \cosh 2x - 1)e^x$ (*xi*) $\frac{2 \sinh x \sin x}{(\sinh x+\sin x)^2}$

16. (*b*) (*i*) $\frac{2}{\sqrt{1+4x^2}}$ (*ii*) $\frac{2}{\sqrt{4x^2-9}}$ (*iii*) $\frac{1}{x^2-2x}$ (*iv*) $\sec x$

(*v*) $\frac{4x}{1-4x^4}$ (*vi*) $\frac{-2}{x\sqrt{1-x^4}}$ (*vii*) $\frac{-1}{\sqrt{1+e^{2x}}}$ (*viii*) $\frac{2 \sec^2 x}{4-\tan^2 x}$

(*ix*) $\sec x$.

17. (*i*) $\frac{m \cos (m \sin^{-1} \sqrt{x})}{2\sqrt{x}\sqrt{1-x}}$ (*ii*) $-2x e^{x^2} \sin (e^{x^2})$

(*iii*) $\cot x \sec^2 (\log \sin x)$ (*iv*) $\frac{\sqrt{\sec \sqrt{x}} \tan \sqrt{x}}{4\sqrt{x}}$

(v) $e^{e^{x^2}} . e^{x^2} . 2x$

(vi) $\dfrac{-4e^{4\tan^{-1}x} . \operatorname{cosec}(e^{4\tan^{-1}x}) . \cot(4e^{\tan^{-1}x})}{1+x^2}$

(vii) $20x(1-x^2)^9 \operatorname{cosec}^2(1-x^2)^{10}$ (viii) $\dfrac{1}{\sqrt{1+x^2}}$

(ix) $\dfrac{\cos x}{\sin x \log \sin x}$ (x) $\dfrac{2(\sin^{-1}x)\, e^{(\sin^{-1}x)^2}}{\sqrt{1-x^2}}$

(xi) $\dfrac{-5 \sin x [\sin^{-1}(2\cos x)]^{3/2}}{\sqrt{1-4\cos^2 x}}$ (xii) $\dfrac{1}{\sqrt{x^2-1}\,(x-\sqrt{x^2-1})}$

(xiii) $\dfrac{-4\sin^3 x \cos x}{\sqrt{1-\sin^8 x}}$ (xiv) $\dfrac{-5(\operatorname{cosec}^{-1}\sqrt{x})^9}{x\sqrt{x-1}}$

(xv) $7(e^{2x+3}+3x^2)^6(2e^{2x+3}+6x)$ (xvi) $-10x(e^{x^2}-1)^{-6} . e^{x^2}$

(xvii) $\dfrac{2e^x \tan^{-1}(e^x)}{1+e^{2x}}$ (xviii) $24x \cos(4x^2)\sin^2(4x^2)$

(xix) $8 \cosh^3 2x \sinh 2x$ (xx) $-16x\, \operatorname{sech}^2(4x^2) \tanh(4x^2)$

(xxi) $3\sec^3(\sec x)\tan(\sec x)\sec^2 x$ (xxii) $\dfrac{\sec^2(\tanh\sqrt{x})\operatorname{sech}^2\sqrt{x}}{2\sqrt{x}}$

(xxiii) $\dfrac{9[\sinh^{-1}(3x)]^2}{\sqrt{9x^2+1}}$ (xxiv) $\dfrac{2\sec^2 x \tan x}{1-\sec^4 x}$

18. (i) $\operatorname{cosec}^{-1}\dfrac{1}{x}+\dfrac{x}{\sqrt{1-x^2}}$ (ii) $\dfrac{4}{4+x^2}$

(iii) $\sqrt[3]{x^2+3} . \dfrac{x}{\sqrt{x^2-5}}+\frac{2}{3}x\sqrt{x^2-5} . (x^2+3)^{-2/3}$

(iv) $4(2x+1)(x^2+x+1)^3\tan^{-1}(x^3)+\dfrac{3x^2(x^2+x+1)^4}{1+x^6}$

(v) $-ae^{-ax}\tan^2(x^2+1)+4x\,e^{-ax} . \tan(x^2+1) . \sec^2(x^2+1)$

(vi) $-5\sin 5x \sin^{-1}(\tan x)+\dfrac{\cos 5x . \sec^2 x}{\sqrt{1-\tan^2 x}}$

(vii) $\dfrac{(2x\cos x\log x-1)e^{\sin x}}{2x(\log x)^{3/2}}$ (viii) $\dfrac{-2x}{\sqrt{1-x^2}(1+x^2)^{3/2}}$

(ix) $2\sec^2 2x$ (x) $\dfrac{16x(2x^2-1)(4-x)+(2x^2-1)^2}{2(4-x)^{3/2}}$

(xi) $\dfrac{6(1+e^{-x})\sin^2 2x \,.\, \cos 2x+e^{-x} \,.\, \sin^3 2x}{(1+e^{-x})^2}$

(xii) $\dfrac{1}{(x+2)^2}\sec^2\left(\dfrac{x+1}{x+2}\right)$ (xiii) $\sqrt{a^2-x^2}$ (xiv) $\log (x+\sqrt{1+}$

(xv) $7\cos^{10} 3x \,.\, \sin^6 x\cos x-30\sin^7 x\cos^9 3x\sin 3x$

(xvi) $6x\tan^2(x^2)\sec^2(x^2) \,.\, \sqrt{\sin (x^3)}+\dfrac{3x^2\cos (x^3) \,.\, \tan^3 (x^2)}{2\sqrt{\sin (x^3)}}$

(xvii) $-e^{\frac{1}{x}}\dfrac{\sin^2 4x}{x^2}+8e^{\frac{1}{x}} \,.\, \cos 4x\sin 4x$

(xviii) $-\log (2-x^2) \,.\, \dfrac{1}{\sqrt{\cot^{-1} 4\sqrt{x}}} \cdot \dfrac{1}{1+16x} \cdot \dfrac{1}{\sqrt{x}}-\dfrac{2\sqrt{\cot^{-1} 4\sqrt{x}}}{2-x^2}$

(xix) $3\sinh 2x\sinh 3x+2\cosh 3x\cosh 2x$

(xx) $2\cos^3 x\cosh x\sinh x-3\cosh^2 x\cos^2 x\sin x$

(xxi) $9\cosh 2x\sin^2 3x\cos 3x+2\sin^3 3x\sinh 2x$

(xxii) $\log (\sinh x)\,\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}+e^{\sqrt{x}}\coth x$

(xxiii) $-\sqrt{2}\,/\,[\sqrt{1+x^2} \,.\, (1+x)^2]$

(xxiv) $\dfrac{(a^2-b^2)\sin x}{(a+b\cos x)(b+a\cos x)}$ (xxv) $\dfrac{-x}{\sqrt{1-x^4}}$

(xxvi) $2\tanh^{-1} x\tan x\sec^2 x+\dfrac{\tan x}{1-x^2}$

(xxvii) $\dfrac{x^2\,\text{sech}^{-1} x-\sqrt{1-x^2}}{x(1-x^2)^{3/2}}$ (xxviii) $\dfrac{2}{(2x^2-1)\sqrt{1-x^2}}$

19. (i) $\dfrac{\cos^{-1}x-x\sqrt{1-x^2}}{(1-x^2)^{3/2}}$

(ii) $\dfrac{\sin 3x\left[e^{-x} \cdot \dfrac{2x}{1+x^2}-2x\log (1+x^2) \,.\, e^{-x^2}\right]-3\log (1+x^2) \,.\, e^{-x^2} \,.\, \cos}{\sin^2 3x}$

(iii) $\dfrac{(2x-1)\left[\log (\cos^{-1} x) \,.\, \sec^2(\log x) \,.\, \dfrac{1}{x}-\dfrac{\tan (\log x) \,.}{\sqrt{1-x^2} \,.\, \cos^{-1} x}\right]-2\tan (\log x)\log (\cos^{-1}}{(2x-1)^2}$

(iv)
$$\frac{(3x^2+5)^2\left[(x^2+2)^4 . 12(4x-1)^2+(4x-1)^3 . 8x\,(x^2+2)^3\right] -(4x-1)^3 . (x^2+2)^4 . 12x\,(3x^2+5)}{(3x^2+5)^4}$$

(v)
$$e^{3x}\left[3\sqrt{\cos x}\,\sin^2 x\cos x-\frac{(1+\sin^3 x)\sin x}{2\sqrt{\cos x}}\right] +3e^{3x} . (1+\sin^3 x)\sqrt{\cos x}$$

(vi)
$$\frac{(1-x^5)^{\frac{1}{3}}\left[\frac{\log(1-\sqrt{x})}{x\{1+(\log x)^2\}}-\frac{\tan^{-1}(\log x)}{2\sqrt{x}(1-\sqrt{x})}\right]+\frac{5}{3}x^4(1-x^5)^{-2/3} . \tan^{-1}(\log x) . \log(1-\sqrt{x})}{(1-x^5)^{3/2}}$$

(vii)
$$\frac{\sqrt{\log x}\left[\sin^2 x . \frac{1}{x\sqrt{4x^2-1}}+\sec^{-1}2x . 2\sin x\cos x\right] -\sec^{-1}(2x) . \sin^2 x . \frac{1}{2x\sqrt{\log x}}}{\log x}$$

(viii)
$$\frac{(1-3x)^2\left[-\sqrt{\cot x}\,e^{cosec^{-1}x} . \frac{1}{x\sqrt{x^2-1}}-e^{cosec^{-1}x} . \frac{1}{2\sqrt{\cot x}}\,cosec^2 x\right]+e^{cosec^{-1}x} . \sqrt{\cot x} . 6(1-3x)}{(1-3x)^4}$$

(ix)
$$\frac{(1+x^2)\sqrt{1+x}\left(\frac{\sec^2 x}{1+\tan x}\right)-\log(1+\tan x) . \left[2x\sqrt{1+x}+\frac{(1+x^2)}{2\sqrt{1+x}}\right]}{(1+x^2)^2(1+x)}$$

(x)
$$\frac{\tan^3 x\left[-\frac{3\cos 2x}{2\sqrt{x}} . cosec^3\sqrt{x}\cot\sqrt{x}-2\,cosec^3\sqrt{x} . \sin 2x\right] -3\,cosec^3\sqrt{x} . \cos 2x\tan^2 x . \sec^2 x}{\tan^6 x}$$

(xi)
$$\frac{\cos(\log x)\left[-\sqrt[4]{\sin x} . \frac{x}{\sqrt{1-x^2}}+\sqrt{1-x^2}) . \frac{1}{4}(\sin x)^{-\frac{3}{4}} . \cos x\right] +\sqrt{1-x^2}\;\sqrt[4]{\sin x}\,\frac{\sin(\log x)}{x}}{\cos^2(\log x)}$$

(*xii*) $$\frac{\sqrt{1+x^2}\,.\left[\tan^{-1}x\,.\,e^{x^2}\,.\,2x+e^{x^2}\,.\,\frac{1}{1+x^2}\right]-\frac{e^{x^2}\tan^{-1}x}{\sqrt{1+x^2}}}{1+x^2}$$

(*xiii*) $3\sin 3x\cos 4x\, e^{3x}+3e^{3x}\cos 4x\cos 3x-4e^{3x}\sin 3x\sin 4x$

(*xiv*) $$\frac{\tan(3x^2)\,[-2\sec^2 4x\,\text{cosec}\,2x\cot 2x+8\,\text{cosec}\,2x\sec^2 4x\,.\,\tan 4x]-6x\,\text{cosec}\,2x\sec^2 4x\sec^2(3}{\tan^2(3x^2)}$$

(*xv*) $$\frac{\sin^4 x\,[\cosh 5x\,\frac{1}{2\sqrt{x}}+5\sqrt{x}\sinh 5x]-4\sqrt{x}\cosh 5x\sin^3 x}{\sin^8 x}$$

(*xvi*) $$\frac{5x\,e^x\,\frac{1}{\tanh 5x}\,.\,\text{sech}^2\,5x-\log\tanh 5x\,.\,e^x(1+x)}{x^2e^{2x}}$$

21. (*i*) $\frac{2}{1+x^2}$ (*ii*) $\frac{3}{\sqrt{1-x^2}}$ (*iii*) $\frac{1}{1+x^2}$ (*iv*) $\frac{2}{1+x^2}$ (*v*) $\frac{2}{\sqrt{1-x^2}}$

(*vi*) $-\frac{2}{\sqrt{1-x^2}}$ (*vii*) $-\frac{3}{\sqrt{1-x^2}}$ (*viii*) $\frac{-2}{1+x^2}$ (*ix*) $\frac{-1}{x\sqrt{x^2-1}}$

(*x*) $\frac{2}{1+x^2}$ (*xi*) $\frac{3}{1+x^2}$ (*xii*) $\frac{1}{2(1+x^2)}$ (*xiii*) $-\frac{1}{1+x^2}$

(*xiv*) $\frac{1}{1+x^2}$ (*xv*) $-\frac{1}{2\sqrt{1-x^2}}$ (*xvi*) $\frac{1}{\sqrt{1-x^2}}$

2.2 Implicit Differentiation

1. Find $\frac{dy}{dx}$ if (*i*) $x^2+y^2=a^2$ (*ii*) $y^2=4ax$ (*iii*) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
(*iv*) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$.

2. Find $\frac{dy}{dx}$ if (*i*) $2x^2-3xy+4y^2=1$ (*ii*) $x^2y^2=x^2+y^2$
(*iii*) $x^3y^3=x+y$ (*iv*) $x^3+y^3=3axy$
(*v*) $ax^2+2hxy+by^2+2gx+2fy+c=0$
(*vi*) $x^3+y^3+3x^2y^2-4x^2y+3xy^2=0$
(*vii*) $3x^3-x^2y+2xy^2-y^3-3x^2+y^2=1$

(*viii*) $x^2+2xy+3y^2-x-y-8=0$ when $x=2,\ y=1$

(*ix*) $x^2y^3+2x^2y+3x^4-y=1$ when $x=1,\ y=-1$

(*x*) $x+\sqrt{xy}=x^2$ (*xi*) $x=y\sqrt{1-y^2}$

3. Find $\frac{dy}{dx}$ if (*i*) $a\cos mx+b\sin ny=0$

(*ii*) $x\sin y+y\sin x=10$ (*iii*) $e^{x+y}=e^x+e^y$

(*iv*) $y+x\log y=5$ (*v*) $\sin(\sqrt{xy})=\sin(x+y)-x+y$

(*vi*) $\log(x+y)=\tan^{-1}\frac{x}{y}$

(*vii*) $e^{-x\cos y}+\sin y=e$ when $y=\pi$ $x=1$

(*viii*) $x\cosh y+y\sinh x=5$

(*ix*) $xy=\sin(x-y)$ (*x*) $x^2\log y+y^2\log x=2$

(*xi*) $\tan^{-1}\frac{y}{x}=\log(x^2+y^2)$

4. (*i*) If $x^my^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$

(*ii*) If $\sin y=x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

(*iii*) If $\sec\left(\frac{x+y}{x-y}\right)=a$, prove that $\frac{dy}{dx}=\frac{y}{x}$

(*iv*) If $x\sqrt{1+y}+y\sqrt{1+x}=0,\ y\neq x$, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

(*v*) If $\sqrt{1-x^2}-\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(*vi*) If $y=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{\ldots}}}}$, prove that $\frac{dy}{dx}=\frac{1}{2y-1}$

Answers

1. (*i*) $-\frac{x}{y}$ (*ii*) $\frac{2a}{y}$ (*iii*) $\frac{b^2x}{a^2y}$ (*iv*) $-\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

2. (*i*) $\frac{4x-3y}{3x-8y}$ (*ii*) $\frac{x-y^2x}{x^2y-y}$ (*iii*) $\frac{1-3x^2y^3}{3x^3y^2-1}$ (*iv*) $\frac{x^2-ay}{ax-y^2}$

(*v*) $\frac{ax+by+g}{hx+by+f}$ (*vi*) $\frac{8xy-3x^2-6xy^2-3y^2}{3y^2+6x^2y-4x^2+6xy}$

(*vii*) $\frac{9x^2-2xy+2y^2-6x}{x^2-4xy+3y^2-2y}$ (*viii*) $-\frac{5}{9}$ (*ix*) $-\frac{3}{2}$

(*x*) $\frac{2\sqrt{xy}(2x-1)-y}{x}$ (*xi*) $\frac{\sqrt{1-y^2}}{1-2y^2}$

3. (*i*) $\frac{am \sin mx}{bn \cos ny}$ (*ii*) $-\frac{\sin y+y \cos x}{x \cos y+\sin x}$ (*iii*) $\frac{e^x(1-e^y)}{e^y(e^x-1)}$

(*iv*) $-\frac{y \log y}{x+y}$ (*v*) $\frac{2\sqrt{xy}\cos(x+y)-y\cos(\sqrt{xy})-1}{x\cos\sqrt{xy}-2\sqrt{xy}\cos(x+y)-1}$

(*vi*) $\frac{xy-x^2}{2x^2+xy+y^2}$ (*vii*) e (*viii*) $\frac{-\cosh y(1+y)}{\sinh x(1+x)}$

(*ix*) $\frac{\cos(x-y)-y}{\cos(x-y)+x}$ (*x*) $-\frac{(2x^2 \log y+y^2)y}{(2y^2 \log x+x^2)x}$ (*xi*) $\frac{2x+y}{x-2y}$

2.3 Parametric Differentiation

1. Find $\frac{dy}{dx}$ if (*i*) $x=at^2, y=2at$ (*ii*) $x=a \sec \theta, y=b \tan \theta$

(*iii*) $x=a \cos^n \theta, y=b \sin^n \theta$

(*iv*) $x=a \cos(\log t), y=a \log(\cos t)$

(*v*) $x=e^t \sin t, y=e^{-t} \cos t$

(*vi*) $x=a(\cos \theta+\theta \sin \theta), y=a(\sin \theta-\theta \cos \theta)$

(*vii*) $x=3 \cos t-2 \cos^3 t, y=3 \sin t-2 \sin^3 t$

(*viii*) $x=a\left(t-\frac{1}{t}\right), y=a\left(t+\frac{1}{t}\right)$

(*ix*) $x=\frac{1-t^2}{1+t^2}, y=\frac{2t}{1+t^2}$ (*x*) $x=\frac{3at}{1+t^3}, y=\frac{3at^2}{1+t^3}$

(*xi*) $x=\sqrt{t}, y=1-\frac{1}{\sqrt{t}}$ when $t=4$ (*xii*) $x=\frac{t-1}{t+1}, y=\frac{2t-}{t-}$

2. (*i*) If $x=a\left(\cos t+\log \tan \frac{t}{2}\right), y=a \sin t$,

prove that $\frac{dy}{dx}=\tan t$

(*ii*) If $x=3 \cos \theta-\cos^3 \theta, y=3 \sin \theta-\sin^3 \theta$,

prove that $\frac{dy}{dx}=-\cot^3 \theta$

(*iii*) If $x=3\sin 2\theta+2\sin 3\theta$, $y=2\cos 3\theta-3\cos 2\theta$, prove that $\frac{dy}{dx}=-\tan\frac{\theta}{2}$

(*iv*) If $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$, prove that $\frac{dy}{dx}=\tan\frac{\theta}{2}$

(*v*) If $x=a(t-\sin t)$, $y=a(1+\cos t)$, evaluate $\frac{dy}{dx}$ when $t=\frac{\pi}{2}$

(*vi*) Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ if $x=a\cos^3\theta$, $y=b\sin^3\theta$

(*vii*) If $x=a\cos^4 t$, $y=a\sin^4 t$ prove that $\frac{dy}{dx}=-1$ when $t=\frac{3\pi}{4}$

(*viii*) Evaluate $\frac{dy}{dx}$ when $t=\frac{\pi}{4}$ if $x=2\cos t-\cos 2t$,
$y=2\sin t-\sin 2t$.

3. Differentiate
(*i*) $\cos 2t$ w.r.t. $2\sin t$
(*ii*) $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t. $\cos^{-1}(1-2x^2)$
(*iii*) $\log \operatorname{sech} x$ w.r.t. $\cosh x$
(*iv*) $\sin(\log t)$ w.r.t. $\tan(e^t)$
(*v*) $\cosh^{-1}(t)$ w.r.t. $\sinh t$

Answers

1. (*i*) $\frac{1}{t}$ (*ii*) $\frac{b\operatorname{cosec}\theta}{a}$ (*iii*) $-\frac{b}{a}\tan^{n-2}\theta$ (*iv*) $\frac{t\tan t}{\sin\log t}$
(*v*) $-e^{-2t}$ (*vi*) $\tan\theta$ (*vii*) $\cot t$ (*viii*) $\frac{t^2-1}{t^2+1}$ (*ix*) $\frac{t^2-1}{2t}$
(*x*) $\frac{2t-t^4}{1-2t^3}$ (*xi*) $\frac{1}{4}$ (*xii*) $-\frac{3(t+1)^2}{2(t-2)^2}$

2. (*v*) -1 (*vi*) $-b/a$ (*viii*) $\sqrt{2}+1$

3. (*i*) $-2\sin t$ (*ii*) 1 (*iii*) $-\operatorname{sech} x$ (*iv*) $\frac{\cos(\log t)}{t\cdot e^t\sec^2(e^t)}$ (*v*) $\frac{1}{\sqrt{t^2-1}\cosh t}$

2.4 Logarithmic Differentiation

1. Find $\frac{dy}{dx}$ if

(*i*) $y=\frac{x(1-x^2)^2}{\sqrt{1+x^2}}$ (*ii*) $y=\frac{x^{\frac{1}{2}}(1-2x)^{\frac{2}{3}}}{(2-3x)^{\frac{3}{4}}(3-4x)^{\frac{4}{5}}}$

(*iii*) $y=\dfrac{(2x-1)^2\,(1-4x)}{(1-x)^3\,(5+7x)}$ (*iv*) $y=x^2\,e^{-3x}\cos 4x\tanh x$

(*v*) $y=\dfrac{x^4\sin(e^x)}{e^{2x}(1-x^2)^2}$ and all the problems in (8) and (19) of Art.

2. Prove the product rule and quotient rule of differentiat using logarithmic differentiation.

3. Find $\dfrac{dy}{dx}$ if

(*i*) $y=x^x$ (*ii*) $y=x^{\log x}$ (*iii*) $y=x^{\cot x}$ (*iv*) $y=(\sin x)^x$

(*v*) $y=(\sin x)^{\cos x}$ (*vi*) $y=(\tan x)^{\log x}$ (*vii*) $y=(\log x)$

(*viii*) $y=(x^x)^x$ (*ix*) $y=(\cosh x)^{x^2}$ (*x*) $y=(\tanh^{-1}x)^{x^2}$

4. Find $\dfrac{dy}{dx}$ if y is equal to

(*i*) e^{x^x} (*ii*) x^{x^x} (*iii*) $x^x+x^a+a^x+a^a$

(*iv*) $(\tan x)^{\cot x}+(\cot x)^{\tan x}$ (*v*) $(\log x)^x+(\sin^{-1}x)^{\sin x}$

(*vi*) $x^{\sinh x}+(\sinh x)^x$ (*vii*) $\sin(x^x)$

5. Find $\dfrac{dy}{dy}$ if (*i*) $x^x=e^{x-y}$ (*ii*) $(\sin x)^{\cos y}=(\sin y)^{\cos x}$

(*iii*) $x^y\,y^x=1$

6. (*i*) If $x^m y^n=(x+y)^{m+n}$, prove that $\dfrac{dy}{dx}=y/x$

(*ii*) If $x^y=y^x$, prove that $\dfrac{dy}{dx}=\dfrac{y\,(y-x\log y)}{x\,(x-y\log x)}$

(*iii*) If $x^y=e^{x-y}$, prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$

(*iv*) If $y=x^{x^{x^{\cdot^{\cdot^{\cdot\infty}}}}}$, prove that $x\dfrac{dy}{dx}=\dfrac{y^2}{1-y\log x}$

Answers

(*i*) $y\left[\dfrac{1}{x}-\dfrac{4x}{1-x^2}-\dfrac{x}{1+x^2}\right]$ (*ii*) $y\left[\dfrac{1}{2x}-\dfrac{4}{3\,(1-2x)}+\dfrac{9}{4(2-3x)}+\dfrac{16}{5(3-}\right.$

(*iii*) $y\left[\dfrac{4}{2x-1}-\dfrac{4}{1-4x}+\dfrac{3}{1-x}-\dfrac{7}{5+7x}\right]$

(iv) $y\left[\frac{2}{x}-3-4\tan 4x+\frac{\operatorname{sech}^2 x}{\tanh x}\right]$

(v) $y\left[\frac{4}{x}+e^x\cot(e^x)-2+\frac{4x}{1-x^2}\right]$

3. (i) $y(1+\log x)$ (ii) $\frac{2y\log x}{x}$ (iii) $y[\frac{\cot x}{x}-\operatorname{cosec}^2x\log x]$

(iv) $y[\log\sin x+x\cot x]$ (v) $y[-\sin x\log\sin x+\cos x\cot x]$

(vi) $y[\frac{\log\tan x}{x}+\frac{\sec^2x\log x)}{\tan x}]$

(vii) $y[\cos x\log(\log x)+\frac{\sin x}{x\log x}]$ (viii) $y[2x\log x+x]$

(ix) $y[2x\log\cosh x+x^2\tanh x]$

(x) $y[2x\log\tanh^{-1}x+\frac{x^2}{(1-x^2)\tanh^{-1}x}]$

4. (i) $yx^x[1+\log x]$ (ii) $yx^{x-1}[1+x\log x\log(ex)]$

(iii) $x^x\log(ex)+ax^{a-1}+a^x\log a$

(iv) $(\tan x)^{\cot x}\operatorname{cosec}^2x\log(e\cot x)-(\cot x)^{\tan x}\sec^2x\log(e\tan x)$

(v) $(\log x)^x\left[\log\log x+\frac{1}{\log x}\right]+(\sin^{-1}x)^{\sin x}$

$\left[\cos x\log\sin^{-1}x+\frac{\sin x}{\sqrt{1-x^2}\sin^{-1}x}\right]$

(vi) $x^{\sinh x}\left[\cosh x\log x+\frac{\sinh x}{x}\right]+(\sinh x)^x$

$[\log\sinh x+x\coth x]$ (vii) $\cos(x^x).x^x\log(ex)$

5. (i) $-\log x$ (ii) $(\cot y\cos x+\sin x\log\sin y)/(\cos x\cot y+\sin y\log\sin x)$ (iii) $-y(x\log y+y)/x(y\log x+x)$

CHAPTER 3

SUCCESSIVE DIFFERENTIATION OF ORDER 2

1. Find the second derivative of the following w.r.t. x

(i) $x+x^{-1}$ (ii) $x^3-3x^2+4x+\frac{7}{x}$ (iii) $\frac{x^2+1}{\sqrt{x}}$

(iv) $\cos 4x$ (v) $e^{-\sqrt{x}}$ (vi) $\tan(1-3x)$ (vii) $\tan^{-1}x$

(*viii*) $\frac{\log x}{x}$ (*ix*) $e^{4x} \sin 3x$ (*x*) $e^{3x} \cos 2x$ (*xi*) $x^2 \log x$

(*xii*) $\tan^5 x$ (*xiii*) $\sin 6x \cos 4x$ (*xiv*) $\cos 4x \cos 2x$

(*xv*) $x^2 \sin 3x$ (*xvi*) $\frac{x}{\sqrt{x-1}}$ (*xvii*) $\coth 3x$

2. (*i*) If $y^2=4\,ax$, prove that $\frac{d^2y}{dx^2}=\frac{-4a^2}{y^3}$

(*ii*) If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, prove that $\frac{d^2y}{dx^2}=\frac{-b^4}{a^2y^3}$

(*iii*) If $x^2+xy+y^2=a^2$, prove that $\frac{d^2y}{dx^2}=-\frac{-6a^2}{(x+2y)^3}$

(*iv*) If $x^2+2xy+3y^2=1$, prove that $\frac{d^2y}{dx^2}=\frac{-2}{(x+3y)^3}$

(*v*) If $xy+4y=4x$, prove that $\frac{d^2y}{dx^2}=\frac{-24}{(x+4)^3}$

(*vi*) If $ax^2+2hxy+by^2=1$, prove that $\frac{d^2y}{dx^2}=\frac{h^2-ab}{(hx+by)^3}$

(*vii*) If $x^2-xy+y^2=3$, prove that $y''=\frac{18}{(x-2y)^3}$

(*viii*) If $x^2-y^2-x-1=0$, prove that y'' at $(2, 1)$ is $-5/4$

(*ix*) If $x^3+y^3-3axy=0$, prove tha $\frac{d^2y}{dx^2}=\frac{-2a^3xy}{(y^2-ax)^3}$

3. (*i*) If $x=at^2$, $y=2at$, prove that $\frac{d^2y}{dx^2}=\frac{-1}{2at^3}$

(*ii*) If $x=a\cos\theta, y=b\sin\theta$, prove that $\frac{d^2y}{dx^2}=\frac{-b}{a^2}\text{cosec}^3\theta$

(*iii*) If $x=a\cos^3 t, y=a\sin^3 t$, prove that $\frac{d^2y}{dx^2}=\frac{4\sqrt{2}}{3a}$ when $t=$

(*iv*) If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$, prove that $\frac{d^2y}{dx^2}=\frac{\sec^4}{4a}$

(*v*) If $x=e^t(\cos t+\sin t), y=e^t(\cos t-\sin t)$, prove that $\frac{d^2y}{dx^2}=-\frac{1}{2}e^{-t}\sec^3 t$

(vi) If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$, prove that $\dfrac{d^2y}{dx^2}=\dfrac{\sec^3\theta}{a\theta}$

(vii) If $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$, $y=a\sin t$, prove that $\dfrac{d^2y}{dx^2}=\dfrac{\sin t}{a\cos^4 t}$

(viii) If $x=\dfrac{1-t}{1+t}$, $y=\dfrac{2t}{1+t}$, prove that $\dfrac{d^2y}{dx^2}=0$

(ix) If $x=t+\dfrac{1}{t}$, $y=t+1$, prove that $\dfrac{d^2y}{dx^2}=\dfrac{-2t}{(t^2-1)^3}$.

4. (i) If $y=a\cos mx+b\sin mx$, prove that $\dfrac{d^2y}{dx^2}+m^2y=0$

(ii) If $y=a\,e^{mx}+b\,e^{-mx}$, prove that $\dfrac{d^2y}{dx^2}=m^2y$

(iii) If $y=x\cos x$, prove that $x^2y_2-2xy_1+(x^2+2)\,y=0$

(iv) If $y=x^2\cos x$, prove that $x^2y_2-4xy_1+(x^2+6)\,y=0$

(v) If $y=ax^{n+1}+\dfrac{b}{x^n}$, prove that $x^2y_2=n\,(n+1)y$

(vi) If $y=(x^2+a^2)^6$, prove that $(x^2+a^2)\,y_2-10xy_1=12y$

(vii) If $y=e^{ax}\sin bx$, prove that $y_2-2ay_1+(a^2+b^2)y=0$

(viii) If $y=\sin(\sin x)$, prove that $y_2+(\tan x)y_1+(\cos^2 x)y=0$

(ix) If $y=x\cos\log x+x\log x$, prove that $x^2y_2-xy_1+2y=x\log x$

(x) If $xy=ae^x+be^{-x}$, prove that $xy''+2y'-y=0$

(xi) If $y=x\cosh x$, prove that $xy''-2y'-xy+2\cosh x=0$

5. (i) If $y=\sin^{-1}x$, prove that $(1-x^2)y_2-xy_1=0$

(ii) If $y=\tan^{-1}x$, prove that $(1+x^2)y_2+2xy_1=0$

(iii) If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)y_2-xy_1-2=0$

(iv) If $y=(\cos^{-1}x)^2$, prove that $(1-x^2)y_2-xy_1-2=0$

(v) If $y=(\tan^{-1}x)^2$, prove that $(1+x^2)^2y_2+2x(x^2+1)y_1=2$

(vi) If $y=(\sinh^{-1}x)^2$, prove that $(1+x^2)y_2+xy_1=2$

(vii) If $y=e^{m\sin^{-1}x}$, prove that $(1-x^2)y_2-xy_1-m^2y=0$

(viii) If $y=e^{m\cos^{-1}x}$, prove that $(1-x^2)y_2-xy_1-m^2y=0$

(ix) If $y=e^{\tan^{-1}x}$, prove that (a) $(1+x^2)y_2+(2x-1)y_1=0$
(b) $(1+x^2)^2y_2+2x(1+x^2)y_1=y$

(*x*) If $y=\sin(a\sin^{-1}x)$, prove that $(1-x^2)y_2-xy_1+a^2y=0$

(*xi*) If $y=\cos(a\sin^{-1}x)$, prove that $(1-x^2)y_2-xy_1+a^2y=0$

(*xii*) If $y=\sin(a\cos^{-1}x)$, prove that $(1-x^2)y_2-xy_1+a^2y=0$

(*xiii*) If $y=\cos(a\tan^{-1}x)$, prove that $(1+x^2)^2y_2+2x(1+x^2)y_1+a^2y=0$

(*xiv*) If $y=\sin(a\sinh^{-1}x)$, prove that $(1+x^2)y_2+xy_1+a^2y=0$.

6. (*i*) If $y=\sin(\log x)$, prove that $x^2y_2+xy_1+y=0$

(*ii*) If $y=e^x\log x$, prove that $xy_2-(2x-1)y_1+(x-1)y=0$

(*iii*) If $y=(x+\sqrt{x^2+1})^m$, prove that $(x^2+1)y_2+xy_1-m^2y=0$

(*iv*) If $y=\log(x+\sqrt{x^2-1})$, prove that $(x^2-1)y_2+xy_1=0$

(*v*) If $y=a\cos\log x+b\sin\log x$, prove that $x^2y_2+xy_1+y=0$

(*vi*) If $x=\sin\theta$, $y=\cos p\theta$, prove that $(1-x^2)y_2-xy_1+p^2y=0$

Answers

1. (*i*) $2x^{-3}$ (*ii*) $6x-6+\frac{14}{x^3}$ (*iii*) $\frac{3}{4}(x^{-\frac{1}{2}}+x^{-\frac{5}{2}})$ (*iv*) $-16\cos 4x$

(*v*) $\frac{1}{4}e^{-\sqrt{x}}\frac{(\sqrt{x}+1)}{x\sqrt{x}}$ (*vi*) $18\sec^2(1-3x)\tan(1-3x)$ (*vii*) $\frac{-2x}{(1+x^2)}$

(*viii*) $\frac{2\log x-3}{x^3}$ (*ix*) $e^{4x}(24\cos 3x+7\sin 3x)$

(*x*) $e^{3x}(5\cos 2x-12\sin 2x)$ (*xi*) $3+2\log x$

(*xii*) $10\sec^2x\tan^3x(\tan^2x+2\sec^2x)$

(*xiii*) $-2(25\sin 10x+\sin 2x)$ (*xiv*) $-2(\cos 2x+9\cos 6x)$

(*xv*) $(2-9x^2)\sin 3x+12x\cos 3x$ (*xvi*) $\frac{4-x}{4(x-1)^{5/2}}$

(*xvii*) $18\,\mathrm{cosech}^3 3x\coth 3x$

CHAPTER 4

APPLICATION OF DIFFERENTIATION

4.1 Tangent and Normal

1. (*a*) Explain the geometrical meaning of $\frac{dy}{dx}$

(*b*) Find the equations of tangent and normal to the following curves at the given points:

(*i*) $y=x^2+1$ at $(1, 2)$ (*ii*) $y=\dfrac{6x}{x^2-1}$ at $(2, 4)$

(*iii*) $3x^2-xy+y^2=3$ at $(1, 1)$ (*iv*) $2x^3+2y^3-9xy=0$ at $(2, 1)$

(*v*) $\sqrt{x}-\sqrt{y}=1$ at $(9, 4)$ (*vi*) $xy=c^2$ at $(ct, c/t)$

(*vii*) $y(x^2+1)=x+3$ at $(2, 1)$ (*viii*) $2x-xy+3y^2=6$ at $(3, 1)$

(*ix*) $x=3t, y=4/t$ at $t=2$ (*x*) $x=a\cos^3\theta, y=a\sin^3\theta$ at $\theta=\pi/4$

2. Find the equations of tangent and normal to the following curves at the points where they cross (*a*) x-axis (*b*) y-axis:
   (*i*) $(1+x^2)y=2-x$ (*ii*) $y(x-2)(x-3)=x-1$

3. Find the points on the curve (*i*) $y=2x^3-3x^2+5$
   (*ii*) $y=x^3-2x^2+x$ (*iii*) $y=x^3-12x+18$ where the tangent is parallel to x-axis.

4. (*i*) Find the point on the curve $y=6x-x^2$ where the tangent has slope -4.

   (*ii*) Find the points on the circle $x^2+y^2=5$ at which the tangents are parallel to the line $2x-y+1=0$.

   (*iii*) Find a point on the curve $x=a\cos^3\theta, y=a\sin^3\theta$ at which the tangent is parallel to x-axis.

5. Find the equations of tangent and normal to the curve
   (*i*) $y=3x^2+2$ where the slope of the tangent is -2
   (*ii*) $y=5x^2-8$ where the slope of the normal is $\frac{1}{10}$
   (*iii*) $y^2=3x+2$ where the tangent is parallel to $3x-4y+1=0$
   (*iv*) $y=6-x^2$ where the normal is parallel to $x-4y+3=0$
   (*v*) $y=5x^3-2x+7$ where the tangent is $\perp$ to $x+13y=0$
   (*vi*) $2y=3x^2+6x$ where the normal is $\perp$ to $6x+2y=5$
   (*vii*) $y=4x^2$ where the normal is parallel to $y=4x+3$

6. The tangent to the curve $y=5-2x^2$ at the point $x=1$ cuts the coordinate axes at A and B. Find the area of $\triangle$ AOB.

7. The normal to the curve $x^2=4y-12$ at $(2, 4)$ cuts the coordinate axes at A and B. Show that the area of $\triangle$AOB is 18 sq. units.

8. If $y=4x-5$ touches $y^2=ax^3+b$ at $(2, 3)$, find a and b.

9. If the equation of the tangent to the curve $y=3x^2-4x$ at $(1,-1)$ is of the form $ax=y+b$, find a and b.

Answers

1. (*i*) $2x-y=0$; $x+2y-5=0$ (*ii*) $10x+3y-32=0$; $3x-10y+34=$
(*iii*) $5x+y-6=0$; $x-5y+4=0$ (*iv*) $5x-4y-6=0$; $4x+5y-13=$
(*v*) $2x-3y-6=0$; $3x+2y-35=0$ (*vi*) $x+t^2y-2ct=0$;
$t^3x-ty+c-ct^4=0$ (*vii*) $3x+5y-11=0$; $5x-3y-7=0$
(*viii*) $x+3y-6=0$, $3x-y-8=0$ (*ix*) $x+3y-12=0$, $3x-y-16=$
(*x*) $\sqrt{2}\,x+\sqrt{2}\,y-a=0$; $x-y=0$.

2. (*a*) (*i*) $x+5y-2=0$; $5x-y-10=0$ (*ii*) $x-2y-1=0$; $2x+y-2=$
(*b*) (*i*) $x+y-2=0$; $x-y+2=0$;
(*ii*) $x-36y-6=0$; $216x+6y+1=0$

3. (*i*) (0, 5), (1, 4) (*ii*) (1, 0), $\left(\frac{1}{3}, \frac{4}{27}\right)$ (*iii*) (2, 2), (–2, 34)

4. (*i*) (5, 5) (*ii*) (–2, 1), (2, –1) (*iii*) (a, 0)

5. (*i*) $6x+3y-5=0$; $x-2y+5=0$ (*ii*) $10x+y+13=0$; $x-10y-29=$
(*iii*) $3x-4y+6=0$; $12x+9y-26=0$ (*iv*) $4x+y-10=0$; $x-4y+6=$
(*v*) $13x-y-3=0$ and $13x-y+17=0$; $x+13y-131=0$ and $x+13$
$-51=0$ (*vi*) $3x+y+6=0$; $x-3y+2=0$ (*vii*) $64x+256y-$
$1=0$; $1024x-256y+33=0$.

6. $\frac{49}{8}$ sq. units 8. $a=2$, $b=-7$ 9. $a=2$, $b=3$.

4.2 Angle between two curves

1. (*a*) Explain how will you find the angle between two cur
(*b*) Find the angle (acute) between the following curves:
(*i*) $y=x^2+3x-7$ and $y^2=2x+5$ at (2, 3)
(*ii*) $x^2+2y^2=14x-16$ and $y^2=2x$ at (2, 2)
(*iii*) $y=x(x^2-4)$ and $y=2x^2-3x-2$ at (1, –3)
(*iv*) $y^2=4x$ and $y^2=4x^3$ at (1, 2)
(*v*) $4y=x^3$ and $y=6-x^2$ at (2, 2)
(*vi*) $y=\dfrac{x+3}{x^2+1}$ and $y=\dfrac{x^2-7x+11}{x-1}$ at (2, 1)
(*vii*) $y=2\sin^2 x$ and $y=\cos 2x$ at $\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right)$

2. Find the angle (acute) of intersection of the curves:
(*i*) $x^2+y^2=8$ and $x^2=2y$
(*ii*) $y=x^2-1$ and $y=-2x^2+2$
(*iii*) $y^2=4x$ and $x^2+y^2=5$

(*iv*) $y^2=x$ and $x^2=y$ at the point other than origin

(*v*) $y^2=x$ and $y^2=x^3$ at the points other than (0, 0)

(*vi*) $x^2+y^2=13$ and $x^2+y^2-8x+3=0$ at one of their points of intersection.

(*vii*) $x^2=y$ and $y=8-x^2$ (*viii*) $y=x^2+x-21$ and $y=x^2-3x-5$

(*ix*) $x^2-4x+y^2=0$ and $x^2+y^2=8$

3. (*i*) Show that the curves $y=x^3+x+1$ and $2y=x^3+5x$ touch at the point (1, 3) and also find the equation of the common tangent.

(*ii*) Show that the curves $xy+3x=1$ and $y^2+x+5y+5=0$ touch each other at (1, –2). Also find the equations of the common tangent and common normal.

(*iii*) Show that the curves (*a*) $y^2=8x$ and $x^2=4y-12$ (*b*) $y=6+x-x^2$ and $y(x-1)=x+2$ touch at (2, 4). Also find the equations of the common tangent and common normal.

(*iv*) Show that the curves $y=x^3+2$ and $y=2x^2+2$ touch each other at (0, 2) and intersect at an angle $\tan^{-1}(\frac{4}{97})$ at (2, 10).

4. Show that the following curves cut each other orthogonally at the given points:

(*i*) $2y=3x+x^2$ and $2x+3y-y^3=0$ at (0, 0)

(*ii*) $y=x^3$ and $6y=7-x^2$ at (1, 1)

(*iii*) $x^3+y^3+x+2y=0$ and $xy+2x=y$ at origin

(*iv*) $y=2x^2+3x$ and $3y=3x^2-x$ at the origin

5. Show that the curves $x^3+2=3xy^2$ and $3x^2y-y^3=0$ cut orthogonally (at any point of intersection).

6. Show that the curves $\frac{x^2}{A}+\frac{y^2}{B}=1$ and $\frac{x^2}{a}+\frac{y^2}{b}=1$ cut each other orthogonally if A–B=a–b.

Answers

1. (*b*) (*i*) $\tan^{-1} 2$ (*ii*) $\tan^{-1}\frac{6}{13}$ (*iii*) 90° (*iv*) $\tan^{-1}\frac{1}{2}$ (*v*) $\tan^{-1}\frac{7}{11}$ (*vi*) 45° (*vii*) $\tan^{-1}3\sqrt{3}$ 2. (*i*) $\tan^{-1}$ 3. (*ii*) $\tan^{-1}\frac{6}{7}$ (*iii*) $\tan^{-1} 3$ (*iv*) $\tan^{-1}\frac{3}{4}$ (*v*) $\tan^{-1}\frac{4}{7}$ (*vi*) $\tan^{-1}\frac{12}{5}$ (*vii*) $\tan^{-1}\frac{8}{15}$ (*viii*) $\tan^{-1}\frac{2}{23}$ (*ix*) 45°

3. (*i*) $4x-y-1=0$ (*ii*) $x+y+1=0$; $x-y-3=0$ (*iii*) (*a*) $x-y+2=0$; $x+y-6=0$ (*b*) $3x+y-10=0$; $x-3y+10=0$.

4.3 Subtangent and Subnormal

1. Define subtangent and subnormal and obtain expressio for the same.
2. Find the lengths of subtangent and subnormal at the poi
   - (*i*) (1, 4) on the curve $5x^2-xy-1=0$
   - (*ii*) (2, 1) on the curve $xy(x+y)=6$
   - (*iii*) $(1, \frac{1}{2})$ on $y(x+1)=x$
   - (*iv*) (2, 1) on $y=x^3+x^2-11$
   - (*v*) (1, –3) on $x^2+xy+y^2=7$
   - (*vi*) where the *y*–axis cuts the curve $y=5x-4x^3+2$
   - (*vii*) $\theta=\pi/2$ on $x=a(1-\cos\theta)$, $y=a(\theta+\sin\theta)$
3. At any point on the curve $y=b\sin(x/a)$, find the lengths the ST and SN (subtangent and subnormal).
4. Show that in the parabola $y^2=4ax$, the subtangent at a point is double the abscissa and the subnormal is consta
5. Show that for the curve $x^2-y^2=a^2$, the length of SN is eq to the abscissa.
6. Prove that in the curve $x^m y^n=a^{m+n}$, the subtangent at a point varies as the abscissa.
7. In the curve $x^3y^2=a^4$, show that the ST at any point var as the abscissa of the point.
8. Prove that in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the subnormal varies the abscissa.
9. In the curve $y=be^{\frac{x}{a}}$, prove that SN varies as the square the ordinate.
10. Prove that the subtangent at any point on the curve (*i*) $y^m=kx^n$ (*ii*) $x=ay^n$ varies as the abscissa.
11. Show that in the curve $ay^2=(x+b)^3$, the square of the varies as the SN.
12. Prove that for the curve $x^5=ay^6$, the square of the ST va as the cube of the SN.
13. Prove that in the curve $x^{m+n}=a^{m-n}y^{2n}$, the m^{th} power ST varies as the n^{th} power of SN.

14. Prove that in the curve $xy=c^2$, the subnormal (*i*) varies as the cube of the ordinate and (*ii*) varies inversely as the cube of the obscissa.

15. In the curve $x^2y^2=a^2(x^2-a^2)$, show that SN varies inversely as the cube of the abscissa.

Answers

2. (*i*) $\frac{2}{3}$; 24 (*ii*) $\frac{8}{5}$; $\frac{5}{8}$ (*iii*) 2; $\frac{1}{8}$ (*iv*) $\frac{1}{16}$; 16 (*v*) 15; $\frac{3}{5}$ (*vi*) $\frac{2}{5}$, 10 (*vii*) $a\left(\frac{\pi}{2}+1\right)$; $a\left(\frac{\pi}{2}+1\right)$ 3. $a \tan \frac{x_1}{a}$; $\frac{b^2}{a} \sin \frac{x_1}{a} \cos \frac{x_1}{a}$

4.4 Derivative as a rate measurer

1. Define $\frac{dy}{dx}$ as a rate measurer and express velocity and accelaration as derivatives; with the usual notation if $s=4t^3-6t^2+t-7$, find the acceleration and velocity when $t=2$.

2. The displacement s cms. of a particle at time t secs. is given by $s=2t^3-3t^2-36t+90$. Find (*i*) the velocity after 3 secs. (*ii*) acceleration after 4 secs. (*iii*) the displacement and the acceleration when the velocity vanishes (*iv*) the displacement and the acceleration when the velocity is 36 cm./sec. (*v*) the displacement and the velocity when the acceleration is zero.

3. With the usual notation if $s=2t^3-9t^2+12t+6$, find when the acceleration is zero and find the velocity at the instant.

4. When breakes are applied to a moving car, the car travels a distance s feet in t secs. given by $s=20t-40t^2$. When does the car stop?

5. A particle shot vertically upwards rises 's' feet in t secs. where $s=40t-16t^2$. Find the initial velocity and the greatest height attained by the particle.

6. If $s=t^3-6t^2+9t+8$ (*i*) find the initial velocity (*ii*) when will the body be at rest momentarily (*iii*) find the acceleration at that instant.

7. If $s=160t-16t^2$, show that the velocities at $t=1$ and $t=9$ are equal in magnitude but opposite in direction. Also prove that the displacement is 400 units when the velocity is zero.

8. A prticle moves according to the law $s=t^3+at^2+bt$. Find a and b if the initial velocity is 5 units and when $t=1$ it is moving with a velocity which is 4 times the initial velocity.

9. If $s=at^3+bt$, find a and b given that when $t=3$ velocity is 0 and acceleration is 14 units.

10. A particle moves along a straight line so that its distiance 's' from a fixed point on it at any time t is given by $s=2\sin t+3\cos t$. Show that the acceleration is always numerically equal to the distance from the fixed point. Also find the velocity at $t=\pi$ and acceleration when $t=\pi/2$.

11. A point moves on a straight line so that its distance 's' feet from a fixed point on the line at time t is $s=5 \cos 2t$. Find its velocity and acceleration in terms of s.

12. If $s=kv^2$, where k is constant, prove that acceleration is constant.

13. Prove that $\frac{dv}{dt}=v\frac{dv}{ds}$. If $v^2=s^2+4s+4$, find the acceleration when $s=10$.

14. If $s^2=at^2+2bt+c$, prove that

(*i*) the acceleration is inversely proportional to s^3

(*ii*) the acceleration is $\frac{a-v^2}{s}$ where v is the velocity.

15. Express each of the following statements as an equatio involving differential coefficient:

(*i*) The volume increases uniformly at the rate of 6 cm^3/mi

(*ii*) A metal plate is expanding at the rate of 5 sq. cm./sec.

(*iii*) Water is flowing out of a conical filter at 0.5 cc./sec.

16. (*i*) The radius of a sphere is increasing at the rate o 12 ft./minute. Find the rates of increase of its volum and of surface area (*a*) when its radius is 5 ft. (*b*) when i volume is 36π c. ft. (*c*) when its surface area is 64π sq. f (*d*) after 1 second.

(*ii*) The radius of a sphere is increasing at the rate of 0.5 mts/se Find the rates of increase of the surface area and of th volume after 3 secs.

(*iii*) The surface area of a spherical bubble is increasing at th rate of 0.8 sq. cm/sec At what rates are the radius a the volume of the bubble increasing when its surface area is 25π square cm.

(iv) The volume of a spherical ball is increasing at the rate of 4π c.c./sec. Find the rates of increase of the radius and of the surface area when the volume is 288π c.c.

(v) The volume of a sphere is increasing at the rate of 4π c.c./sec. Find the rate at which the area of its surface increases when its radius is 10 cms.

(vi) The surface area of a spherical balloon is decreasing at the rate of 35 sq. cm/sec. Find the rate of decrease of the volume of the balloon when its diameter is 4 cm.

(vii) The volume of a sphere is increasing at a constant ratio. Prove that its radius is increasing at a rate inversely proportional to the square of the radius of the sphere.

(viii) A spherical iron ball 8 cms. in diameter is coated with a layer of wax of uniform thickness. If the wax melts at the rate of 10 c.c./sec., prove that the thickness of the wax decreases at the rate of $5/72\pi$ cm./sec. when it is 2 cms. thick.

17. (i) A circular patch of oil spreads on water, the area growing at the rate of 2 sq. cm./min. How fast are the radius and the circumference increasing when the diameter is 24 cm.

(ii) A metal plate in the form of a circle expands uniformly at the rate of 2 cm^2/sec. Find the rate at which its radius is increasing when the area is 5 cm^2.

(iii) A drop of ink spreads over a plotting paper so that the circumference of the blot, which is circular, increases at a rate of 3 cm./sec. Find the rates of increase of the radius and of the area of the blot when its circumference is 4π cm.

(iv) A circular plot of ink increases in area in such a way that the radius r inches at time t is given by $r=2t^2-t^3/4$. What is the rate of increase of the area when $t=4$.

(v) A stone is dropped into a pond, waves in the form of circles are generated and the radius of the outer most ripple increases at the rate of 2 inches/sec. How fast is the area increasing when the radius is 5 inches? after 5 secs.?

18. (*i*) A square plate expands uniformly, the side increasing at the rate of 5 cm./sec. What is the rate at which the area is increasing when the side is 20 cms.

(*ii*) The sides of an equilateral triangle are increasing at the rate of 2 cm./sec. How fast is its area increasing when the sides are 10 cms.

(*iii*) A cylinderical tank is 10 metres in diameter. Water is flowing in it at the rate of 24 m^3/sec. At what rate is the height of the water rising.

(*iv*) Water is being poured into a cylinderical tub at the rate of v cc/min. If the tub has a circular base of r cms., find the rate at which the level of water rising in the tub.

(*v*) A metal cube is heated such that its edge expands at the rate of 0.02 cm./min. Prove that the volume is increasing at the rate of 6 cc/min., when the edge is 10 cm.

19. (*i*) The height of a cone is 30 cms. and it is constant and the radius of the base is increasing at the rate of 0.25 cm./sec. Find the rate of increase of volume of the cone when the radius is 10 cms.

(*ii*) Water flows out of a conical filter at the steady rate of 4 cubic inches/sec. The height of the filter is 3 inches and the radius of the circular base is 4 inches. Find the rate at which the level of water falls when the depth of the water is 2 inches.

(*iii*) A filter paper in the form of a right circular cone with its altitude 5 inches and the radius of the base 3 inches. Water is being poured into it in such a way that the height of the water cone is increasing at the rate of $\frac{25}{27\pi}$ inches/min. Find the rate at which (a) the radius of the water cone increases (*b*) the volume of the water cone increases (*c*) the area of the water surface increases, when the height of the water is 3 inches.

(*iv*) Sand is poured on the ground at the rate of 2 cubic metres per minute and formed a conical pile whose height is equal to the radius of the base. At what rate is the height increasing when the height is one metre.

20. (*i*) A ladder 20 ft. long rests with its ends on a smooth horizontal floor and against a smooth vertical wall. If the lower end is moved at the rate of 5 ft./sec., find the rate at which the upper end moves when the lower end is 12′ from the wall.

(*ii*) A rod 13 ft. long slides with its ends A and B on two straight lines at right angles which meet at O. If A is moved away from O with a uniform speed of 4 ft./sec., find the speed of the end B, when A is 5 feet from O.

21. (*i*) A man of height 1.5 m walks directly away from a source of light 5 metres above the ground level at the rate of 6 metres/sec. Find the rate at which his shadow lengthens and the rate at which the tip of shadow moves.

(*ii*) A street lamp is hung 12 feet above a straight horizontal floor on which a man of 5 feet is walking. How fast is his shadow lengthening when he is walking away from the lamp post at the rate of 175 ft./min.? Also find the rate at which the tip of his shadow is moving.

22. Two persons start at the same time from a place. One travels west at 40 km./hr. and the other travels north at 50 km./hr. How fast is the distance between them increasing after 3 hours.

23. At constant temperature. pv=constant, where p is the pressure and v is the volume. Find the rate at which the volume is increasing w.r.t. pressure when (*a*) $v=3$ (*b*) $p=4$. It is given that when $v=4, p=6$.

24. Prove that the rate of increase of the area of the circle w.r.t. its radius is proportional to its radius and the rate of increase of its circumference is a constant.

25. Find the rates at which the volume and the surface area of a spherical balloon are increasing w.r.t. its radius when its radius is 3 cms.

Answers

1. 36; 25 units 2. (*i*) 0 (*ii*) 42 cm/sec^2 (*iii*) 9 cms; 30 cm/sec^2
(*iv*) 26 cms; 42 cm/sec^2 (*v*) $\frac{143}{2}$ cms; $-\frac{75}{2}$ cm/sec
3. $\frac{8}{2}$; $-\frac{8}{2}$ units 4. $\frac{1}{4}$ secs. 5. 40′/sec; $\frac{5}{4}$ secs; 25 feet
6. 9; 1, 3; -6, 6 units 8. $a=6, b=5$ 9. $a=\frac{7}{9}, b=-21$

10. -2 units each 11. $v=-2\sqrt{25-s^2}$; $-4s$ 13. 12 units

15. (*i*) $\frac{dv}{dt}=6$ cm³/min (*ii*) $\frac{dA}{dt}=5$ cm²/sec (*iii*) $\frac{dv}{dt}=-0.5$ c.c./sec.

16. (*i*) (*a*) $200\,\pi$ ft³/min.; $480\,\pi$ ft²/min.
 (*b*) $432\,\pi$ ft³/min.; $288\,\pi$ ft²/min.
 (*c*) $768\,\pi$ ft³/min.; $384\,\pi$ ft²/min.
 (*d*) $\frac{48\,\pi}{25}$ ft³/min.; $\frac{96\,\pi}{5}$ ft/min.

(*ii*) $6\,\pi$ m²/sec; 4.5π m³/sec. (*iii*) $\frac{1}{25\,\pi}$ cm/sec.; 1 cc./sec

(*iv*) $\frac{1}{36}$ cm/sec; $\frac{4\pi}{3}$ sq cm/sec. (*v*) $\frac{4\pi}{5}$ sq.cm/sec (*vi*) 35 cc./sec.

17. (*i*) $\frac{1}{12\pi}$ cm./min.; $\frac{1}{6}$ cm./min. (*ii*) $\frac{1}{\sqrt{5\pi}}$ cm/sec.

(*iii*) $\frac{3}{2\pi}$ cm/sec; 6 cm²/sec (*iv*) $128\,\pi$ sq. inches/unit time

(*v*) 20π inches²/sec.; 40π inches²/sec.

18. (*i*) 200 cm²/sec. (*ii*) $10\sqrt{3}$ cm²/sec. (*iii*) $\frac{24}{25\pi}$ m/sec.

(*iv*) $\frac{v}{\pi r^2}$ cm²/min. 19. (*i*) 50π cm³/sec. (*ii*) $\frac{9}{16\pi}$ inches/sec.

(*iii*) (*a*) 3 cubic inches/sec. (*b*) $\frac{5}{9\pi}$ inches/sec. (*c*) 2 inches/sec.

(*iv*) $\frac{2}{\pi}$ m/min. 20. (*i*) $\frac{15}{4}'$/sec .(slides down)

(*ii*) $\frac{5}{3}'$/sec. (slides down) 21. (*i*) $\frac{18}{7}$ m/sec; $\frac{60}{7}$ m/sec.

(*ii*) 125′/sec.; 300′/sec. 22. $10\sqrt{41}$ km/hr.

23. (*a*) $-\frac{3}{8}$ units (*b*) $-\frac{3}{2}$ units 25. 36π units; 24π units.

4.5 Maxima and Minima

1. Define critical values of a function.
2. Find the critical values of the function
 (*i*) $\frac{1}{3}x^3+\frac{1}{2}x^2-6x+8$ (*ii*) $(x+3)^3(x-4)^4$ (*iii*) x^3+3/x.
3. Define (relative) maximum and minimum points of a function and prove that they are critical points.

4. State the second derivative test for maxima and minima.

5. Examine the following functions for maxima and minima:

(*i*) x^2+16/x (*ii*) $x \log x$ (*iii*) $3+2x-x^2$ (*iv*) $\dfrac{\log x}{x}$

(*v*) $x^3-3x^2-9x+17$ (*vi*) $2x^3-15x^2+36x+12$

(*vii*) x^3-6x^2+9x-8 (*viii*) $x(12-2x)^2$ (*ix*) $\dfrac{x}{1+x^2}$

(*x*) $x+\dfrac{1}{x+1}$ (*xi*) $x^2 e^{-x}$ (*xii*) $2 \sin x-\sin 2x,\ 0 \leqslant x < 2\pi$

(*xiii*) $4 \sin^2 x+3 \cos^2 x,\ 0 \leqslant x < \pi$.

6. Find the minimum point of the catenary $y=a \cosh \dfrac{x}{a}$.

7. Prove that x^x is minimum for $x=1/e$.

8. The sum of two positive integers is 30. Prove that the numbers must be equal if (*i*) their product is maximum (*ii*) the sum of their squares is minimum.

9. The product of two positive integers is 16. Find the numbers if (*i*) their sum is least (*ii*) the sum of one and the square of the other is least.

10. Show that a right angled triangle of a given perimeter has the greatest area when it is isosceles.

11. (*i*) Prove that among all rectangles of a given area, the square has the minimum perimeter.

(*ii*) Prove that among all rectangles with a fixed perimeter, the square is the one with (*a*) maximum area (*b*) shortest diagonal.

(*iii*) Find the maximum area of the rectangle that can be inscribed in a circle of radius *r*.

(*iv*) What is the largest size rectangle that can be inscribed in a semicircle of radius 1 so that two vertices lie on the diameter.

(*v*) Find the rectangle of greatest perimeter which can be inscribed in a circle of radius *r*.

(*vi*) A sheet of paper for a poster contains 18 square feet. The margin at the top and bottom are 9″ and at the sides 2″. What are the dimensions if the printed area is a maximum.

12. (*i*) The slant height of a right circular cone is 4 units. Find the dimensions of the cone which has maximum volume.

(*ii*) Show that the semivertical angle of a right circular cone of maximum volume and of a given slant height is $\tan^{-1}\sqrt{2}$.

(*iii*) Show that a right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of its base.

(*iv*) The surface area of a cone including the base is 4π square feet. Find the dimensions of the cone when the volume is maximum.

13. (*i*) Show that the volume of the greatest right circular cylinder which can be inscribed in a sphere of radius r is $4\pi r^3/3\sqrt{3}$.

(*ii*) Find the maximum volume of the right circular cylinder that can be inscribed in a cone of base radius r and height h.

(*iii*) A right circular cylinder closed at both ends is to be made from 12 square metres of sheet metal. What must be the area of the base if the volume is to be maximum.

(*iv*) A right circular cylinder has a volume of 64 cc. Find the dimensions so that the surface area is minimum when the cylinder is (*a*) open at one side (*b*) a closed one.

Answers

2. (*i*) $-3, 2$ (*ii*) $-3, 0, 4$ (*iii*) $-1, 1$

5.

	Maximum point	Maximum value	Minimum point	Minimum value
(*i*)	—	—	2	12
(*ii*)	—	—	$1/e$	$-1/e$
(*iii*)	1	4	—	—
(*iv*)	e	$1/e$	—	—
(*v*)	-1	22	3	-10
(*vi*)	2	40	3	39
(*vii*)	1	-4	3	-8
(*viii*)	2	128	6	0

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(*ix*)	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$
(*x*)	-2	-3	0	1
(*xi*)	2	$4e^{-2}$	0	0
(*xii*)	$2\pi/3$	$3\sqrt{3}/2$	$4\pi/3$	$-3\sqrt{3}/2$
(*xiii*)	$\pi/2$	4	0	3

6. $(0, a)$ 9. (*i*) 4, 4 (*ii*) 8, 2 11. (*iii*) and (*v*) square of side $\sqrt{2}r$ (*iv*) $\sqrt{2}$, $1/\sqrt{2}$ units (*vi*) $2\sqrt{3}'$, $3\sqrt{3}'$ 12. (*i*) $h=4/\sqrt{3}$ units; $r=4\sqrt{2}/\sqrt{3}$ units (*iv*) $r=1'$; semivertical angle$=\sin^{-1}(\frac{1}{3})$

13. (*ii*) $\frac{4\pi r^2 h}{27}$ (*iii*) 2 sq.m. (*iv*) (*a*) $h=r=4/\sqrt[3]{\pi}$ cm.

(*b*) $h=2r=4\sqrt[3]{4/\pi}$ cm.

CHAPTER 5

INTEGRATION

5.1 Standard forms

1. Define indefinite integral of a function as anti-derivative and obtain the fundamental integration formulas including integrals of hyperbolic functions.

2. Integrate the following w.r.t. x

(*i*) x^4 (*ii*) 1 (*iii*) $x^{\frac{7}{5}}$ (*iv*) $\frac{1}{\sqrt{x}}$ (*v*) $\sqrt[3]{x^2}$ (*vi*) $\frac{1}{x^2}$

(*vii*) $\frac{1}{\sin^2 x}$ (*viii*) $\frac{\cos x}{\sin^2 x}$ (*ix*) 5^x (*x*) $\frac{1}{\cosh^2 x}$

3. (a) Prove that $\int kf(x)dx=k\int f(x)dx$ and $\int[f(x)\pm g(x)]\,dx$ $=\int f(x)dx\pm\int g(x)dx$; obtain the integrals of

(*i*) $3x^2$ (*ii*) $\frac{4}{x}$ (*iii*) $2x+3$ (*iv*) $5\cos x+3\sin x$ w.r.t. x

(b) Integrate the following w.r.t. x

(*i*) ax^2+bx+c (*ii*) $3\sec x-4\tan x$ (*iii*) $x^2-\cot x-\operatorname{cosec} x$

(*iv*) $\frac{x^5+3x^2-4x}{x^7}$ (*v*) $\frac{x^2-3x+1}{\sqrt{x}}$ (*vi*) $\frac{x+1}{x}$ (*vii*) $\frac{x^3+3x-2}{x}$

(*viii*) $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ (*ix*) $\left(x^2-\frac{1}{x}\right)^3$ (*x*) $(1-x)\sqrt{x}$ .... ..

(*xi*) $\dfrac{(1+x)^2}{\sqrt{x}}$ (*xii*) $\dfrac{\sin x+\cos x}{\cos x}$ (*xiii*) $(1+\tan x)^2$

(*xiv*) $e^x-x^e+e^e$ (*xv*) $2\sinh x+4\cosh x$ (*xvi*) $\dfrac{3\cosh x-4}{\sinh^2 x}$

(*xvii*) $(2x-1)(3x+2)$ (*xviii*) $\dfrac{(x+1)(x^2-1)}{x}$

(*xix*) $3x^2-4x+8x^0+\dfrac{2}{x}-\dfrac{3}{x^2}+\dfrac{2}{x^3}$

4. Integrate the following w.r.t. x

(*i*) $\sqrt{1+\sin 2x}$ (*ii*) $\sqrt{1-\sin 2x}$ (*iii*) $\sqrt{1+\cos 2x}$

(*iv*) $\sqrt{1-\cos 2x}$ (*v*) $\sqrt{\dfrac{1+\cos 2x}{1-\cos 2x}}$ (*vi*) $\dfrac{1}{1+\cos x}$

(*vii*) $\dfrac{1}{1-\cos x}$ (*viii*) $\dfrac{1}{1+\sin x}$ (*ix*) $\dfrac{1}{1-\sin x}$

(*x*) $\dfrac{\sin^2 x}{1+\cos x}$ (*xi*) $\dfrac{1}{\sin^2 x\cos^2 x}$ (*xii*) $\dfrac{\cos 2x}{\sin^2 x\cos^2 x}$

(*xiii*) $\dfrac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}$

5. (*a*) If $\int f(x)\,dx=g(x)+c$, then prove that $\int f(ax+b)\,dx=\dfrac{1}{a}g(ax+b)+c,\ a\neq 0$; using this obtain the integrals of $(ax+b)^n$, $\sin(ax+b)$, $\cos(ax+b)$ etc., e^{ax+b}, $\sinh(ax+b)$, $\cosh(ax+b)$ etc. w.r.t. x.

(*b*) Integrate the following w.r.t. x

(*i*) $\sin 13x$ (*ii*) $\cos(14x+5)$ (*iii*) $\dfrac{\sin 3x}{\cos 3x}$ (*iv*) $\dfrac{1}{\sin^2(4x+3)}$

(*v*) $\dfrac{1}{\cos^2 8x}$ (*vi*) $\dfrac{\sin 3x}{\cos^2 3x}$ (*vii*) $\dfrac{\cos 5x}{\sin^2 5x}$ (*viii*) $\dfrac{\cos 14x}{\sin 14x}$

(*ix*) $\dfrac{1}{\sin 2x}$ (*x*) $\dfrac{1}{\cos 3x}$ (*xi*) $(2+3x)^5$ (*xii*) $\dfrac{1}{3+4x}$

(*xiii*) $\dfrac{1}{(13x+1)^5}$ (*xiv*) $\sqrt{1-4x}$ (*xv*) $\dfrac{1}{\sqrt[4]{8-x}}$ (*xvi*) $\dfrac{1}{1-x}$

(*xvii*) $\dfrac{1}{\sqrt{2x-3}}$ (*xviii*) $e^{-x/2}+\dfrac{1}{e^x}+e^{2x}$ (*xix*) $\dfrac{1}{4x-3}$

(*xx*) $\sqrt{1+\sin x}$ (*xxi*) e^{4x-3} (*xxii*) $e^{(x-4)/3}$

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(*xxiii*) $\sinh(2x+1)$ (*xxiv*) $\cosh\left(\frac{x}{2}-3\right)$ (*xxv*) $\tanh 3x$

(*xxvi*) $\coth 2x$ (*xxvii*) $\operatorname{sech}^2 \frac{2x}{3}$ (*xxviii*) $\operatorname{cosech}^2(1+x)$

(*xxix*) $\operatorname{sech} 3x \tanh 3x$ (*xxx*) $\operatorname{cosech} 2x \coth 2x$

(*xxxi*) $e^x \sinh x$ (*xxxii*) $e^{2x} \cosh x$

6. Integrate the following w.r.t. x

(*i*) $\sin 15x \cos 13x$ (*ii*) $\cos 4x \cos 2x$ (*iii*) $\sin 4x \cos 3x$
(*iv*) $\cos 5x \sin 3x$ (*v*) $\sin 5x \sin 3x$ (*vi*) $\sin 4x \sin 6x$
(*vii*) $\cos 10x \cos 14x$ (*viii*) $\sin 5x \cos 2x$ (*ix*) $\cos 5x \cos 3x$
(*x*) $\cos 3x \cos 2x \cos x$

7. Integrate the following w.r.t. x

(*i*) $\sin^2 x$ (*ii*) $\cos^2 x$ (*iii*) $\tan^2 x$ (*iv*) $\cot^2 x$

(*v*) $\frac{1+\cos x}{1-\cos x}$ (*vi*) $\frac{\sin^2 x}{(1+\cos x)^2}$ (*vii*) $\tanh^2 x$ (*viii*) $\coth^2 x$

8. Integrate the following w.r.t. x

(*i*) $\sin^2 3x$ (*ii*) $\cos^2 5x$ (*iii*) $\tan^2 6x$ (*iv*) $\sin^2 x \cos^2 x$ (*v*) $\cot^2 8x$
(*vi*) $\sin^2 8x$ (*vii*) $\cos^2 15x$ (*viii*) $\tan^2 16x$ (*ix*) $\cot^2 15x$
(*x*) $(\tan x-\sec x)^2$ (*xi*) $(\tan x+\cot x)^2$ (*xii*) $(\coth x-\tanh x)^2$

9. Integrate the following w.r.t. x

(*i*) $\sin^3 x$ (*ii*) $\cos^3 x$ (*iii*) $\sin^4 x$ (*iv*) $\cos^4 x$ (*v*) $\sin^3 2x$
(*vi*) $\cos^3 2x$ (*vii*) $\sin^4 2x$.

10. Integrate the following w.r.t. x

(*a*) (*i*) $\frac{1}{x^2+a^2}$ (*ii*) $\frac{1}{x^2+5}$ (*iii*) $\frac{1}{x^2+16}$

(*b*) (*i*) $\frac{1}{\sqrt{a^2-x^2}}$ (*ii*) $\frac{1}{\sqrt{25-x^2}}$ (*iii*) $\frac{1}{\sqrt{7-x^2}}$

(c) (*i*) $\frac{1}{\sqrt{x^2+a^2}}$ (*ii*) $\frac{1}{\sqrt{x^2+4}}$ (*iii*) $\frac{1}{\sqrt{x^2+3}}$

(*d*) (*i*) $\frac{1}{\sqrt{x^2-a^2}}$ (*ii*) $\frac{1}{\sqrt{x^2-9}}$ (*iii*) $\frac{1}{\sqrt{x^2-5}}$

(e) (*i*) $\frac{1}{a^2-x^2}$ (*ii*) $\frac{1}{4-x^2}$ (*iii*) $\frac{1}{16-x^2}$

(*f*) (*i*) $\frac{1}{x^2-a^2}$ (*ii*) $\frac{1}{x^2-25}$ (*iii*) $\frac{1}{x^2-9}$

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Answers

(Add arbitrary constant c to all the answers.)

2. (*i*) $\frac{x^5}{5}$ (*ii*) x (*iii*) $\frac{5}{12}x^{12/5}$ (*iv*) $2x^{\frac{1}{2}}$ (*v*) $\frac{3}{5}x^{\frac{5}{3}}$ (*vi*) $-\frac{1}{x}$
(*vii*) $-\cot x$ (*viii*) $-\operatorname{cosec} x$ (*ix*) $5^x/\log 5$ (*x*) $\tanh x$

3. (*a*) (*i*) x^3 (*ii*) $4\log x$ (*iii*) x^2+3x (*iv*) $5\sin x-3\cos x$
(*b*) (*i*) $a\frac{x^3}{3}+b\frac{x^2}{2}+cx$ (*ii*) $3\log(\sec x+\tan x)-4\log\sec x$
(*iii*) $\frac{x^3}{3}-\log\sin x+\log(\operatorname{cosec} x-\cot x)$ (*iv*) $-\frac{1}{x}-\frac{3}{4x^4}+\frac{4}{5x}$
(*v*) $\frac{2}{5}x^{\frac{5}{2}}-2x^{\frac{3}{2}}+2x^{\frac{1}{2}}$ (*vi*) $x+\log x$ (*vii*) $\frac{x^3}{3}+3x-2\log$
(*viii*) $\frac{x^2}{2}+\log x+2x$ (*ix*) $\frac{x^7}{7}-\frac{3}{4}x^4+3x+\frac{1}{2x^2}$ (*x*) $\frac{2}{3}x^{\frac{3}{2}}-$
(*xi*) $2x^{\frac{1}{2}}+\frac{4}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}$ (*xii*) $\log\sec x+x$ (*xiii*) $\tan x+2\log s$
(*xiv*) $e^x-\frac{x^{e+1}}{e+1}+e^e x$ (*xv*) $2\cosh x+4\sinh x$
(*xvi*) $-3\operatorname{cosech} x+4\coth x$ (*xvii*) $2x^3+\frac{x^2}{2}-2x$
(*xviii*) $\frac{x^3}{3}+\frac{x^2}{2}-x-\log x$ (*xix*) $x^3-2x^2+8x+2\log x+\frac{3}{x}-$

4. (*i*) $\sin x-\cos x$ (*ii*) $-(\cos x+\sin x)$ (*iii*) $\sqrt{2}\sin x$ (*iv*) $-\sqrt{2}\,\mathrm{co}$
(*v*) $\log\sin x$ (*vi*) $-\cot x+\operatorname{cosec} x$ (*vii*) $-(\cot x+\mathrm{cosec}$
(*viii*) $\tan x-\sec x$ (*ix*) $\tan x+\sec x$ (*x*) $x-\sin x$
(*xi*) $\tan x-\cot x$ (*xii*) $-\cot x-\tan x$ (*xiii*) $\sec x-\operatorname{cosec} x$

5. (*i*) $-\frac{1}{13}\cos 13x$ (*ii*) $\frac{1}{14}\sin(14x+5)$ (*iii*) $\frac{1}{3}\log\sec 3x$
(*iv*) $-\frac{1}{4}\cot(4x+3)$ (*v*) $\frac{1}{8}\tan 8x$ (*vi*) $\frac{1}{3}\sec 3x$ (*vii*) $-\frac{1}{5}\,\mathrm{cosec}$
(*viii*) $\frac{1}{14}\log\sin 14x$ (*ix*) $\frac{1}{2}\log(\operatorname{cosec} 2x-\cot 2x)$
(*x*) $\frac{1}{3}\log(\sec 3x+\tan 3x)$ (*xi*) $\frac{1}{18}(2+3x)^6$ (*xii*) $\frac{1}{4}\log(3+$
(*xiii*) $-\frac{1}{52}(13x+1)^{-4}$ (*xiv*) $-\frac{1}{6}(1-4x)^{\frac{3}{2}}$ (*xv*) $-\frac{4}{3}(8-x)^{\frac{3}{2}}$
(*xvi*) $-\log(1-x)$ (*xvii*) $\sqrt{2x-3}$ (*xviii*) $-2e^{-x/2}-e^{-x}+\frac{1}{2}e^{2x}$
(*xix*) $\frac{1}{4}\log(4x-3)$ (*xx*) $-2\cos\frac{x}{2}+2\sin\frac{x}{2}$ (*xxi*) $\frac{1}{4}e^{4x}$

(*xxii*) $3e^{(x-4)/3}$ (*xxiii*) $\frac{1}{2}\cosh(2x+1)$ (*xxiv*) $2\sinh\left(\frac{x}{2}-3\right)$

(*xxv*) $\frac{1}{3}\log\cosh 3x$ (*xxvi*) $\frac{1}{2}\log\sinh 2x$ (*xxvii*) $\frac{3}{2}\tanh\frac{2x}{3}$

(*xxviii*) $-\coth(x+1)$ (*xxix*) $-\frac{1}{3}\operatorname{sech} 3x$ (*xxx*) $-\frac{1}{2}\operatorname{cosech} 2x$

(*xxxi*) $\frac{1}{4}e^{2x}-\frac{1}{2}x$ (*xxxii*) $\frac{1}{6}e^{3x}+\frac{1}{2}e^{x}$.

6. (*i*) $-\frac{1}{2}\left[\frac{\cos 28x}{28}+\frac{\cos 2x}{2}\right]$ (*ii*) $\frac{1}{2}\left[\frac{\sin 6x}{6}+\frac{\sin 2x}{2}\right]$

(*iii*) $-\frac{1}{2}\left[\frac{\cos 7x}{7}+\cos x\right]$ (*iv*) $-\frac{1}{2}\left[\frac{\cos 8x}{8}-\frac{\cos 2x}{2}\right]$

(*v*) $-\frac{1}{2}\left[\frac{\sin 8x}{8}-\frac{\sin 2x}{2}\right]$ (*vi*) $-\frac{1}{2}\left[\frac{\sin 10x}{10}-\frac{\sin 2x}{2}\right]$

(*vii*) $\frac{1}{2}\left[\frac{\sin 24x}{24}+\frac{\sin 4x}{4}\right]$ (*viii*) $-\frac{1}{2}\left[\frac{\cos 7x}{7}+\frac{\cos 3x}{3}\right]$

(*ix*) $\frac{1}{2}\left[\frac{\sin 8x}{8}+\frac{\sin 2x}{2}\right]$ (*x*) $\frac{1}{4}\left[\frac{\sin 6x}{6}+\frac{\sin 4x}{4}+\frac{\sin 2x}{2}+x\right]$

7. (*i*) $\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]$ (*ii*) $\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]$ (*iii*) $\tan x-x$

(*iv*) $-(\cot x+x)$ (*v*) $-(2\cot\frac{x}{2}+x)$ (*vi*) $2\tan\frac{x}{2}-x$

(*vii*) $x-\tanh x$ (*viii*) $x-\coth x$

8. (*i*) $\frac{1}{2}\left[x-\frac{\sin 6x}{6}\right]$ (*ii*) $\frac{1}{2}\left[x+\frac{\sin 10x}{10}\right]$ (*iii*) $\frac{1}{6}\tan 6x-x$

(*iv*) $\frac{1}{8}\left[x-\frac{\sin 4x}{4}\right]$ (*v*) $-\left[\frac{\cot 8x}{8}+x\right]$ (*vi*) $\frac{1}{2}\left[x-\frac{\sin 16x}{16}\right]$

(*vii*) $\frac{1}{2}\left[x+\frac{\sin 30x}{30}\right]$ (*viii*) $\frac{\tan 16x}{16}-x$ (*ix*) $-\left[\frac{\cot 15x}{15}+x\right]$

(*x*) $2\tan x-2\sec x-x$ (*xi*) $\tan x-\cot x$ (*xii*) $-\tanh x-\coth x$

9. (*i*) $\frac{1}{4}\left[\frac{\cos 3x}{3}-3\cos x\right]$ (*ii*) $\frac{1}{4}\left[3\sin x+\frac{\sin 3x}{3}\right]$

(*iii*) $\frac{1}{4}\left[\frac{3}{2}x-\sin 2x+\frac{\sin 4x}{8}\right]$ (*iv*) $\frac{1}{4}\left[\frac{3}{2}x+\sin 2x+\frac{\sin 4x}{8}\right]$

(*v*) $\frac{1}{4}\left[\frac{\cos 6x}{6}-\frac{3\cos 2x}{2}\right]$ (*vi*) $-\frac{1}{4}\left[\frac{3\sin 2x}{2}+\frac{\sin 6x}{6}\right]$

(*vii*) $\frac{1}{4}\left[\frac{3}{2}x-\frac{\sin 4x}{2}+\frac{\sin 8x}{16}\right]$

10. (*a*) (*i*) $\frac{1}{a}\tan^{-1}\frac{x}{a}$ (*ii*) $\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}$ (*iii*) $\frac{1}{4}\tan^{-1}\frac{x}{4}$

(*b*) (*i*) $\sin^{-1}\frac{x}{a}$ (*ii*) $\sin^{-1}\frac{x}{5}$ (*iii*) $\sin^{-1}\frac{x}{\sqrt{7}}$ (*c*) (*i*) $\sinh^{-1}\frac{x}{a}$

(*ii*) $\sinh^{-1}\frac{x}{2}$ (*iii*) $\sinh^{-1}\frac{x}{\sqrt{3}}$ (*d*) (*i*) $\cosh^{-1}\frac{x}{a}$

(*ii*) $\cosh^{-1}\frac{x}{3}$ (*iii*) $\cosh^{-1}\frac{x}{\sqrt{5}}$ (*e*) (*i*) $\frac{1}{a}\tanh^{-1}\frac{x}{a}$

(*ii*) $\frac{1}{2}\tanh^{-1}\frac{x}{2}$ (*iii*) $\frac{1}{4}\tan^{-1}\frac{x}{4}$ (*f*) (*i*) $-\frac{1}{a}\coth^{-1}\frac{x}{a}$

or $\frac{1}{a}\tanh^{-1}\frac{x}{a}$ (*ii*) $-\frac{1}{5}\coth^{-1}\frac{x}{5}$ (*iii*) $-\frac{1}{3}\coth^{-1}$

5.2 Method of substitution

1. Explain the method of evaluating $\int f(g(x))g'(x)dx$; in partic

$\int [f(x)]^n f'(x)dx$; $\int \frac{f'(x)}{f(x)}dx$ and $\int \frac{f'(x)}{\sqrt{f(x)}}dx$

2. Integrate the following w.r.t. x

(*i*) $(3x^2+x+1)^{59}.(6x+1)$ (*ii*) $(2x^2-6x+4)^{\frac{3}{2}}(2x-3)$

(*iii*) $(4x^3-12x^2+24x+7)^3(x^2-2x+2)$ (*iv*) $(3x^2-4x+1)^{\frac{100}{8}}(3x$

(*v*) $\frac{x}{x^2+1}$ (*vi*) $\frac{2x+5}{x^2+5x-7}$ (*vii*) $\sqrt{5x^3+1}\,x^2$

(*viii*) $\frac{x^3}{\sqrt{1+x^4}}$ (*ix*) $\frac{ax+b}{\sqrt[3]{ax^2+2bx+c}}$ (*x*) $\frac{x^4}{\sqrt{10x^5+11}}$

(*xi*) $\frac{2ax+b}{(ax^2+bx+c)^n}$, (n≠1) (*xii*) $\frac{x+2}{\sqrt{x^2+4x+3}}$ (*xiii*) $\frac{x}{\sqrt{a^2}}$

3. Integrate the following w.r.t. x

(*i*) $\cot x$ (*ii*) $\tan x$ (*iii*) $\cos^{5/2}x\sin x$ (*iv*) $\tan^5 x\sec^2 x$

(*v*) $\sqrt{\sin x}\cos x$ (*vi*) $\frac{\cos x}{\sqrt[3]{\sin x}}$ (*vii*) $\frac{\sin x}{1-2\cos x}$

(*viii*) $(1+2\sin x)^5\cos x$ (*ix*) $\dfrac{\sin x}{\cos^4 x}$ (*x*) $\dfrac{\sin 3x}{\cos^5 3x}$

(*xi*) $\sin^3 x\cos x$ (*xii*) $\sec^3 x\tan x$ (*xiii*) $\tan^3 x\sec^2 x$

(*xiv*) $\sin x\sqrt[3]{\cos x}$ (*xv*) $\dfrac{\sec^2 x}{(4+\tan x)^4}$ (*xvi*) $\dfrac{1-\tan x}{1+\tan x}$

(*xvii*) $\sqrt{\cot x}\,\operatorname{cosec}^2 x$ (*xviii*) $\dfrac{\sqrt[3]{\tan x}}{\cos^2 x}$ (*xix*) $\sec^5 x\tan x$

(*xx*) $\dfrac{\cos x}{\sqrt[3]{1-9\sin x}}$

4. Integrate the following w.r.t. x

(*i*) $\sqrt{\dfrac{\sin^{-1}x}{1-x^2}}$ (*ii*) $\dfrac{(\cos^{-1}x)^3}{\sqrt{1-x^2}}$ (*iii*) $\dfrac{(\tan^{-1}x)^2}{1+x^2}$ (*iv*) $\dfrac{(\cot^{-1}x)^5}{1+x^2}$

(*v*) $\dfrac{\sec^{-1}x}{x\sqrt{x^2-1}}$ (*vi*) $\dfrac{\log x}{x}$ (*vii*) $\dfrac{1}{x}(\log x)^5$ (*viii*) $\dfrac{(\log x)^n}{x}$

(*ix*) $(e^x+1)^5 e^x$ (*x*) $\dfrac{e^x}{e^x-1}$ (*xi*) $\dfrac{1}{1+e^x}$

5. Integrate the following w.r.t. x

(*i*) $\sinh^3 x\cosh x$ (*ii*) $\tanh x\,\operatorname{sech}^2 x$ (*iii*) $\operatorname{sech}^3 x\tanh x$

(*iv*) $\dfrac{(\sinh^{-1}x)^2}{\sqrt{x^2+1}}$ (*v*) $\dfrac{\sqrt{\tanh^{-1}x}}{1-x^2}$ (*vi*) $\dfrac{(\operatorname{sech}^{-1}x)^5}{x\sqrt{1-x^2}}$

6. Integrate the following w.r.t. x

(*i*) $\dfrac{\sec^2 x}{a+b\tan x}$ (*ii*) $\dfrac{\sin x+\cos x}{\sin x-\cos x}$ (*iii*) $\left(1+\dfrac{1}{x}\right)(x+\log x)$

(*iv*) $\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$ (*v*) $\dfrac{1}{x\sqrt{1-\log x}}$ (*vi*) $\dfrac{1}{\sqrt{x}\,(1-\sqrt{x})}$

(*vii*) $\tan x\sqrt{\sec x}$ (*viii*) $(x-x^2)^4(2x-1)$ (*ix*) $\dfrac{\sin x}{5+3\cos x}$

(*x*) $\dfrac{\sin 2x}{\sqrt{1+\sin^2 x}}$ (*xi*) $\dfrac{\sqrt{\tan x}}{\cos^2 x}$ (*xii*) $\dfrac{1}{\sqrt{1+x^2}\,\sinh^{-1}x}$

(*xiii*) $\dfrac{x^{e-1}+e^{x-1}}{x^e+e^x}$ (*xiv*) $\dfrac{e^{-x}}{\sqrt[4]{1+e^{-x}}}$ (*xv*) $(2x^2-3x)^{\frac{8}{5}}(4x-3)$

7. Integrate the following w.r.t. x

(i) $x^2 \cos(x^3)$ (ii) $x \sin(1-3x^2)$ (iii) $\frac{\sin\sqrt{x}}{\sqrt{x}}$ (iv) $\frac{\cos}{\sqrt{\ }}$

(v) $\frac{\cos(\log x)}{x}$ (vi) $e^x \sin(1+e^x)$ (vii) $e^{-4x^3+1}\, x^2$

(viii) $\frac{e^{2\tan^{-1} x}}{1+x^2}$ (ix) $\frac{e^{-\sqrt{x}}}{\sqrt{x}}$ (x) $e^{4x^2+9}\, x$ (xi) $\frac{e^{1/x^2}}{x^3}$

(xii) $\frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}$ (xiii) $\cos x \cot(\sin x)$ (xiv) $\frac{1}{x} e^{2\log x+1}$

(xv) $e^{-x} \operatorname{cosec}^2(e^{-x})$ (xvi) $x \cosh x^2$ (xvii) $\frac{\operatorname{sech}^2\sqrt{x}}{\sqrt{x}}$

(xviii) $\frac{\cos(3+2\log x)}{x}$ (xix) $x^3 \operatorname{sech}(x^4) \tanh(x^4)$

(xx) $x^2 \operatorname{cosec}(x^3) \cot(x^3)$

8. Integrate the following w.r.t. x

(i) $\frac{\sin x}{1+\cos^2 x}$ (ii) $\frac{e^x}{1+e^{2x}}$ (iii) $\frac{x^3}{1+x^8}$ (iv) $\frac{\sec^2 x}{4+\tan^2 x}$

(v) $\frac{\sin x}{\sqrt{4-\cos^2 x}}$ (vi) $\frac{\cos x}{\sqrt{8-3\sin^2 x}}$ (vii) $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

(viii) $\frac{e^x}{\sqrt{9-e^{2x}}}$ (ix) $\frac{x}{\sqrt{1-x^4}}$ (x) $\frac{\sin x}{\sqrt{9\sin^2 x-5}}$

(xi) $\frac{1}{x[1+(\log x)^2]}$ (xii) $\frac{1}{x\sqrt{4-\log^2 x}}$

(xiii) $\frac{\sinh x}{4+\cosh^2 x}$ (xiv) $\frac{\log\cosh x}{\sqrt{9-4\tanh^2 x}}$

9. Integrate the following w.r.t. x

(i) $\frac{1}{4+9x^2}$ (ii) $\frac{1}{9+16x^2}$ (iii) $\frac{1}{1+(2x-7)^2}$ (iv) $\frac{1}{25+(4}$

(v) $\frac{1}{4x^2+4x+5}$ (vi) $\frac{1}{x^2+4x+5}$ (vii) $\frac{1}{4x^2-12x+13}$

(viii) $\frac{1}{9x^2+6x+17}$

10. Integrate the following w.r.t. x

(i) $\frac{1}{\sqrt{9-4x^2}}$ (ii) $\frac{1}{\sqrt{4-(3x+2)^2}}$ (iii) $\frac{1}{\sqrt{3-(x+1)^2}}$ (iv) $\frac{1}{\sqrt{5+4x-x^2}}$

(v) $\frac{1}{\sqrt{2x-x^2}}$ (vi) $\frac{1}{\sqrt{5-4x-4x^2}}$ (vii) $\frac{1}{\sqrt{9+24x-9x^2}}$

11. Integrate the following w.r.t. x

Hyperbolic

(i) $\frac{\cos x}{\sqrt{\sin^2 x+1}}$ (ii) $\frac{e^x}{\sqrt{e^{2x}+4}}$ (iii) $\frac{1}{\sqrt{x^2-2x+17}}$ (iv) $\frac{\sec^2 x}{\sqrt{\tan^2 x-9}}$

(v) $\frac{1}{\sqrt{4x^2-9}}$ (vi) $\frac{1}{\sqrt{x^2-2x}}$ (vii) $\frac{\sec x\tan x}{144-9\sec^2 x}$ (viii) $\frac{1}{4x^2+12x+5}$

12. Integrate the following w.r.t. x

(i) $\sin^3 x$ (ii) $\cos^3 x$ (iii) $\tan^3 x$ (iv) $\cot^3 x$ (v) $\tan^4 x$ (vi) $\cot^4 x$
(vii) $\sec^4 x$ (viii) $\operatorname{cosec}^4 x$ (ix) $\cos^5 x$ (x) $\tan^5 x$

13. Integrate the following w.r.t. x

(i) $\frac{1}{9\sin^2 x+5}$ (ii) $\frac{1}{1+3\sin^2 x}$ (iii) $\frac{1}{5+3\cos x}$

(iv) $\frac{1}{4\cos^2 x+9\sin^2 x}$ (v) $\frac{1}{a^2\cos^2 x+b^2\sin^2 x}$ (vi) $\frac{1}{5+4\cos^2 x}$

14. Explain the method of evaluating integrals involving $\sqrt{a^2-x^2}$, $\sqrt{x^2+a^2}$ and $\sqrt{x^2-a^2}$ using trigonometric substitution.

15. Integrate the following w.r.t. x

(i) $\sqrt{a^2-x^2}$ (ii) $x^2\sqrt{1-x^2}$ (iii) $\frac{x^3}{\sqrt{1-x^2}}$

16. Integrate the following w.r.t. x

(i) $\frac{1}{x\sqrt{x^2+1}}$ (ii) $\frac{\sqrt{x^2+1}}{x}$ (iii) $\frac{1}{\sqrt{x^2+a^2}}$

17. Integrate the following w.r.t. x

(i) $\frac{1}{x\sqrt{x^2-9}}$ (ii) $\frac{1}{x\sqrt{x^2-2}}$ (iii) $\frac{1}{\sqrt{x^2-a^2}}$

Answers (Add c to every answer)

2. (i) $\frac{(3x^2+x+1)^{60}}{60}$ (ii) $\frac{(2x^2-6x+4)^{5/2}}{5}$ (iii) $\frac{(4x^3-12x^2+24x+7)^4}{48}$

(*iv*) $\frac{3(3x^2-5x+1)^{103/3}}{206}$ (*v*) $\frac{1}{2}\log(x^2+1)$ (*vi*) $\log(x^2+5x$

(*vii*) $\frac{2}{45}(5x^3+1)^{3/2}$ (*viii*) $\frac{1}{2}(1+x^4)^{1/2}$ (*ix*) $\frac{3}{4}(ax^2+2bx+c)$

(*x*) $\frac{1}{25}\sqrt{10x^5+11}$ (*xi*) $\frac{(ax^2+bx+c)^{1-n}}{1-n}$ (*xii*) $\sqrt{x^2+4x}$

(*xiii*) $-\sqrt{a^2-x^2}$

3. (*i*) $\log\sin x$ (*ii*) $\log\sec x$ (*iii*) $-\frac{2}{7}(\cos x)^{7/2}$ (*iv*) $\frac{1}{6}\tan$

(*v*) $\frac{2}{3}(\sin x)^{3/2}$ (*vi*) $\frac{3}{2}(\sin x)^{2/3}$ (*vii*) $\frac{1}{2}\log(1-2\cos x)$

(*viii*) $\frac{1}{12}(1+2\sin x)^6$ (*ix*) $\frac{1}{3\cos^3 x}$ (*x*) $\frac{1}{12}(\cos 3x)^{-4}$ (*xi*) $\frac{1}{4}\sin$

(*xii*) $\frac{\sec^3 x}{3}$ (*xiii*) $\frac{\tan^4 x}{4}$ (*xiv*) $-\frac{3}{4}(\cos x)^{4/3}$ (*xv*) $-\frac{1}{3}(4+\tan x$

(*xvi*) $\log(\sin x+\cos x)$ (*xvii*) $-\frac{2}{3}(\cot x)^{3/2}$ (*xviii*) $\frac{3}{4}(\tan x$

(*xix*) $\frac{\sec^5 x}{5}$ (*xx*) $-\frac{1}{6}(1-9\sin x)^{2/3}$

4. (*i*) $\frac{2}{3}(\sin^{-1}x)^{3/2}$ (*ii*) $-\frac{(\cos^{-1}x)^4}{4}$ (*iii*) $\frac{(\tan^{-1}x)^3}{3}$ (*iv*) $-\frac{(\cot}{6}$

(*v*) $\frac{(\sec^{-1}x)^2}{2}$ (*vi*) $\frac{(\log x)^2}{2}$ (*vii*) $\frac{(\log x)^6}{6}$ (*viii*) $\frac{(\log x)^{n+1}}{n+1}, n \neq$

(*ix*) $\frac{(e^x+1)^6}{6}$ (*x*) $\log(e^x-1)$ (*xi*) $-\log(1+e^{-x})$

5. (*i*) $\frac{\sinh^4 x}{4}$ (*ii*) $\frac{\tanh^2 x}{2}$ (*iii*) $-\frac{\operatorname{sech}^3 x}{3}$ (*iv*) $\frac{(\sinh^{-1}x)^3}{3}$

(*v*) $\frac{2}{3}(\tanh^{-1}x)^{\frac{3}{2}}$ (*vi*) $-\frac{(\operatorname{sech}^{-1}x)^6}{6}$

6. (*i*) $\frac{1}{b}\log(a+b\tan x)$ (*ii*) $\log(\sin x-\cos x)$ (*iii*) $\frac{1}{2}(x+\log$

(*iv*) $\log(e^x+e^{-x})$ (*v*) $-2\sqrt{1-\log x}$ (*vi*) $-2\log(1-\sqrt{x})$ (*vii*) $2\sqrt{s}$

(*viii*) $-\frac{(x-x^2)^5}{5}$ (*ix*) $-\frac{1}{3}\log(5+3\cos x)$ (*x*) $2\sqrt{1+\sin^2 x}$

(*xi*) $\frac{2}{3}(\tan x)^{\frac{3}{2}}$ (*xii*) $\log\sinh^{-1}x$ (*xiii*) $\frac{1}{e}\log(x^e+e^x)$

(*xiv*) $-\frac{4}{3}(1+e^{-x})^{\frac{3}{4}}$ (*xv*) $\frac{3}{8}(2x^2-3x)^{\frac{8}{3}}$

7. (*i*) $\frac{1}{3}\sin(x^3)$ (*ii*) $\frac{1}{6}\cos(1-3x^2)$ (*iii*) $-2\cos\sqrt{x}$ (*iv*) $2\sin\sqrt{x}$
(*v*) $\sin(\log x)$ (*vi*) $-\cos(1+e^x)$ (*vii*) $-\frac{1}{12}e^{-4x^3+1}$
(*viii*) $\frac{1}{2}e^{2\tan^{-1}x}$ (*ix*) $-2e^{-\sqrt{x}}$ (*x*) $\frac{1}{8}e^{4x^2+9}$ (*xi*) $-\frac{1}{2}e^{1/x^2}$
(*xii*) $\log\sec(\sin^{-1}x)$ (*xiii*) $\log\sin(\sin x)$ (*xiv*) $\frac{1}{2}e^{2\log x+3}$
(*xv*) $\cot(e^{-x})$ (*xvi*) $\frac{1}{2}\sinh x^2$ (*xvii*) $2\tanh\sqrt{x}$
(*xviii*) $\frac{1}{2}\sin(3+2\log x)$ (*xix*) $-\frac{1}{4}\operatorname{sech}(x^4)$ (*xx*) $-\frac{1}{3}\operatorname{cosec}(x^3)$.

8. (*i*) $-\tan^{-1}(\cos x)$ (*ii*) $\tan^{-1}(e^x)$ (*iii*) $\frac{1}{4}\tan^{-1}(x^4)$
(*iv*) $\frac{1}{2}\tan^{-1}\left(\frac{\tan x}{2}\right)$ (*v*) $-\sin^{-1}\left(\frac{\cos x}{2}\right)$
(*vi*) $\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}\sin x}{\sqrt{8}}\right)$ (*vii*) $\sin^{-1}(\tan x)$ (*viii*) $\sin^{-1}\left(\frac{e^x}{3}\right)$
(*ix*) $\frac{1}{2}\sin^{-1}(x^2)$ (*x*) $-\frac{1}{3}\sin^{-1}\left(\frac{3\cos x}{2}\right)$ (*xi*) $\tan^{-1}(\log x)$
(*xii*) $\sin^{-1}(\frac{1}{2}\log x)$ (*xiii*) $\frac{1}{2}\tan^{-1}(\cosh x)$ (*xiv*) $\frac{1}{6}\sin^{-1}(\frac{2}{3}\tanh x)$.

9. (*i*) $\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$ (*ii*) $\frac{1}{12}\tan^{-1}\left(\frac{4x}{3}\right)$ (*iii*) $\frac{1}{2}\tan^{-1}(2x-7)$
(*iv*) $\frac{1}{20}\tan^{-1}\left(\frac{4x-3}{5}\right)$ (*v*) $\frac{1}{4}\tan^{-1}(2x+1)$ (*vi*) $\tan^{-1}(x+2)$
(*vii*) $\frac{1}{4}\tan^{-1}\left(\frac{2x-3}{2}\right)$ (*viii*) $\frac{1}{12}\tan^{-1}\left(\frac{3x+1}{4}\right)$

10. (*i*) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$ (*ii*) $\frac{1}{3}\sin^{-1}\left(\frac{3x+2}{2}\right)$ (*iii*) $\sin^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$
(*iv*) $\sin^{-1}\left(\frac{x-2}{3}\right)$ (*v*) $\sin^{-1}(x-1)$ (*vi*) $\frac{1}{2}\sin^{-1}\left(\frac{2x+1}{\sqrt{6}}\right)$
(*vii*) $\frac{1}{3}\sin^{-1}\left(\frac{3x+4}{5}\right)$

11. (*i*) $\sinh^{-1}(\sin x)$ (*ii*) $\sin^{-1}\left(\frac{e^x}{2}\right)$ (*iii*) $\sinh^{-1}\left(\frac{x-1}{4}\right)$
(*iv*) $\cosh^{-1}\left(\frac{\tan x}{3}\right)$ (*v*) $\frac{1}{2}\cosh^{-1}\frac{2x}{3}$ (*vi*) $\cosh^{-1}(x-1)$
(*vii*) $\frac{1}{36}\tanh^{-1}\left(\frac{\sec x}{4}\right)$ (*viii*) $-\frac{1}{4}\coth^{-1}\left(\frac{2x+3}{2}\right)$ or
$-\frac{1}{4}\tanh^{-1}\left(\frac{2x+3}{2}\right)$

12. (*i*) $\frac{\cos^3 x}{3} - \cos x$ (*ii*) $\sin x - \frac{(\sin x)^3}{3}$ (*iii*) $\frac{1}{2}\tan^2 x + \log\cos$

(*iv*) $-\frac{1}{2}\cot^2 x - \log\sin x$ (*v*) $\frac{\tan^3 x}{3} - \tan x + x$

(*vi*) $-\frac{\cot^3 x}{3} + \cot x + x$ (*vii*) $\frac{\tan^3 x}{3} + \tan x$ (*viii*) $-\frac{\cot^3 x}{3} - \cot$

(*ix*) $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x$ (*x*) $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log\sec$

13. (*i*) $\frac{1}{\sqrt{70}}\tan^{-1}\left(\frac{\sqrt{14}\tan x}{\sqrt{5}}\right)$ (*ii*) $\frac{1}{2}\tan^{-1}(2\tan x)$

(*iii*) $\frac{1}{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{2}\right)$ (*iv*) $\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)$

(*v*) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$ (*vi*) $-\frac{1}{3\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{5}\tan x}{3}\right)$

15. (*i*) $\frac{a^2}{2}\left[\sin^{-1}\frac{x}{a} + \frac{x}{a}\frac{\sqrt{a^2-x^2}}{a}\right]$

(*ii*) $\frac{1}{8}\left[\theta - \frac{\sin 4\theta}{4}\right]$ where $\theta = \sin^{-1} x$ or

$\frac{1}{8}\left[\sin^{-1} x - x\sqrt{1-x^2}\,(1-2x^2)\right]$ (*iii*) $\frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2}$

16. (*i*) $\log(\operatorname{cosec}\theta - \cot\theta)$ where $\theta = \tan^{-1} x$ or $\log\left(\frac{\sqrt{1+x^2}-1}{x}\right.$

(*ii*) $\log(\operatorname{cosec}\theta - \cot\theta) + \sec\theta$ where $\theta = \tan^{-1} x$

or $\log\left(\frac{\sqrt{1+x^2}-1}{x}\right) + \sqrt{1+x^2}$ (*iii*) $\log(x + \sqrt{x^2+$

17. (*i*) $\frac{1}{3}\sec^{-1}\frac{x}{3}$ (*ii*) $\frac{1}{\sqrt{2}}\sec^{-1}\frac{x}{\sqrt{2}}$ (*iii*) $\log(x + \sqrt{x^2+a^2}$

5.3 Method of Partial fraction

1. Integrate the following w.r.t. x

(*i*) $\frac{4x-3}{(x-2)(x+3)}$ (*ii*) $\frac{x^2-10x+13}{(x-1)(x-2)(x-3)}$ (*iii*) $\frac{x^2+1}{x(x^2-4)}$

(*iv*) $\frac{3x-8}{2x^2+x-6}$ (*v*) $\frac{5x^2-1}{x-x^3}$ (*vi*) $\frac{5x-2}{6x^2-x-15}$

(*vii*) $\dfrac{1}{x^2(x-1)}$ (*viii*) $\dfrac{3x^2+2x+1}{x^2\,(x+1)}$ (*ix*) $\dfrac{x^2-7x+7}{(x-3)^2\,(x+2)}$

(*x*) $\dfrac{1}{x(x^2+1)}$ (*xi*) $\dfrac{3x^2-x+2}{(x^2+1)\,(x-1)}$ (*xii*) $\dfrac{3x^2-1}{x^3+x}$

(*xiii*) $\dfrac{5x-1}{(x+1)\,(x^2+2)}$ (*xiv*) $\dfrac{x+1}{x-1}$ (*xv*) $\dfrac{x^3}{2x-1}$ (*xvi*) $\dfrac{x^2}{x+3}$

(*xvii*) $\dfrac{x^2}{x^2+a^2}$ (*xviii*) $\dfrac{x^2+1}{x^2-1}$ (*xix*) $\dfrac{x^4}{x^2+1}$

2. Integrate the following w.r.t. x

(*i*) $\dfrac{1}{x^2-a^2}$ (*ii*) $\dfrac{1}{a^2-x^2}$ (*iii*) $\dfrac{1}{x^2-4}$ (*iv*) $\dfrac{1}{9-x^2}$ (*v*) $\dfrac{1}{x^2-5}$ (*vi*) $\dfrac{1}{8-x^2}$

3. If $f'(x)=\dfrac{\cos x}{(1+\sin x)\,(2+\sin x)}$, find $f(x)$

Answers (Add c to every answer)

1. (*i*) $\log(x-2)+3\log(x+3)$ (*ii*) $2\log(x-1)+3\log(x-2)-4\log(x-3)$ (*iii*) $-\frac{1}{4}\log x+\frac{5}{8}\log(x+2)+\frac{5}{8}\log(x-2)$

(*iv*) $2\log(x+2)-\frac{1}{2}\log(2x-3)$ (*v*) $-\log x-2\log(1-x)-2\log(1+x)$

(*vi*) $\frac{1}{2}\log(2x+3)+\frac{1}{3}\log(3x-5)$ (*vii*) $\log(x-1)-\log x+\dfrac{1}{x}$

(*viii*) $\log x-\dfrac{1}{x}+2\log(x+1)$ (*ix*) $\dfrac{1}{x-3}+\log(x+2)$

(*x*) $\log x-\frac{1}{2}\log(x^2+1)$ (*xi*) $\frac{1}{2}\log(x^2+1)+2\log(x-1)$

(*xii*) $2\log(x^2+1)-\log x$ (*xiii*) $-2\log(x+1)+\log(x^2+2)+\dfrac{3}{\sqrt{2}}\tan^{-1}\dfrac{x}{\sqrt{2}}$ (*xiv*) $x+2\log(x-1)$ (*xv*) $\frac{1}{6}x^3+\frac{1}{8}x^2+\frac{1}{8}x+\frac{1}{16}\log(2x-1)$ (*xvi*) $\dfrac{x^2}{2}-3x+9\log(x+3)$ (*xvii*) $x-a\tan^{-1}\dfrac{x}{a}$

(*xviii*) $x+\log(x-1)-\log(x+1)$ (*xix*) $\dfrac{x^3}{3}-x+\tan^{-1}x.$

2. (*i*) $\dfrac{1}{2a}\log\dfrac{x-a}{x+a}$ (*ii*) $\dfrac{1}{2a}\log\dfrac{a+x}{a-x}$ (*iii*) $\frac{1}{4}\log\dfrac{x-2}{x+2}$

(*iv*) $\frac{1}{6}\log\dfrac{3+x}{3-x}$ (*v*) $\dfrac{1}{2\sqrt{5}}\log\dfrac{x-\sqrt{5}}{x+\sqrt{5}}$ (*vi*) $\dfrac{1}{4\sqrt{2}}\log\dfrac{2\sqrt{2}+x}{2\sqrt{2}-x}$

3. $\log(1+\sin x)-\log(2+\sin x)$

5.4 Integration by parts

1. State and prove the rule of integration by parts

2. Integrate the following w.r.t. x

(*i*) $x \sin x$ (*ii*) $x \cos x$ (*iii*) $x \sin 3x$ (*iv*) $x \cos 5x$
(*v*) $x \sin 4x \sin 2x$ (*vi*) $x \cos 5x \sin 7x$ (*vii*) $x \cos x \cos$
(*viii*) $x \sin^2 x$ (*ix*) $x \cos^2 x$ (*x*) $x \sin^3 x$ (*xi*) $x \cos^3 x$
(*xii*) $x \sec^2 x$ (*xiii*) $x \tan^2 x$ (*xiv*) $x \sec x \tan x$
(*xv*) $x \operatorname{cosec} x \cot x$ (*xvi*) $x \operatorname{cosec}^2 x$

3. Integrate the following w.r.t. x

(*i*) $x^2 \sin x$ (*ii*) $x^2 \cos x$ (*iii*) $x^2 \sin 3x$ (*iv*) $(x^2+x-1) \cos$

4. Integrate the following w.r.t. x

(*i*) $x e^x$ (*ii*) $x e^{3x}$ (*iii*) $x^2 e^{-x}$ (*iv*) $x^3 e^{2x}$ (*v*) (x^2+2x-3)
(*vi*) $x^2 e^{5x}$

5 Integrate the following w r t. x

(*i*) $\log x$ (*ii*) $x \log x$ (*iii*) $x^3 \log x$ (*iv*) $x^n \log x, n \neq -1$
(*v*) $\frac{\log x}{x^2}$ (*vi*) $(\log x)^2$

6. Integrate the following w.r.t. x

(*i*) $\sin^{-1} x$ (*ii*) $\cos^{-1} x$ (*iii*) $\tan^{-1} x$ (*iv*) $\operatorname{cosec}^{-1} x$ (*v*) $\sec$
(*vi*) $\cot^{-1} x$ (*vii*) $x \sin^{-1} x$ (*viii*) $x \cos^{-1} x$ (*ix*) $x \tan^{-1} x$
(*x*) $x \cot^{-1} x$ (*xi*) $x \sec^{-1} x$ (*xii*) $x^2 \tan^{-1} x$

7. Integrate the following w.r.t. x

(*i*) $x \sinh x$ (*ii*) $x \cosh x$ (*iii*) $x \tanh^{-1} x$ (*iv*) $\sinh^{-1} x$.

8. Integrate the following w.r.t. x

(*i*) $\sec^3 x$ (*ii*) $e^x \cos x$ (*iii*) $\frac{x e^x}{(1+x)^2}$

Answers (Add c to every answer)

2. (*i*) $-x \cos x + \sin x$ (*ii*) $x \sin x + \cos x$

(*iii*) $-\frac{x \cos 3x}{3} + \frac{\sin 3x}{9}$ (*iv*) $\frac{x \sin 5x}{5} + \frac{\cos 5x}{25}$

(*v*) $\frac{x}{2}\left(-\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right) - \frac{1}{2}\left(\frac{\cos 6x}{36} - \frac{\cos 2x}{4}\right)$

(*vi*) $-\frac{x}{2}\left[\frac{\cos 2x}{2} + \frac{\cos 12x}{12}\right] + \frac{1}{2}\left[\frac{\sin 2x}{4} + \frac{\sin 12x}{144}\right]$

(*vii*) $\frac{x}{2}\left[\frac{\sin 6x}{6}+\frac{\sin 4x}{4}\right]+\frac{1}{2}\left[\frac{\cos 6x}{36}+\frac{\cos 4x}{16}\right]$

(*viii*) $\frac{x}{2}\left(x-\frac{\sin 2x}{2}\right)-\frac{1}{2}\left(\frac{x^2}{2}+\frac{\cos 2x}{4}\right)$

(*ix*) $\frac{x}{2}\left(x+\frac{\sin 2x}{2}\right)-\frac{1}{2}\left(\frac{x^2}{2}-\frac{\cos 2x}{4}\right)$

(*x*) $\frac{1}{4}x\left[-3\cos x+\frac{\cos 3x}{3}\right]-\frac{1}{4}\left[-3\sin x+\frac{\sin 3x}{9}\right]$

(*xi*) $\frac{1}{4}x\left[3\sin x+\frac{\sin 3x}{3}\right]+\frac{1}{4}\left[3\cos x+\frac{\cos 3x}{9}\right]$

(*xii*) $x\tan x-\log\sec x$ (*xiii*) $x\tan x-\log\sec x-\frac{x^2}{2}$

(*xvi*) $x\sec x-\log(\sec x+\tan x)$ (*xv*) $-x\,\mathrm{cosec}\,x$
$+\log(\mathrm{cosec}\,x-\cot x)$ (*xvi*) $-x\cot x+\log\sin x$.

3. (*i*) $-x^2\cos x+2x\sin x+2\cos x$ (*ii*) $x^2\sin x+2x\cos x-2\sin x$

(*iii*) $-x^2\frac{\cos 3x}{3}+2x\frac{\sin 3x}{9}+\frac{2\cos 3x}{27}$

(*iv*) $(x^2+x-1)\sin x+(2x+1)\cos x-2\sin x$

4. (*i*) xe^x-e^x (*ii*) $x\frac{e^{3x}}{3}-\frac{e^{3x}}{9}$ (*iii*) $-x^2e^{-x}-2xe^{-x}-2e^{-x}$

(*iv*) $\frac{x^3e^{2x}}{2}-\frac{3x^2e^{2x}}{4}+\frac{3xe^{2x}}{4}-\frac{3e^{2x}}{8}$

(*v*) $(x^2+2x-3)\frac{e^{5x}}{5}-(2x+2)\frac{e^{5x}}{25}+\frac{2e^{2x}}{125}$

(*vi*) $\frac{x^2e^{5x}}{5}-\frac{2xe^{5x}}{25}+\frac{2e^{5x}}{125}$

5. (*i*) $x\log x-x$ (*ii*) $\frac{x^2}{2}\log x-\frac{x^2}{4}$ (*iii*) $\frac{x^4}{4}\log x-\frac{x^4}{16}$

(*iv*) $\frac{x^{n+1}}{n+1}\log x-\frac{x^{n+1}}{(n+1)^2}$ (*v*) $-[x^{-1}\log x+x^{-1}]$

(*vi*) $x(\log x)^2-2(x\log x-x)$.

6. (*i*) $x\sin^{-1}x+\sqrt{1-x^2}$ (*ii*) $x\cos^{-1}x-\sqrt{1-x^2}$

(*iii*) $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)$ (*iv*) $x\,\mathrm{cosec}^{-1}x+\log(x+\sqrt{x^2-1})$

(*v*) $x\sec^{-1}x-\log(x+\sqrt{x^2-1})$ (*vi*) $x\cot^{-1}x+\frac{1}{2}\log(x^2+1)$

(vii) $\frac{x^2}{2}\sin^{-1}x-\frac{\sin^{-1}x}{4}+\frac{x\sqrt{1-x^2}}{4}$

(viii) $\frac{x^2}{2}\cos^{-1}x+\frac{\cos^{-1}x}{4}-\frac{x\sqrt{1-x^2}}{4}$ (ix) $\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}(x-\tan^{-1}$

(x) $\frac{x^2}{2}\cot^{-1}x+\frac{1}{2}(x-\tan^{-1}x)$ (xi) $\frac{x^2}{2}\sec^{-1}x-\frac{1}{2}\sqrt{x^2-1}$

(xii) $\frac{x^3}{3}\tan^{-1}x-\frac{x^2}{6}+\frac{1}{6}\log(1+x^2)$

7. (i) $x\cosh x-\sinh x$ (ii) $x\sinh x-\cosh x$

(iii) $\frac{x^2}{2}\tanh^{-1}x-\frac{1}{2}(\tanh^{-1}x-x)$ (iv) $x\sinh^{-1}x-\sqrt{1+x^2}$

8. (i) $\frac{1}{2}[\sec x\tan x+\log(\sec x+\tan x)]$ (ii) $\frac{1}{2}e^x(\sin x+\cos$

(iii) $\frac{e^x}{1+x}$.

5.5 Definite integrals

1. State the fundamental theorem of integral calculus and evalu

(i) $\int_0^1(x^2-5)\,dx$ (ii) $\int_0^3(x^2+3)\,dx$

(iii) $\int_{-2}^0 x^4\,dx$ (iv) $\int_{-1}^1(x^2-5x+2)\,dx$ (v) $\int_0^9\frac{dx}{\sqrt{x}}$

(vi) $\int_4^9\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ (vii) $\int_1^2\left(3-\frac{2}{x^2}\right)^2dx$

(viii) $\int_0^1(x^2+1)^2\,dx$ (ix) $\int_1^2\frac{x^2-3x+2}{\sqrt{x}}\,dx$

(x) $\int_1^4\frac{x+1}{\sqrt{x}}\,dx$ (xi) $\int_{-3}^{-1}\left(\frac{1}{x^3}-\frac{1}{x^2}\right)dx$

(xii) $\int_0^2 x^2(x^3+1)\,dx$ (xiii) $\int_0^1 x(1-\sqrt{x})^2\,dx$

2. Evaluate: (i) $\int_0^2(x+1)^3\,dx$ (ii) $\int_1^2(2x+3)^2\,dx$

(iii) $\int_0^2\sqrt{4x+1}\,dx$ (iv) $\int_{-3}^{-1}\frac{1}{(x-1)^2}\,dx$

(v) $\int_0^1(4-3x)^3\,dx$ (vi) $\int_2^3\frac{3}{4+x}\,dx$

(*vii*) $\int_0^3 \frac{dx}{\sqrt{1+x}}$ (*viii*) $\int_5^9 \frac{dx}{\sqrt{x-5}}$ (*ix*) $\int_0^\infty e^{-x}\,dx$

(*x*) $\int_0^1 e^{-2x}\,dx$ (*xi*) $\int_0^1 e^{x/2}\,dx$ (*xii*) $\int_{-1}^1 e^{2x+3}dx$

3. Evaluate: (*i*) $\int_0^{\pi/4} \sin 2x\,dx$ (*ii*) $\int_0^{\pi} \cos\frac{x}{2}\,dx$

(*iii*) $\int_{-\pi}^{\pi} \sin\frac{x}{2}\,dx$ (*iv*) $\int_0^{\pi/3} \sec^2 x\,dx$

(*v*) $\int_0^{\pi/4} \sec^2 3x\,dx$ (*vi*) $\int_0^{\pi/6} \sec 2x \tan 2x\,dx$

(*vii*) $\int_0^{\pi/2} \cos^2 x\,dx$ (*viii*) $\int_0^{\pi/4} \sin^2 x\,dx$

(*ix*) $\int_0^{\pi/4} \tan^2\theta\,d\theta$ (*x*) $\int_0^{\pi/2} \cos^3 x\,dx$

(*xi*) $\int_0^{\pi/2} \sin^3 x\,dx$ (*xii*) $\int_0^{\pi/2} (\sin x+\cos x)^2\,dx$

(*xiii*) $\int_0^{\pi} \sin^2\frac{x}{3}\,dx$ (*xiv*) $\int_0^{\pi/2} \cos^2 3x\,dx$

(*xv*) $\int_0^{\pi/3} \frac{1}{1-\sin x}\,dx$ (*xvi*) $\int_0^{\pi/3} \tan x\,dx$

4. Evaluate (*i*) $\int_0^{\pi/2} \sin 3x \cos x\,dx$ (*ii*) $\int_0^{\pi} \sin 3x \sin x\,dx$

(*iii*) $\int_0^{\pi/4} \cos 3x \sin x\,dx$ (*iv*) $\int_0^{\pi/2} \cos 5x \cos 3x\,dx$

(*v*) $\int_0^{\pi/2} \sin 3\theta \cos 5\theta\,d\theta$ (*vi*) $\int_0^{\pi} \sin 5t \sin 3t\,dt$

(*vii*) $\int_0^{\pi/2} \cos 3t \cos 2t\,dt$ (*viii*) $\int_0^{\pi} \sin\frac{x}{2} \cos\frac{x}{2}\,dx$

5. Evaluate (*i*) $\int_0^1 \frac{1}{1+x^2}\,dx$ (*ii*) $\int_0^2 \frac{1}{4+x^2}\,dx$

(*iii*) $\int_0^1 \frac{dx}{x^2+2x+5}$ (*iv*) $\int_0^1 \frac{1}{4x^2+4x+5}\,dx$

6. Evaluate (*i*) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ (*ii*) $\int_0^{\sqrt{3}} \frac{1}{\sqrt{3-x^2}}\,dx$

(*iii*) $\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$ (*iv*) $\int_1^2 \frac{dx}{\sqrt{2x-x^2}}$ (*v*) $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}}$

7. **Evaluate** (*i*) $\int_2^5 \frac{dx}{x^2+4x+3}$ (*ii*) $\int_2^4 \frac{dx}{x^2+x}$

(*iii*) $\int_0^2 \frac{x^4+1}{x^2+1}\,dx$

8. **Evaluate** (*i*) $\int_0^2 (x+1)\sqrt{x^2+2x+3}\,dx$

(*ii*) $\int_0^1 x(x^2+1)^3\,dx$ (*iii*) $\int_1^2 \frac{x}{\sqrt{x^2+3}}\,dx$

(*iv*) $\int_0^1 \frac{2x+3}{x^2+3x+1}\,dx$ (*v*) $\int_0^1 \frac{x^2+1}{x^3+3x+4}\,dx$

(*vi*) $\int_4^8 \frac{x\,dx}{\sqrt{x^2-15}}$ (*vii*) $\int_2^3 \frac{x\,dx}{1+x^2}$

(*viii*) $\int_0^1 (x^2-2x+2)^3(x-1)\,dx$ (*ix*) $\int_0^3 x\sqrt{x^2+16}\,dx$

(*x*) $\int_0^4 \left(\frac{x}{\sqrt{x^2+9}}+x\sqrt{x^2+9}\right)dx$ (*xi*) $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}$

9. **Evaluate** (*i*) $\int_0^{\pi/2} \sin x\cos^3 x\,dx$ (*ii*) $\int_0^{\pi/2} \sin^5 x\cos x$

(*iii*) $\int_0^{\pi/2} (1+2\cos\theta)^3\sin\theta\,d\theta$ (*iv*) $\int_0^{\pi/2} \frac{\cos x\,dx}{3+4\sin x}$

(*v*) $\int_1^e \frac{\log x}{x}\,dx$ (vi) $\int_0^{\pi/4} \tan^3 x\sec^2 x\,dx$

(*vii*) $\int_0^1 \frac{(\tan^{-1}x)^2}{1+x^2}\,dx$ (*viii*) $\int_0^1 \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\,dx$

(*ix*) $\int_0^{\pi/4} \sec^2 x\tan^4 x\,dx$ (*x*) $\int_0^{\pi} \cos^2 x\sin x\,dx$

(*xi*) $\int_0^{\pi/4} \sin^3 x\cos x\,dx$ (*xii*) $\int_0^1 x\,e^{x^2}\,dx$

(*xiii*) $\int_0^{\pi/2} \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$

10. **Evaluate** (*i*) $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

(*ii*) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$ (*iii*) $\int_0^{\pi/2} \frac{\cos x\,dx}{4+\sin^2 x}$

(*iv*) $\int_0^{\pi/2} \frac{\cos\theta\,d\theta}{\sqrt{4-\sin^2\theta}}$ (*v*) $\int_0^1 \frac{dx}{e^x+e^{-x}}$

(*vi*) $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ (*vii*) $\int_{\pi/4}^{5\pi/4} \frac{dx}{1+2\cos^2 x}$

11. Evaluate (*i*) $\int_0^a \sqrt{a^2-x^2}\,dx$ (*ii*) $\int_0^{\pi/4} \tan^4\theta\,d\theta$

(*iii*) $\int_0^{\pi/4} \sec^4\theta\,d\theta$ (*iv*) $\int_0^1 \frac{dx}{(1+x^2)^2}$

12. Evaluate (*i*) $\int_0^{\pi} x\sin x\,dx$ (*ii*) $\int_0^{\pi/2} x\cos x\,dx$

(*iii*) $\int_0^{\pi/2} x^2\cos x\,dx$ (*iv*) $\int_0^{\pi/2} x^2\sin x\,dx$

(*v*) $\int_0^1 x e^x\,dx$ (*vi*) $\int_0^1 x^2 e^{-x}\,dx$ (*vii*) $\int_0^1 x^2 e^{2x}\,dx$

(*viii*) $\int_1^2 \log x\,dx$ (*ix*) $\int_1^e x^2\log x\,dx$ (*x*) $\int_0^1 x\tan^{-1}x\,dx$

(*xi*) $\int_0^{\pi/4} x\sec^2 x\,dx$ (*xii*) $\int_0^{\sqrt{2}} e^{x^2} x^3\,dx$

Answers

1. (*i*) $-\frac{14}{5}$ (*ii*) 18 (*iii*) $\frac{33}{5}$ (*iv*) $\frac{14}{3}$ (*v*) 6 (*vi* $\frac{44}{3}$ (*vii*) $\frac{25}{6}$
(*viii*) $\frac{28}{15}$ (*ix*) $\frac{8\sqrt{2}-12}{5}$ (*x*) $\frac{20}{3}$ (*xi*) $-\frac{10}{9}$ (*xii*) $\frac{40}{3}$ (*xiii*) $\frac{1}{30}$

2 (*i*) 20 (*ii*) $\frac{109}{3}$ (*iii*) $\frac{18}{3}$ (*iv*) $\frac{1}{4}$ (*v*) $\frac{85}{4}$ (*vi*) $3\log\frac{7}{6}$ (*vii*) 2
(*viii*) 4 (*ix*) 1 (*x*) $\frac{1}{2}[1-e^{-2}]$ (*xi*) $2(\sqrt{e}-1)$ (*xii*) $\frac{1}{2}(e^5-e)$

3. (*i*) $\frac{1}{2}$ (*ii*) 2 (*iii*) 0 (*iv*) $\sqrt{3}$ (*v*) $-\frac{1}{3}$ (*vi*) $\frac{1}{2}$ (*vii*) $\frac{\pi}{4}$
(*viii*) $\frac{\pi}{8}-\frac{1}{4}$ (*ix*) $1-\frac{\pi}{4}$ (*x*) $\frac{2}{3}$ (*xi*) $\frac{2}{3}$ (*xii*) $1+\frac{\pi}{2}$
(*xiii*) $\frac{1}{2}(\pi-\frac{3\sqrt{3}}{4})$ (*xiv*) $\frac{\pi}{4}$ (*xv*) $\sqrt{3}+1$ (*xvi*) log 2.

4. (*i*) $\frac{1}{2}$ (*ii*) 0 (*iii*) 0 (*iv*) 0 (*v*) $-\frac{1}{2}$ (*vi*) 0 (*vii*) $\frac{3}{5}$ (*viii*) 1.

5. (*i*) $\frac{\pi}{4}$ (*ii*) $\frac{\pi}{8}$ (*iii*) $\frac{1}{2}(\frac{\pi}{4}-\tan^{-1}\frac{1}{2})$ (*iv*) $\frac{1}{4}[\tan^{-1}\frac{3}{2}-\tan^{-1}\frac{1}{2}]$.

6. (*i*) $\frac{\pi}{2}$ (*ii*) $\frac{\pi}{2}$ (*iii*) $\frac{\pi}{3}$ (*iv*) $\frac{\pi}{2}$ (*v*) $\frac{\pi}{6}$.

7. (*i*) $\frac{1}{2}\log\frac{5}{4}$ (*ii*) $\log\frac{6}{5}$ (*iii*) $\frac{2}{3}+2\tan^{-1}2$.

8. (*i*) $\frac{1}{3}[11^{\frac{3}{2}}-3^{\frac{3}{2}}]$ (*ii*) $\frac{15}{8}$ (*iii*) $\sqrt{7}-2$ (*iv*) $\log 5$ (*v*) $\frac{1}{3}\log$
(*vi*) 6 (*vii*) $\frac{1}{2}\log 2$ (*viii*) $-\frac{15}{8}$ (*ix*) $\frac{61}{3}$ (*x*) $\frac{104}{3}$ (*xi*) log

9. (*i*) $\frac{1}{4}$ (*ii*) $\frac{1}{6}$ (*iii*) 10 (*iv*) $\frac{1}{4}\log\frac{7}{3}$ (v) $\frac{1}{2}$ (*vi*) $\frac{1}{4}$ (*vii*)
(*viii*) $\frac{\pi^4}{64}$ (*ix*) $\frac{1}{5}$ (*x*) $\frac{2}{3}$ (*xi*) $\frac{1}{16}$ (*xii*) $\frac{1}{2}(e-1)$ (*xiii*) $e-1$.

10. (*i*) $\frac{\pi}{4}$ (*ii*) $\frac{\pi}{4}$ (*iii*) $\frac{1}{2}\tan^{-1}(\frac{1}{2})$ (*iv*) $\frac{\pi}{6}$ (*v*) $\tan^{-1}(e)$ -
(*vi*) $\frac{\pi}{2ab}$ (*vii*) 0. 11. (*i*) $\frac{\pi a^2}{4}$ (*ii*) $\frac{\pi}{4}-\frac{2}{3}$ (*iii*) $\frac{4}{3}$ (*iv*) $\frac{\pi}{8}+$

12. (*i*) π (*ii*) $\frac{\pi}{2}-1$ (*iii*) $\frac{\pi^2}{4}-2$ (*iv*) $\pi-2$ (*v*) 1 (*vi*) 2 -
(*vii*) $\frac{e^2}{4}-\frac{1}{4}$ (*viii*) $2\log 2-1$ (*ix*) $\frac{2e^3}{9}+\frac{1}{9}$ (*x*) $\frac{\pi}{4}-\frac{1}{2}$
(*xi*) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (*xii*) $\frac{1}{2}(e^2+1)$.

5.6 Properties of Definite Integrals

1. Prove that

(*i*) $\int_a^b f(x)dx=\int_a^b f(t)dt$ (*ii*) $\int_a^b f(x)dx=-\int_b^a f(x)$

(*iii*) $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx,\ a<c<b$

(*iv*) $\int_a^a f(x)dx=0$ (*v*) $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ and

deduce that $\int_0^{\pi/2} f(\sin x)=dx\int_0^{\pi/2} f(\cos x)dx$

2. Prove that

(*i*) $\int_0^{\pi/2}\sin^2 x dx=\int_0^{\pi/2}\cos^2 x dx=\frac{\pi}{4}$

(*ii*) $\int_0^{\pi/2}\log\tan\theta\, d\theta=0$ (*iii*) $\int_0^{\pi/2}\frac{(\sin x-\cos x)dx}{1+\sin x\cos x}$

(*iv*) $\int_0^{\pi}\frac{\cos x}{1+\sin^2 x}dx=0$.

3. Evaluate

(*i*) $\int_0^{\pi/2} \frac{\sin x}{\cos x+\sin x}\,dx$ (*ii*) $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

(*iii*) $\int_0^{\pi/2} \frac{1}{1+\tan x}\,dx$ (*iv*) $\int_0^{\pi/2} \frac{\cot x}{1+\cot x}\,dx$

(*v*) $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x}\,dx$ (*vi*) $\int_0^{\pi/2} \frac{a\sin x+b\cos x}{\sin x+\cos x}\,dx.$

4. Evaluate

(*i*) $\int_0^{a} \frac{dx}{x+\sqrt{a^2-x^2}}$ (*ii*) $\int_0^{\infty} \frac{x\,dx}{(1+x)(1+x^2)}$

(*iii*) $\int_0^{\pi/4} \log(1+\tan x)\,dx$ (*iv*) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$

(*v*) $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$ (*vi*) $\int_0^{1} x(1-x)^n\,dx$

Answers

3. (*i*) to (*v*) $\frac{\pi}{4}$ (*vi*) $\frac{\pi}{4}(a+b)$. 4. (*i*) $\frac{\pi}{4}$ (*ii*) $\frac{\pi}{4}$ (*iii*) $\frac{\pi}{8}\log 2$

(*iv*) $\frac{\pi^2}{4}$ (*v*) π (*vi*) $\frac{1}{(n+1)\,(n+2)}$.

5.7 Application of Definite Integrals

1. Define definite integral $\int_a^b f(x)dx$ as a limit of an infinite sum and hence prove the following:

(*i*) $\int_1^2 x\,dx=\frac{3}{2}$ (*ii*) $\int_1^2 x^2\,dx=\frac{7}{3}$ (*iii*) $\int_1^3 x^3\,dx=20$

2. Deduce from 1. $\int_0^1 f(x)dx$ as a limit of a sum and hence prove the following:

(*i*) $\underset{n\to\infty}{\text{Lt}} \left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots\ldots+\frac{1}{2n}\right]=\frac{\pi}{4}$

(*ii*) $\underset{n\to\infty}{\text{Lt}} \left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots\ldots+\frac{1}{2n}\right]=\log 2$

(*iii*) $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} \frac{1}{\sqrt{4n^2-r^2}}=\frac{\pi}{6}$

(*iv*) $\underset{n\to\infty}{\text{Lt}} \frac{1}{n^3}(1+4+9+\ldots\ldots+n^2)=\frac{1}{3}$

(*v*) $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} \frac{1}{3n+r}=\log\frac{4}{3}$.

3. Using the definition of definite integral as a limit of a sum, interpret $\int_a^b f(x)dx$ geometrically.

4. Find the area of the figure enclosed by the following curves the *x*-axis and the ordinates at the given points:

(*i*) $4y=x^2$; $x=1$, $x=3$ (*ii*) $x^2=4ay$; $x=0$, $x=4$

(*iii*) $y^2=4ax$, $(a>0)$; $x=4$, $x=9$ (*iv*) $xy=1$; $x=1$, $x=2$

(*v*) $y=2\cos x-\sin x$; $x=0$, $x=\pi/4$

(*vi*) $y=x+\sin 2x$; $x=0$, $x=\pi/2$

(*vii*) $y=e^{x/a}+e^{-x/a}$; $x=\pm a$ (*viii*) $y=x^2-7x+6$; $x=2$, $x=$

(*ix*) $y=\frac{1}{1+x^2}$; $x=\pm 1$ (*x*) $y=\frac{x^2}{1+x^2}$; $x=0$, $x=1$

5. Find the area of the region bounded by the following curves the *y*-axis and the given lines:

(*i*) $x^2=4y$; $y=0$, $y=1$ (*ii*) $3y=4x+3$; $y=1$, $y=$

6. Find the area bounded by the curve

(*i*) $y=x^2-7x+10$ and the *x*-axis

(*ii*) $y=4x-x^2-3$ and the x-axis (*iii*) $y=4x-x^2$ and the *x*-axis

(*iv*) $y=x^2-5x+6$ and the *x*-axis (*v*) $y=6+x-x^2$ and the *x*-ax

(*vi*) $x=4-y^2$ and the *y*-axis.

7. Find the area enclosed between the parabola

(*i*) $y=11x-24-x^2$ and the line $y=x$

(*ii*) $y^2=4x$ and the line $x=2y$

(*iii*) $y=x^2$ and the line $2x-y+3=0$

(*iv*) $y=2-x^2$ and the line $x+y=0$.

8. Find the area enclosed between the curves

(*i*) $y^2=6x$ and $x^2=6y$ (*ii*) $y=8-x^2$ and $y=x^2$

(*iii*) $y=x^2$ and $y=x^3$ (*iv*) $y=2x^2+1$ and $y=x^2+5$

(*v*) $x^2=y$ and $y^2=8x$ (*vi*) $y^2=4ax$ and $y=2x$

(*vii*) $y^2=4x$ and $x^2=4y$ (*viii*) $4y^2=9x$ and $3x^2=16y$

(*ix*) $y=x^2+3x+2$ and $y=6x$ (*x*) $y=x^2$ and $3y=4x$

(*xi*) $y^2=4x$ and $y=4x$ (*xii*) $y=x^2$ and $x=y^2$

(*xiii*) $y=6x-x^2$ and $y=x^2-2x$ (*xiv*) $y=x^2-4$ and $y=8-2x^2$.

9. Find the area enclosed between the curves $y^2=4x$ and $y=2x-4$
10. Find the area of the circle $x^2+y^2=r^2$ by integration
11. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by integration.

Answers (in square units)

4. (*i*) $\frac{13}{6}$ (*ii*) $\frac{16a^2}{3}$ (*iii*) $\frac{76}{3}\sqrt{a}$ (*iv*) $\log 2$ (*v*) $\frac{3}{\sqrt{2}}-1$ (*vi*) $\frac{\pi^2}{8}+1$ (*vii*) $2a\left(e-\frac{1}{e}\right)$ (*viii*) $\frac{56}{3}$ (*ix*) $\frac{\pi}{2}$ (*x*) $1-\frac{\pi}{4}$

5. (*i*) $\frac{4}{3}$ (*ii*) $\frac{8}{3}$. 6. (*i*) $\frac{9}{2}$ (*ii*) $\frac{4}{3}$ (*iii*) $\frac{32}{3}$ (*iv*) $\frac{1}{6}$ (*v*) $\frac{125}{6}$ (*vi*) $\frac{32}{3}$. 7. (*i*) $\frac{4}{3}$ (*ii*) $\frac{64}{3}$ (*iii*) $\frac{32}{3}$ (*iv*) $\frac{9}{2}$. 8. (*i*) 12 (*ii*) $\frac{64}{3}$ (*iii*) $\frac{1}{12}$ (*iv*) $\frac{32}{3}$ (*v*) $\frac{8}{3}$ (*vi*) $\frac{a^2}{3}$ (*vii*) $\frac{16}{3}$ (*viii*) 4 (*ix*) $\frac{1}{6}$ (*x*) $\frac{32}{81}$ (*xi*) $\frac{1}{24}$ (*xii*) $\frac{1}{3}$ (*xiii*) $\frac{64}{3}$ (*xiv*) 32. 9. 9. 10. πr^2. 11. πab.

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SECTION V: OBJECTIVE TYPE PROBLEMS

5.1 ALGEBRA

5.11 Cross products, relations and functions

1. If the ordered pairs $(x+y, 1)$ and $(3, x-y)$ are equal, then (x, y) i
(a) (1, 2) (b) (1, 1) (c) (2, 2) [✓ (d) (2, 1)]

2. If the number of elements in A is n, then the number of elemen
in A×A is (a) n (b) $2n$ [✓ (c) n^2] (d) none of these

3. If A={1, 2, 3} and B={1}, then A×B is
(a) {(1, 1)} (b) {(1, 1), (2, 1), (1, 3)} (c) {(1, 1), (2, 1), (3, 1
(d) none of these

4. If $X=\{x \mid x^2-4x+3=0\}$, then X×X is
[✓ (a) { (1, 1), (1, 3), (3, 1), (3, 3)}] (b) { (1, 1), (3, 1), (1, 3)
(c) { (–1, –1), (–1, –3), (–3, –1), (–3, –3) } (d) none

5. If A={a, b, c, d} and B={a, b, c}, then (A–B)×(B–A) i
[✓ (a) ϕ] (b) {(a, a), (b, b), (c, c)} (c) { (d, d)} (d) none of the

6. If $A=\{x \mid 0<x<3,\ x\in N\}$ and $B=\{x \mid 1<x<4,\ x\in N\}$, th
(A×A) ∩ (B×B) is
(a) { (1, 1), (2, 2)} [✓ (b) { (2, 2) }] (c) { (1, 1), (2, 1) }
(d) { (1, 2), (1, 3), (2, 1), (2, 3) }

7. If A={a, b}, B={2, 3} and C={3, 5}, then (B∩C)×A is
(a) { (a, 3), (b, 3) } (b) { (3, a), (3, b) }
(c) { (a, 3), (b, 3), (a, 5), (b, 5) } (d) none

8. If A={1, 2} and B={–1, 0, 1}, then (A×B) ∩ (B×A) is
(a) { (1, 1), (1, 2) } (b) { (2, 1), (1, 1) , (2, 2) }
(c) { (1, 1) } (d) none of these

9. If A={a, b, c} and B={ b, c, d}, then (A×A) ∩ (B×B) is
(a) { (b, c), (c, c), (b, b), (c, b) } (b) {(b, c), (c, c) }
(c) { (b, c), (c, b) } (d) none

*10. The cross products A×B and B×A will be equal only wh
(a) A=B (b) A=ϕ (c) B=ϕ (d) (a), (b) or (c) is true.

11. Let A={1, 2, 3}. The non-empty set B satisfying A×B=B×
is (a) ϕ (b) {1, 2} (c) {1, 2, 3} (d) none

*12. (A×B) ∩ (B×A)=ϕ only when
(a) A=ϕ (b) B=ϕ (c) A=ϕ and B=ϕ (d) A∩B=

13. **On $S=\{1, 2, 3, 4\}$, a relation R is given by $\{(1, 1), (2, 4), (3, 3), (4, 1), (4, 4)\}$, then R is**
(*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) anti-symmetric

14. **On a set of people on earth, a relation R is defined by xRy iff x loves y. Then R is**
(*a*) symmetric (*b*) transitive (*c*) anti-symmetric (*d*) none

15. **On the set of all subsets of a non-empty set, the relation R, "is a sub-set of", is**
(*a*) equivalence (*b*) reflexive but not transitive (*c*) reflexive, transitive and anti-symmetric (*d*) reflexive & transitive only

16. **Which of the following relations in N is NOT reflexive**
(*a*) "less than or equal to" (*b*) "divides"
(*c*) "relatively prime to" (*d*) "equal to"

17. **The statement "a relation can be both symmetric and anti-symmetric" is**
(*a*) true (*b*) false (*c*) don't know..

18. **The relation of "divides" from A to B where $A=\{2, 3, 4\}$ and $B=\{3, 5\}$ is**
(*a*) $\{(2, 4)\}$ (*b*) $\{(3, 3)\}$ (*c*) $\{(2, 4), (3, 3)\}$
(*d*) $\{(2, 4), (3, 3), (4, 2)\}$

19. **The relation $R=\{(x, y) \mid x, y \in N, x-y$ is divisible by $5\}$ is**
(*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) equivalence

20. **The relation $R=\{(x, y) \mid x, y \in N, 2x+y=10\}$ is equal to**
(*a*) $\{(1, 8), (4, 2)\}$ (*b*) $\{(2, 6), (3, 4)\}$ (*c*) $\{(1, 8), (3, 4), (4, 2)\}$
(*d*) none

21. **Which of the following relations on $\{a, b, c\}$ is both symmetric and anti-symmetric?**
(*a*) $\{(a, a), (b, b), (a, b)\}$ (*b*) $\{(a, a), (b, b), (c, c)\}$
(*c*) $\{(a, b), (b, a), (a, a), (b, b)\}$ (*d*) none

22. **On the set of people in Karnataka, which of the following relations is NOT a function?**
(*a*) "father of" (*b*) "son of" (*c*) "wife of" (*d*) all of them

23. **Which of the following relations $R=\{(x, y): x, y \in R)$ is NOT a function?**
(*a*) $y=3x-4$ (*b*) $y<x$ (*c*) $y=x^2$ (*d*) none

24. **Which of the following relations in $\{0, 1, 2\}$ is a function?**
(*a*) $\{(0, 1), (1, 1), (2, 1)\}$ (*b*) $\{(0, 1), (0, 2), (1, 1)\}$
(*c*) $\{(0, 1), (1, 0), (1, 1)\}$ (*d*) $\{(0, 1), (1, 2)\}$

25. If $n(A)=n(B)=3$, then the number of one-one functions fro
A to B is (*a*) 3 (*b*) $|\underline{3}$ (*c*) 27 (*d*) none

26. Which of the following functions is one-one? given {1,2,3
(*a*) {(1, 1), (2, 1), (3, 2)} (*b*) {(1, 3), (2, 2), (3, 1)}
(*c*) {(1, 1), (1, 2), (1, 3)} (*d*) none

27. Which of the following functions from {1, 2, 3} to itself
NOT onto?
(*a*) {(1, 1), (2, 2), (3, 3)} (*b*) {(1, 2), (2, 2), (3, 2)}
(*c*) {(1, 3), (2, 1), (3, 2)} (*d*) none

28. Which of the following functions from {*a*, *b*, *c*, *d*} to {*a*, *b*} is
constant function?
(*a*) {(*a*, *a*), (*b*, *a*), (*c*, *a*), (*d*, *b*)} (*b*) {(*a*, *a*), (*b*, *a*), (*c*, *b*), (*d*, *b*)
(*c*) {(*a*, *b*), (*b*, *b*), (*c*, *b*), (*d*, *b*)} (*d*) none

29. Which of the following functions from {1, 2} to {1, 2, 3} is th
identity function?
(*a*) {(1, 1)} (*b*) {(2, 2)} (*c*) {(1, 1), (2, 2)} (*d*) none

30. What is the inverse function of {(2, 3), (3, 4), (4, 5)}?
(*a*) {(3, 2), (4, 3), (5, 4)} (*b*) {(2, 2), (3, 3), (4, 4), (5, 5
(*c*) {(2, 3), (3, 2)} (*d*) none

31. Which of the following functions is NOT its own inverse?
(*a*) {(1, 2), (2, 1), (4, 4)} (*b*) {(1, 2), (2, 2)}
(*c*) {(1, 1), (2, 2), (3, 3)} (*d*) {(1, 2), (2, 1)}

32. Which of the following functions of a real variable x is its ow
inverse? (*a*) $-x$ (*b*) $2x$ (*c*) $x+3$ (*d*) none

33. What is the range of the function $y=5+3\cos x$
(*a*) R (*b*) $2 \leqslant y \leqslant 8$ (*c*) $-2 \leqslant y \leqslant 2$ (*d*) none of these

34. The domain of the function x^2-3x+5 is
(*a*) R (*b*) R_+ (*c*) R–{0} (*d*) none of these

35. The domain of the real valued function $\sin x$ is
(*a*) $0 \leqslant x \leqslant 2\pi$ (*b*) $-\pi \leqslant x \leqslant \pi$ (*c*) $-\pi/2 \leqslant x \leqslant \pi/2$ (*d*)

36. The domain of the function $\tan^{-1} x$ is
(*a*) $-\pi/2 \leqslant x \leqslant \pi/2$ (*b*) $-\infty < x < \infty$ (*c*) $0 \leqslant x < \infty$ (*d*) no

37. The domain of the function $\cosh x$ is
(*a*) $0 < x < \infty$ (*b*) $-\infty < x < \infty$ (*c*) $0 \leqslant x \leqslant 2\pi$ (*d*) none of th

38. The range of the function $y=\log x$ is
(*a*) R (*b*) $0 < y < \infty$ (*c*) $-\infty < y < 0$ (*d*) none of these

39. The domain and the range of the real valued function $\sqrt{x}$ are respectively
(*a*) R; R (*b*) R; R_+ (*c*) R_+; R_+ (*d*) none of these

40. A function $f(x)$ is defined by $f(x)=\begin{cases} 1+x, & x<0 \\ 0, & x \geq 0 \end{cases}$
then $f(2)$ is
(*a*) 3 (*b*) 2 (*c*) 0 (*d*) none

41. If $f(x)=\dfrac{2x+1}{3x-2}$, then $f[f(x)]$ is

(*a*) $\dfrac{2x+1}{3x-2}$ (*b*) $\dfrac{2x+3}{3x+2}$ (*c*) x (*d*) $4x$

42. If $f(x)=x^2+5x+7$, then the value of x for which $f(x)=f(x+1)$ is
(*a*) 2 (*b*) -2 (*c*) 3 (*d*) -3

43. If $f(x)=x^2$ and $g(x)=2x+1$, then $(f o g)$ (2) is
(*a*) 16 (*b*) 17 (*c*) 9 (*d*) 25

44. Which of the following functions satisfies $f(xy)=f(x)+f(y)$
(*a*) $\log x$ (*b*) $\cos x$ (*c*) $\sin^{-1} x$ (*d*) e^x

45. Which of the following properties doesn't define one-one function?
(*a*) $f(a)=f(b)$ implies $a=b$ (*b*) $a=b$ implies $f(a)=f(b)$
(*c*) $a \neq b$ implies $f(a) \neq f(b)$ (*d*) none of these

46. The inverse of a function will exist iff it is
(*a*) onto (*b*) one-one (*c*) either onto or one-one
(*d*) both one-one and onto

47. Which of the following functions on R has inverse?
(*a*) x^2+1 (*b*) x^2 (*c*) $2x-3$ (*d*) $\sin x$

48. The function $f(x)=\sin x$ from R to R is
(*a*) one-one (*b*) onto (*c*) neither one-one nor onto
(*d*) both one-one and onto

49. Which of the following functions from $A=\{x \mid -1 \leq x \leq 1\}$ into itself has inverse?

(*a*) x^4-x^2 (*b*) x^5+x^2 (*c*) $\sin \dfrac{\pi x}{2}$ (*d*) none of these

50. The inverse of the function $\dfrac{x-2}{x-3}$ is

(*a*) $\dfrac{2-3x}{x-2}$ (*b*) $\dfrac{2-3x}{1-x}$ (*c*) $\dfrac{x-3}{x-2}$ (*d*) none of these

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Answers

1. *d* 2. *c* 3. *c* 4. *a* 5. *a* 6. *b* 7. *b* 8. *c* 9.
10. *d* 11. *c* 12. *d* 13. *d* 14. *d* 15. *c* 16. *c* 17. *a* 18.
19. *d* 20. *d* 21. *b* 22. *d* 23. *b* 24. *a* 25. *b* 26. *b* 27.
28. *c* 29. *c* 30. *a* 31. *b* 32. *a* 33. *b* 34. *a* 35. *d* 36.
37. *b* 38. *a* 39. *d* 40. *c* 41. *c* 42. *d* 43. *d* 44. *a* 45.
46. *d* 47. *c* 48. *c* 49. *c* 50. *b*.

5.12 Elements of number theory-Divisibility and congruences

1. **If $a \mid 1$, then 'a' is equal to**
   (*a*) 0 (*b*) 1 (*c*) 0, ±1 (*d*) ±1
2. **If $a \mid b$ and $b \mid a$, then 'a' is**
   (*a*) b (*b*) $-b$ (*c*) $\pm b$ (*d*) none of these
3. **If p is a prime, then its divisors are**
   (*a*) 1 and p (*b*) ±1 (*c*) $\pm p$ (*d*) ±1 and $\pm p$
4. **Divisibility relation on N, the set of natural numbers, is**
   (*a*) reflexive and symmetric (*b*) reflexive and transitive
   (*c*) symmetric and transitive (*c*) equivalence
5. **Divisibility relation on Z, the set of integers, is**
   (*a*) reflexive (*b*) transitive but not reflexive (*c*) reflexive an
   transitive (*d*) none of these
6. **Which of the following statements is true?**
   (*a*) $a \mid 0, \forall a \in Z$ (*b*) $0 \mid 5$ (*c*) $c \mid ab \Rightarrow c \mid a$ or $c \mid b$ (*d*) no
7. **Which of the following pairs of numbers are NOT relativ prime to each other**
   (*a*) 1203 and 450 (*b*) 126 and 111 (*c*) 140 and 91 (*d*) 22 and 11
8. **If $x^2 - y^2$ is a prime number, where $x, y \in N$, then**
   (*a*) $x + y = 1$ (*b*) $x - y = +1$ (*c*) $x^2 - y^2 = 1$ (*d*) $x^2 - y^2 = 2$
9. **Which of the following is NOT true?**
   (*a*) If $x \mid a$ and $x \mid a + b$, then $x \mid b$ (*b*) If $a \mid c$ and $b \mid d$, then $ab \mid$
   (*c*) the sum of even primes is 0 (*d*) If $x \mid y$ and $y \mid x$, then $x =$
10. **The g. c. d. of 475 and 495 is**
    (*a*) 45 (*b*) 5 (*c*) 75 (*d*) none
11. **Which of the following is NOT true?**
    (*a*) $ac \mid bc \Rightarrow a \mid b$ (*b*) $a \mid b \Rightarrow -a \mid b$ (*c*) $a \mid b \Rightarrow -a \mid -$
    (*d*) $b \mid a$ and $c \mid a \Rightarrow bc \mid a$

12. If $p \mid ab$, then
(*a*) $p \mid a$ (*b*) $p \mid a$ or $p \mid b$ (*c*) $p \mid a$ and $p \mid b$ (*d*) none

13. If p is prime, then $(p, a)=1$ only when
(*a*) a is prime (*b*) $a<p$ (*c*) a is composite (*d*) $p \nmid a$

14. $p \mid ab$ implies $p \mid b$ only when
(*a*) $(p, b)=1$ (*b*) $p \mid a$ (*c*) $(p, a)=1$ (*d*) p is prime

15. The g. c. d. of 270 and 324 is
[✓ (*a*) 54] (*b*) 18 (*c*) 6 (*d*) 36

16. If $(a, b)=1$ and $a \mid bx$, then
(*a*) $a \mid b$ (*b*) $a \mid x$ (*c*) $a \nmid x$ (*d*) none of these

17. The g.c.d. $(657, 963)=657x+963\,(-15)$, then x is
(*a*) 21 (*b*) 22 (*c*) 20 (*d*) none

18. The g.c.d. $(1985, 1986)=x \,.\, 1985+y \,.\, 1986$, then (x, y) is
(*a*) $(-1, 1)$ (*b*) $(3, -2)$ (*c*) $(-4, 1)$ (*d*) none

19. The number of divisors of 24 are
[✓ (*a*) 8] (*b*) 9 (*c*) 16 (*d*) 14

20. If $b \equiv a \pmod{m}$, then
(*a*) $m \mid a+b$ (*b*) $m \mid a-b$ (*c*) $(a-b) \mid m$ (*d*) $(b-a) \mid m$

21. Which of the following is NOT true?
(*a*) $20 \equiv 2 \pmod{6}$ (*b*) $-3 \equiv 2 \pmod{5}$ (*c*) $110 \equiv 5 \pmod{15}$
(*d*) $121 \equiv 10 \pmod{11}$

22. An integer satisfying $3x \equiv 4 \pmod{9}$ is
(*a*) 16 (*b*) 15 (*c*) 12 (*d*) none

23. The remainder when $40 \times 41 \times 43$ is divided by 42 is
(*a*) 2 (*b*) 40 (*c*) 1 (*d*) none of these

24. The remainder when 3^{2001} is divided by 5 is
(*a*) 0 (*b*) 3 (*c*) 1 (*d*) 4

25. If $2^{5000} \equiv m \pmod{17}$, then m is
(*a*) 1 (*b*) 0 (*c*) 2 (*d*) 3

26. If $a \equiv 3 \pmod{15}$, $b \equiv -4 \pmod{15}$ and $ab \equiv x \pmod{15}$, then x is
(*a*) 12 (*b*) 2 (*c*) 3 (*d*) 5

27. If $3x \equiv 4 \pmod{5}$, then x can take the value
(*a*) 67 (*b*) 78 (*c*) 51 (*d*) 89

28. If the integers a, b, c, d are such that $ab=c-d$, then
(*a*) $a \equiv b \pmod{c}$ (*b*) $b \equiv c \pmod{d}$ (*c*) $c \equiv d \pmod{a}$
(*d*) none of these

29. If $3^{200} \equiv k \pmod{10}$, then k is
(*a*) 3 (*b*) -9 (*c*) 10 (*d*) none of these

30. If $14 \equiv 172 \pmod{x}$, then x can take the value
(*a*) 79 (*b*) 54 (*c*) 66 (*d*) 38

31. If $ax \equiv bx \pmod{m}$ implies $a \equiv b \pmod{m}$ only when
(*a*) $x \neq 0$ (*b*) x is prime (*c*) $(a, b)=1$ (*d*) none of these

32. Which of the following is true?
(*a*) $21 \equiv -7 \pmod{6}$ (*b*) $-3 \equiv 3 \pmod{3}$ (*c*) $23 \equiv -12$ (mod 15
(*d*) $-11 \equiv -17 \pmod{14}$

33. The least non-negative remainder when 2^{300} is divided by 13
(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

34. The last digit in 7^{1000} is
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

35. The equation $ax \equiv b \pmod{m}$ has a unique solution if
(*a*) $(a, b)=1$ (*b*) $(b, m)=1$ (*c*) $(a, m)=1$ (*d*) none of these

36. The integers 0 and 1 are
(*a*) prime (*b*) composite (*c*) prime or composite (*d*) non

37. A value of x satisfying $1 \equiv 9x \pmod{10}$ is
(*a*) 9 (*b*) 8 (*c*) 7 (*d*) 5

38. If $(a, m)=d$ and $ax \equiv b \pmod{m}$ has no solutions, then
(*a*) $d=1$ (*b*) $d \nmid b$ (*c*) $d \mid b$ (*d*) none of these

39. If $(ab, c)=1$ and $(a, c)=1$, then (b, c) is equal to
(*a*) 1 (*b*) b (*c*) c (*d*) a

40. The least positive integer to which $79 \times 101 \times 125$ is congruen modulo 11 is
(*a*) 8 (*b*) 6 (*c*) 4 (*d*) 5

41. A value of x satisfying $27x \equiv 3 \pmod{5}$ is
(*a*) 18 (*b*) 25 (*c*) 20 (*d*) 19

42. If $a \equiv b \pmod{m}$ and n is a +ve divisor of m, then
(*a*) $a \equiv b \pmod{n}$ (*b*) a and b are divisors of n (*c*) $a \equiv n$ (mod
(*d*) none of these

43. The remainder when $2^{50} \times 5^{23}$ is divided by 100 is
(*a*) 2 (*b*) 5 (*c*) 0 (*d*) none of these

44. When 5^{1209} is divided by 7, the remainder is
(*a*) 4 (*b*) 3 (*c*) 1 (*d*) 5

45. The least non-negative remainder when $175 \times 365 \times 69$ is divid by 17 is
(*a*) 1 (*b*) 4 (*c*) 5 (*d*) 6

46. A value of x satisfying $3 \equiv x \pmod{13}$ is
(*a*) 27 (*b*) 28 (*c*) 29 (*d*) 30

47. $5 \mid ab$ and $5 \nmid a$, then
(*a*) $5 \mid b$ (*b*) $b \mid 5$ (*c*) $(5, b) = 1$ (*d*) none

48. $16 \mid ab$ and $16 \nmid a$, then
(*a*) $16 \mid b$ (*b*) $b \mid 16$ (*c*) $(a, b) = 16$ (*d*) none

49. A value of x satisfying $2(x+3) \equiv (3-x) \pmod{11}$ is
(*a*) 16 (*b*) -12 (*c*) 33 (*d*) none

50. The remainder when $75 \times 80 \times 85$ is divided by 13 is
(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 10

Answers

1. (*d*) 2. (*c*) 3. (*d*) 4. (*b*) 5. (*b*) 6. (*d*) 7. (*d*) 8. (*b*) 9. (*d*) 10. (*b*) 11. (*d*) 12. (*d*) 13. (*d*) 14. (*c*) 15. (*a*) 16. (*b*) 17. (*b*) 18. (*a*) 19. (*c*) 20. (*b*) 21. (*d*) 22. (*d*) 23. (*a*) 24. (*b*) 25. (*a*) 26. (*c*) 27. (*b*) 28. (*c*) 29. (*b*) 30. (*a*) 31. (*d*) 32. (*b*) 33. (*d*) 34. (*a*) 35. (*c*) 36 (*d*) 37 (*a*) 38 (*b*) 39 (*a*) 40 (*d*) 41. (*d*) 42. (*a*) 43. (*c*) 44. (*d*) 45. (*d*) 46. (*c*) 47. (*a*) 48. (*d*) 49. (*b*) 50. (*d*)

5.13 Matrices and determinants

1. If $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$, then $A - 2B =$

(*a*) $\begin{bmatrix} 3 & 2 \\ -3 & -2 \end{bmatrix}$ (*b*) $\begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$ (*c*) $\begin{bmatrix} -3 & -2 \\ 3 & -3 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & -2 \\ 3 & -3 \end{bmatrix}$

2. If A is a matrix of order 2×4 and B is a matrix of order 2×3, then $A'B$ is a matrix of order
(*a*) 3×4 (*b*) 2×2 (*c*) 4×3 (*d*) none of these

3. Which of the following is a column matrix?

(*a*) $(3\ 4\ 5)$ (*b*) (2) (*c*) $\begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{pmatrix}$ (*d*) none of these

4. Which of the following is a zero matrix?

(*a*) (0) (*b*) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (*c*) 0 (*d*) none of these

5. Which of the following is a scalar matrix?

(*a*) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (*b*) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ (*c*) $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ (*d*) $\begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix}$

6. If A is a matrix of order 3×1, $B=\begin{pmatrix}1\\1\\1\end{pmatrix}$ and C is a matri such that $3A-2B+5C=O$, then order of C' is
(a) 3×1 (b) 1×3 (c) 1×1 (d) none of these

7. If the product AB of two matrices is 0, then
(a) $A=0$ (b) $B=0$ (c) $A=0$ or $B=0$ (d) none of these

8. If $A^2-I=0$, then the inverse of A is
(a) I (b) $-A$ (c) A (d) none of these

9. If $B+A=\begin{pmatrix}2&0\\5&2\end{pmatrix}$ and $A-B=\begin{pmatrix}0&0\\1&2\end{pmatrix}$, then A=
(a) $\begin{pmatrix}2&0\\6&4\end{pmatrix}$ (b) $\begin{pmatrix}1&0\\2&0\end{pmatrix}$ [✓ (c) $\begin{pmatrix}1&0\\3&2\end{pmatrix}$] (d) none of thes

10. If $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-x\\-2y\\-3z\end{pmatrix}$, then (x, y, z) is
(a) (1, 2, 3) (b) (−1, −2, −3) (c) (1, 1, 1) (d) (0, 0, 0)

11. If $A=\begin{pmatrix}2&3\\4&1\end{pmatrix}$ and $B=\begin{pmatrix}1&2\\3&1\end{pmatrix}$, then $2A-3B$ is
[✓ (a) $\begin{bmatrix}1&0\\-1&-1\end{bmatrix}$] (b) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ (c) $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ (d) $\begin{bmatrix}0&-1\\0&-1\end{bmatrix}$

12. If $A=\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$, then $A^2=$
[✓ (a) $\begin{bmatrix}2&-2\\-2&2\end{bmatrix}$] (b) $\begin{bmatrix}2&-2\\-2&-2\end{bmatrix}$ (c) $\begin{bmatrix}-2&-2\\-2&2\end{bmatrix}$ (d) $\begin{bmatrix}-2&2\\2&-\end{bmatrix}$

13. If $A=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$, then $A^2=$
[✓ (a) $\begin{bmatrix}8&5\\-5&3\end{bmatrix}$] (b) $\begin{bmatrix}8&-5\\-5&3\end{bmatrix}$ (c) $\begin{bmatrix}8&5\\5&3\end{bmatrix}$ (d) $\begin{bmatrix}8&-5\\-5&-3\end{bmatrix}$

14. If $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$, then A (adj A) is
(a) $\begin{pmatrix}-2&-2\\-2&-2\end{pmatrix}$ (b) $\begin{pmatrix}2&1\\-2&0\end{pmatrix}$ (c) $\begin{pmatrix}-2&0\\0&-2\end{pmatrix}$ (d) none of th

15. Which of the following is a singular matrix?
(a) $\begin{pmatrix}1&2\\3&4\end{pmatrix}$ (b) $\begin{pmatrix}2&3\\4&5\end{pmatrix}$ (c) $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ [✓ (d) $\begin{pmatrix}-15&-5\\10&10\end{pmatrix}$]

16. If the matrix $\begin{pmatrix} 3 & 1 & 4 \\ 1 & x & 2 \\ 4 & 2 & 5 \end{pmatrix}$ is singular, then $x=$

(*a*) 2 (*b*) -2 (*c*) -1 (*d*) 1

17. If $\begin{bmatrix} x^2 & 4 \\ 2 & 3 \end{bmatrix}+\begin{bmatrix} 4x & 2 \\ 0 & -1 \end{bmatrix}=\begin{bmatrix} -4 & 2 \\ 2 & y \end{bmatrix}$, then (x, y) is

(*a*) $(-2, 2)$ (*b*) $(2, -2)$ (*c*) $(2, 2)$ (*d*) $(-2, -2)$

18. Which of the following matrices has multiplicative inverse?

(*a*) $\begin{pmatrix} 4 & 12 \\ 3 & 9 \end{pmatrix}$ (*b*) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (*c*) $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ (*d*) none of these

19. Which of the following statements is true?

(*a*) A matrix always has an inverse

(*b*) There exists no matrix which is both column matrix and row matrix

(*c*) If A and B are non-singular, so is AB

(*d*) A diagonal matrix need not have an inverse A^{-1}

20. If $\begin{bmatrix} x+y & -7 \\ 3 & x-y \end{bmatrix}=\begin{bmatrix} 3 & -7 \\ 3 & -1 \end{bmatrix}$, then $(x, y)=$

(*a*) $(1, 0)$ (*b*) $(0, 2)$ (*c*) $(1, -2)$ (*d*) $(1, 2)$

21. The inverse of $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ is

(*a*) $\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$ (*b*) $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ (*c*) $\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$ (*d*) none

22. The adjoint of the matrix $\begin{pmatrix} 2 & -3 \\ 4 & 3 \end{pmatrix}$ is

(*a*) $\begin{pmatrix} -3 & -3 \\ 4 & -2 \end{pmatrix}$ (*b*) $\begin{pmatrix} 3 & -4 \\ 3 & 2 \end{pmatrix}$ (*c*) $\begin{pmatrix} 2 & -4 \\ 3 & 2 \end{pmatrix}$ (*d*) none

23. If A and B are square matrices of same order, then $(A+B)^2=A^2+2AB+B^2$ holds only when

(*a*) $AB=BA=0$ (*b*) $AB=BA=I$ (*c*) $AB=BA$ (*d*) none

24. If A is a square matrix of order 2×2, then $(-2A)\text{ adj }(-2A)$ is

(*a*) $2|A|I$ (*b*) $-2|A|I$ (*c*) $-4|A|I$ (*d*) $4|A|I$

25. If $\begin{pmatrix} x+y & x-y \\ x-2y & 2x+y \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$, then (x, y) is

(*a*) $(\frac{1}{2}, \frac{1}{2})$ (*b*) $(-2, 3)$ (*c*) $(1, 1)$ (*d*) $(2, 2)$

26. If $A=\begin{bmatrix}-1 & -5\\ 2 & 1\end{bmatrix}$, then A^{-1} is

(*a*) $\begin{bmatrix}\frac{1}{9} & \frac{5}{9}\\ \frac{2}{9} & -\frac{1}{9}\end{bmatrix}$ (*b*) $\begin{bmatrix}-\frac{1}{9} & -\frac{5}{9}\\ \frac{2}{9} & \frac{1}{9}\end{bmatrix}$ (*c*) $\begin{bmatrix}\frac{1}{9} & \frac{5}{9}\\ -\frac{2}{9} & -\frac{1}{9}\end{bmatrix}$ (*d*) $\begin{bmatrix}\frac{1}{9} & -\frac{5}{9}\\ -\frac{1}{9} & \frac{2}{9}\end{bmatrix}$

27. $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$, then $A^4=$

(*a*) 4A (*b*) 2A (*c*) A (*d*) 8A

28. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then adj A is

(*a*) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (*b*) $\begin{bmatrix}\cos\theta & -\sin\theta\\ \cos\theta & \sin\theta\end{bmatrix}$ (*c*) $\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ (*d*) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

29. If A, B, C, are matrices of orders 2×3, 3×5 and 4×5 respectively, then the order of the transpose of $CB'A'$ is

(*a*) 2×4 (*b*) 4×2 (*c*) 2×2 (*d*) 4×4

30. The inverse of $\begin{bmatrix}\sec\theta & -\tan\theta\\ -\tan\theta & \sec\theta\end{bmatrix}$ is

(*a*) $\begin{bmatrix}-\sec\theta & -\tan\theta\\ \tan\theta & -\sec\theta\end{bmatrix}$ (*b*) $\begin{bmatrix}-\sec\theta & -\tan\theta\\ -\tan\theta & -\sec\theta\end{bmatrix}$ (*c*) $\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & \sec\theta\end{bmatrix}$

(*d*) none

31. $\begin{vmatrix}15 & 16 & 17\\ 20 & 21 & 22\\ 17 & 18 & 19\end{vmatrix}=$

(*a*) 4 (*b*) -4 (*c*) 0 (*d*) none of these

32. The value of $\begin{vmatrix}77 & 78 & 79\\ 76 & 75 & 74\\ 75 & 74 & 73\end{vmatrix}$ is

(*a*) 0 (*b*) 130 (*c*) 22 (*d*) 5

33. The value of $\begin{vmatrix}1 & 1 & 1\\ 1 & 1+a & 1\\ 1 & 1 & 1+b\end{vmatrix}$ is

(*a*) $ab-1$ (*b*) 0 (*c*) ab (*d*) $a+b+1$

34. If $A=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$, then

(*a*) $AB=0$ and $BA\neq0$ (*b*) $AB=0=BA$ (*c*) $AB\neq B$

(*d*) none of these

35. In a square matrix the first row is the difference of second third rows, then it is a

(*a*) diagonal matrix (*b*) zero matrix (*c*) singular mat

(*d*) none

36. The cofactor of 5 in $\begin{bmatrix} 1999 & 2000 & 3 \\ 2001 & 2002 & 4 \\ 1 & 1 & 5 \end{bmatrix}$ is

(*a*) 5 (*b*) 2 (*c*) 0 (*d*) none

37. The value of $\begin{vmatrix} 2 & 3 & 1 \\ -3 & -1 & -4 \\ -2 & -5 & -3 \end{vmatrix}$ is

(*a*) –44 (*b*) –24 (*c*) –50 (*d*) none of these

38. The value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ is

(*a*) $a^3+b^3+c^3$ (*b*) $(a-b)(b-c)(c-a)$ (*c*) 0 (*d*) none of these

39. $\begin{vmatrix} 3421 & 3422 \\ 3423 & 3424 \end{vmatrix}$ is equal to

(*a*) –2 (*b*) 1 (*c*) 0 (*d*) 3

40. If $A=\begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix}$, then $AB'=$

(*a*) $\begin{bmatrix} 8 & -7 \\ -12 & 22 \end{bmatrix}$ (*b*) $\begin{bmatrix} 8 & -7 \\ -10 & 20 \end{bmatrix}$ (*c*) $\begin{bmatrix} 8 & 7 \\ -12 & 22 \end{bmatrix}$ (*d*) none

41. In the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & \omega \end{vmatrix}$ where ω is an imaginary cube root of unity, the value of the determinant minor of ω is

(*a*) ω (*b*) ω^2 (*c*) $1+\omega+\omega^2$ (*d*) none

42. If $A=\begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 6 & 7 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & -2 \\ 8 & 6 \\ -4 & 3 \end{bmatrix}$, then $(2A-3B)'$ is

(*a*) $\begin{bmatrix} -7 & -20 & 24 \\ 12 & -8 & 5 \end{bmatrix}$ (*b*) $\begin{bmatrix} -7 & 12 \\ -20 & -8 \\ 24 & 5 \end{bmatrix}$ (*c*) $\begin{bmatrix} -7 & -20 & 0 \\ 0 & 8 & 5 \end{bmatrix}$

(*d*) none

43. For matrices A, B, C, AB=AC will imply B=C only when

(*a*) $A \neq 0$ (*b*) $A=I$ (*c*) A is non-singular (*d*) none

44. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, then A^2 is

(*a*) A (*b*) I (*c*) 0 (*d*) none

45. If A is a square matrix of order 3, then A+A′ is a
(*a*) diagonal matrix (*b*) zero matrix (*c*) symmetric ma
(*d*) none

46. The value of $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ is
(*a*) 0 (*b*) 3 (*c*) –3 (*d*) 4

47. The value of $\begin{vmatrix} 1 & 3 & 6 \\ 3 & 10 & 22 \\ 3 & 11 & 26 \end{vmatrix}$ is
(*a*) –216 (*b*) 280 (*c*) 0 (*d*) none

48. If $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & x \\ 4 & 1 & x^2 \end{bmatrix}$ is a non-singular matrix, then x cannot take the va
(*a*) ±1 (*b*) ±2 (*c*) —1, 2 (*d*) —1, —2

49. The value of $\begin{vmatrix} 2 & 23 & 39 \\ 0 & -1 & 83 \\ 0 & 0 & 5 \end{vmatrix}$ is
(*a*) 249 (*b*) 59 [✓ (*c*) –10] (*d*) none

50. If $A=\begin{bmatrix} 1 & 3 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$, then A′ B′ is
(*a*) $\begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}$ (*b*) $\begin{pmatrix} 3 & -2 \\ 0 & -4 \end{pmatrix}$ (*c*) $\begin{pmatrix} 2 & -3 \\ 0 & 5 \end{pmatrix}$ (*d*) non

51. If A is a square matrix of order 3 and | A | =5, then $|A^{-1}$
(*a*) 5 (*b*) –5 (*c*) $\frac{1}{5}$ (*d*) none

52. If A is a square matrix of order 3 and | A |=3, then | *adj* A
(*a*) 3 (*b*) 3^2 (*c*) 3^3 (*d*) none

53. If $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}(x\ y\ z)=\begin{bmatrix} 2 & 4 & 2 \\ 3 & 6 & 3 \\ 4 & 8 & 4 \end{bmatrix}$, then x, y and z are respecti
(*a*) 1, 2, 1 (*b*) 1, 1, 1 (*c*) 2, 1, 2 (*d*) none

54. If $\begin{vmatrix} 2 & 4 & -6 \\ -1 & x & 3 \\ -3 & 5 & 7 \end{vmatrix}=8$, then x is equal to
(*a*) 4 (*b*) –4 (*c*) –2 (*d*) 2

55. If $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$, then $x=$

(*a*) $\frac{5}{2}$ (*b*) $\frac{2}{5}$ (*c*) $-\frac{5}{2}$ (*d*) $-\frac{2}{5}$

56. If $\begin{vmatrix} x+3 & 5 & 7 \\ 3 & x+5 & 7 \\ 3 & 5 & x+7 \end{vmatrix} = 0$, then x is equal to

(*a*) 0, 15 (*b*) 0, –15 (*c*) 0, –15, 2 (*d*) none

57. If $\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix} = 0$, then $x=0$ and

(*a*) 6 (*b*) –6, 2 (*c*) –2 (*d*) –6

58. The value of $\begin{vmatrix} 80 & 81 & 82 \\ 83 & 84 & 85 \\ 86 & 87 & 89 \end{vmatrix}$ is

(*a*) 0 (*b*) 3 (*c*) –3 (*d*) none.

59. If A and B are square matrices of same order, then $(AB)'=$

(*a*) $A'B'$ (*b*) $A'B$ (*c*) AB' (*d*) $B'A'$.

60. If $A=(x\ 1\ i)$ where $i=\sqrt{-1}$ and $AA^T=0$, then $x=$

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) *i*

61. If A is a square matrix of order 3 and $|A|=5$, then $|5A|$ is

(*a*) 25 (*b*) 125 (*c*) 625 (*d*) none

Answers.

1. *c* 2. *c* 3. *b* 4. *a* 5. *a* 6. *b* 7. *d* 8. *c* 9. *c* 10. *d*
11. *a* 12. *a* 13. *a* 14. *c* 15. *d* 16. *c* 17. *a* 18. *b* 19. *d* 20. *d*
21. *b* 22. *d* 23. *c* 24. *d* 25. *c* 26. *c* 27. *d* 28. *c* 29. *a* 30. *c*
31. *c* 32. *a* 33. *c* 34. *b* 35. *c* 36. *d* 37. *b* 38. *c* 39. *a* 40. *a*
41. *c* 42. *a* 43. *c* 44. *a* 45. *c* 46. *d* 47. *c* 48. *c* 49. *c* 50. *a*
51. *c* 52. *b* 53. *a* 54. *b* 55. *a* 56. *b* 57. *d* 58. *c* 59. *d* 60. *a*
61. *c*

5.14 Groups

1. The subtraction on the set of integers is

(*a*) commutative but not associative (*b*) commutative and associative (*c*) associative but not commutative (*d*) neither associative nor commutative.

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2. The cube roots of unity are in
(*a*) A.P. (*b*) G.P. (*c*) H.P. (*d*) none

3. On Z, the binary operation * is defined by $a*b=a+b+2$;
identity element is
(*a*) 0 (*b*) 1 (*c*) –2 (*d*) none

4. On Z, the binary operation * is defined by $a*b=a+b-1$;
inverse of 5 is
(*a*) –5 (*b*) 3 (*c*) –2 (*d*) none

$a*e = a+e-1 = a$

$\boxed{e=1}$

5. On R, a binary operation * is defined by $a*b=\sqrt{a^2+b^2}$;
identity element is
(*a*) 0 (*b*) 1 (*c*) $\sqrt{2}$ (*d*) none

$\sqrt{a^2+e^2} = a = \sqrt{a^2}$

Squaring

$a^2+e^2=a^2$

$\therefore \boxed{e=0}$

6. Which of the following is not a mathematical system?
(*a*) (Z, +) (*b*) (Q–{0}, ÷) (*c*) (R, +) (*d*) (N, –)

7. On R, a binary operation * is defined by $a*b=ab+1$, then
(*a*) associative (*b*) commutative but not associative
(*c*) commutative and associative (*d*) none

8. In the system {2, 4, 6, 8} under multiplication mod 10, the ident
element is
(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 6

9. The cube roots of unity form a group under
(*a*) addition (*b*) multiplication (*c*) subtraction (*d*) divis

10. The set of integers is a
(*a*) finite group (*b*) additive group (*c*) multiplicative gr
(*d*) both (*b*) and (*c*)

11. Which of the following sets is a group under multiplication mo
(*a*) {1, 3, 4} (*b*) {1, 3, 5} (*c*) {1, 4, 6} (*d*) {1, 2, 4}

12. Which of the following sets is a group under addition mo
(*a*) {0, 2, 4, 6} (*b*) {0, 1, 2, 3} (*c*) {0, 2, 4, 5} (*d*) none

13. Which of the following statements is true?
(*a*) The multiplicative inverse exists for every rational num
(*b*) addition is a binary operation on the set of irratio numbers (*c*) the "vector product" is a binary ope tion on the set of plane vectors (*d*) none

14. On a set of real numbers a binary operation * is defined $a*b=a^b$; (–1) * (–1) is
(*a*) 1 (*b*) –1 (*c*) i (*d*) none

$(-1)^{-1} = \frac{1}{(-1)^1} = -1$

15. On the set of real numbers which of the following definitions of * defines a binary operation

(*a*) $a*b=a/b$ (*b*) $a*b=\sqrt{ab}$ (*c*) $a*b=\dfrac{a+b}{a-b}$ (*d*) $a*b=a$

16. On Q, which of the following is NOT binary?

(*a*) addition (*b*) subtraction (*c*) division (*d*) none

17. On Q, $a*b=\dfrac{a+b}{2}$, then * is

[✓ (*a*)] commutative but not associative (*b*) commutative and associative (*c*) associative but not commutative (*d*) none.

18. On Q, $a*b=\dfrac{ab}{4}$; the identity element and the inverse of 8 are respectively

(*a*) 4; 4 (*b*) 1, 8 (*c*) 4, 2 (*d*) none

19. If G is a group and $a, b, c \in G$, the inverse of $ab^{-1}c$ is

(*a*) $a^{-1}bc^{-1}$ (*b*) $c^{-1}a^{-1}b$ (*c*) $a^{-1}bc$ [✓ (*d*)] $\underline{c^{-1}ba^{-1}}$

20. In the group G={ 1, 2, 3, 4 } under × mod 5, the inverses of 2 and 4 are respectively

(*a*) 2, 3 (*b*) 2, 4 [✓ (*c*)] 3, 4 (*d*) 3, 1

21. The set { ±1 } is a group w.r.t.

(*a*) addition (*b*) addition (mod 2) (*c*) multiplication (*d*) none

22. In the group G of real numbers except –1, under * where $a*b=a+b+ab$, $a, b \in G$, then the solution of $3*x=4*1$ is

(*a*) 3 [✓ (*b*)] 3/2 (*c*) 4/3 (*d*) none

23. Inverse axiom holds in the system

(*a*) Z_2 under multiplication mod 2 (*b*) the set of complex numbers under '.' (*c*) Z_4 under addition mod 4 (*d*) {1, 2, 3} under × mod 4

24. In Z_6, the set of integers mod 6, the additive inverse of 2 is

(*a*) 2 (*b*) –2 (*c*) 3 (*d*) none

25. In which system inverse exists for every non-zero element?

(*a*) $(Z_3, .)$ (*b*) $(Z_4, .)$ (*c*) $(Z_9, .)$ (*d*) $(Z_{12}, .)$

forcomposit number we donot get inverse

26. What is the inverse of 5 in $(Z_8, +)$?

(*a*) –5 (*b*) 5 (*c*) 3 (*d*) doesn't exist

27. What is the inverse of 5 in $(Z_7, \times)$?

(*a*) 1/5 (*b*) 2 (*c*) 3 (*d*) doesn't exist

28. Which of the following is a group?

(*a*) Z_4 under × mod 4 (*b*) Z_5 under × mod 5

(*c*) $Z_4-\{0\}$ under × mod 4 (*d*) $Z_5-\{0\}$ under × mod 5

29. If S is the set of all non-zero 2×2 matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ then multiplicative identity of S is

(*a*) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (*b*) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (*c*) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (*d*) none

30. On the set of non-zero reals, which of the following is NOT binary?

Eg:- $-1+1=0 \notin R$ → (*a*) addition (*b*) multiplication (*c*) division (*d*) none

31. In the group G={ 0, 1, 2, 3, 4 }, under addition modulo 5, the inverse of 4 is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

32. If G is a non-commutative group and *a*, *b*, *c*, *d* ∈ G, then $(ab^{-1}c^{-1}d)^{-1}$ is

(*a*) $a^{-1}bcd^{-1}$ (*b*) $bcd^{-1}a^{-1}$ (*c*) $d^{-1}cba^{-1}$ (*d*) $d^{-1}cb^{-1}a^{-1}$

33. In the group of non-zero real numbers $a*b=\frac{ab}{5}$; the solution of $2*x*5=10$ is

$2*x*5 = \frac{2x}{5}*5 = \frac{10x}{25}$

$\frac{10x}{25} = 10, x = 25$

(*a*) 25 (*b*) 1 (*c*) 25/2 (*d*) 5

34. The set of rational numbers is a group under

(*a*) subtraction (*b*) multiplication (*c*) division (*d*) none

35. Given a binary operation table, which one of the following axioms is NOT obvious from the table?

(*a*) closure (*b*) commutative (*c*) identity (*d*) associative

36. Which one of the following gives a group table?

(*a*) $\begin{array}{c|cc} * & a & b \\ \hline a & a & a \\ b & a & b \end{array}$ (*b*) $\begin{array}{c|cc} * & a & b \\ \hline a & a & b \\ b & a & b \end{array}$ (*c*) $\begin{array}{c|cc} * & a & b \\ \hline a & b & a \\ b & a & b \end{array}$ (*d*) $\begin{array}{c|cc} * & a & b \\ \hline a & a & b \\ b & b & b \end{array}$

37. Which of the following is a subgroup of (Q, +)?

(*a*) (N, +) (*b*) (Z, .) (*c*) (Z, +) (*d*) none

38. Which of the following is a subgroup of (Z_4, +)?

(*a*) {0} under+ (*b*) {0, 1} under+

(*c*) {0, 1, 2} under+ (*d*) none

$\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} =$

39. A subgroup of $Z_5 - \{0\}$ under × mod 5 is
(*a*) $\{0\}$ (*b*) $\{1, 2\}$ (*c*) $\{1, 3\}$ (*d*) $\{1, 4\}$

40. H is a non-empty subset of a group G. Then H is a subgroup of G if the following postulates are true in H
(*a*) closure and associative (*b*) closure and identity
(*c*) closure and inverse (*d*) identity and inverse

41. Which one of the following is NOT a subgroup of the multiplicative group of fourth roots of unity
(*a*) $\{1\}$ (*b*) $\{1, -1\}$ (*c*) $\{1, i, -i\}$ (*d*) none

42. In the multiplicative group $\{1, -1, i, -i\}$ where $i^2 = -1$, the inverse of $-i$ is
(*a*) 1 (*b*) -1 (*c*) i (*d*) $-i$

43. The set of integers modulo 5 is a group under
(*a*) +mod 5 (*b*) × mod 5 (*c*) both (*a*) and (*b*)
(*d*) none of (*a*) and (*b*)

44. On the set of natural numbers, define $a*b = a^b$, then * is
(*a*) commutative (*b*) associative (*c*) not a binary operation
(*d*) none

45. The set of real numbers is not a multiplicative group because of the failure of
(*a*) closure law (*b*) associative law (*c*) identity law
(*d*) inverse law

46. In the multiplicative group $\{1, \omega, \omega^2, \omega^3\}$ of fourth roots of unity, what is the inverse of ω?
(*a*) ω (*b*) ω^2 (*c*) ω^3 (*d*) none

$\because \omega^2 = -1$

47. In a commutative group G, the solution of $axb = ba$, $a, b, x \in G$ is
(*a*) $a^{-1}bab^{-1}$ (*b*) e (*c*) $a^{-1}b^{-1}ab$ (*d*) all of them

$axb = ba$
$x = a^{-1}bab$

48. Which of the following are semi-groups:
(*i*) (N, +) (*ii*) (N, .) (*iii*) (N, −) (*iv*) (N, ÷)
(*a*) (*i*) only (*b*) (*i*) and (*iii*)
(*c*) (*i*) and (*iv*) (*d*) (*i*) and (*ii*)

49. In the group $\{1, 3, 4, 5, 9\}$ under × mod 11, the inverse of 5 is
(*a*) 3 (*b*) 4 (*c*) 9 (*d*) none

50. Which of the following is not a multiplicative group?
(*a*) $Z-\{0\}$ (*b*) $Q-\{0\}$ (*c*) $R-\{0\}$ (*d*) $C-\{0\}$

Answers

1. (*d*) 2. (*b*) 3. (*c*) 4. (*d*) 5. (*d*) 6. (*d*) 7. (*b*) 8. (*d*) 9. (*b*) 10. (*b*) 11. (*d*) 12. (*a*) 13. (*d*) 14. (*b*) 15. (*d*) 16. (*c*) 17. (*a*) 18. (*c*) 19. (*d*) 20. (*c*) 21. (*c*) 22. (*b*) 23. (*c*) 24. (*d*) 25. (*a*) 26. (*c*) 27. (*c*) 28. (*d*) 29. (*c*) 30. (*a*) 31. (*a*) 32. (*c*) 33. (*a*) 34. (*d*) 35. (*d*) 36. (*c*) 37. (*c*) 38. (*a*) 39. (*d*) 40. (*c*) 41. (*c*) 42. (*c*) 43. (*a*) 44. (*d*) 45. (*d*) 46. (*c*) 47. (*d*) 48. (*d*) 49. (*c*) 50. (*a*)

5.15 Vectors

1. **The coordinates of A and B are (1, 2, –1) and (2, 1, –1) respectively. Then $\overrightarrow{BA}$ =**

   (*a*) $3\hat{i}+3\hat{j}-2\hat{k}$ (*b*) $-\hat{i}+\hat{j}$ (*c*) $\hat{i}-\hat{j}$ (*d*) none.

2. **The dot product of the vectors $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ is**

   (*a*) 1 (*b*) 2 (*c*) –1 (*d*) 0

3. **Which of the following is a scalar?**

   (*a*) $(3\hat{i}.4\hat{j})\,3\hat{k}$ (*b*) $(\hat{i}\times\hat{j}).\hat{k}$ (*c*) $\hat{i}\times\hat{i}$ (*d*) $(\hat{i}.\hat{i})\,\hat{i}$

4. **Which of the following is a vector?**

   (*a*) $(\hat{i}\times\hat{i}).(\hat{j}\times\hat{j})$ (*b*) $(\hat{i}\times\hat{i}).\hat{i}$ (*c*) $(\hat{i}+\hat{j})\times(\hat{i}+\hat{j})$ (*d*) none

5. **The angle between the vectors $\hat{i}+4\hat{j}+3\hat{k}$ and $4\hat{i}-2\hat{j}+2\hat{k}$ is**

   (*a*) 90° (*b*) 45° (*c*) $\cos^{-1}(2/\sqrt{5})$ (*d*) none

6. **Which one of the following is a unit vector?**

   (*a*) (1, 1, 1) (*b*) $(\frac{1}{2}, \frac{1}{2}, 0)$ (*c*) $(\frac{1}{2}, -\frac{1}{2}, 0)$ (*d*) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

7. **If $\vec{a}=4\hat{j}$, $\vec{b}=\hat{i}+3\hat{j}$ and $\vec{c}=\hat{k}$, then $|\vec{a}\times\vec{b}+\vec{c}|=$**

   (*a*) 5 (*b*) $\sqrt{5}$ (*c*) $\sqrt{3}$ (*d*) none

8. **If the vectors (2, –1, –1), (–1, 2, –1) and (–1, –1, *m*) are coplanar, then *m*=**

   (*a*) 2 (*b*) –1 (*c*) 1 (*d*) –2

9. **Which of the following is NOT true?**

   (*a*) the angle between two vectors $\vec{a}$ and $\vec{b}$ is $\cos^{-1}(\hat{a}.\hat{b})$

   (*b*) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=a^2-b^2$ (*c*) $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=2(\vec{a}\times\vec{b})$

   (*d*) none

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10. The area of the triangle, two of whose sides are represented by the vectors $4\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is
(*a*) $7\sqrt{2}$ (*b*) $7/2$ (*c*) $7/\sqrt{2}$ (*d*) none

11. The length of the vector $\hat{i}-2\hat{j}+2\hat{k}$ is
(*a*) $\sqrt{14}$ (*b*) 1 (*c*) 3 (*d*) none

12. If A is $(1, -2, 1)$ and B is $(2, 2, -1)$, then $|\overrightarrow{BA}|=$
(*a*) $\sqrt{21}$ (*b*) $\sqrt{20}$ (*c*) 3 (*d*) none

13. If $\vec{a}$ and $\vec{b}$ are unit vectors, the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is
(*a*) 0° (*b*) 60° (*c*) 90° (*d*) 180°

14. If $\vec{a}=2\hat{i}-3\hat{j}$ and $\vec{b}=2\hat{j}+3\hat{k}$, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ is
(*a*) 0 (*b*) -8 (*c*) 9 (*d*) -10

15. If the vectors $(3, -5, 7)$ and $(2, x, 2)$ are orthogonal, then $x=$
(*a*) -2 (*b*) 1 (*c*) -3 (*d*) 4

16. If $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$, then $\vec{a}\times(\vec{a}.\vec{b})$ is
(*a*) 0 (*b*) $3\vec{a}$ (*c*) meaningless (*d*) none

17. The cosine of the angle between the vectors $(2, -1, -2)$ and $(1, -8, -4)$ is (*a*) $\frac{1}{4}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) none

18. If the three unit vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is (*a*) $\frac{3}{2}$ (*b*) 3 (*c*) 0 (*d*) $-\frac{3}{2}$

19. If $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{a}+\vec{b}|=\sqrt{13}$, then $|\vec{a}-\vec{b}|=$
(*a*) 31 (*b*) $\sqrt{29}$ (*c*) $\sqrt{37}$ (*d*) none

use $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2(|\vec{a}|^2+$

20. If the three points $(1, 2, 3)$, $(2, x, 3)$ and $(3, 4, 5)$ are collinear, then $x=$ (*a*) 5 (*b*) $\frac{5}{2}$ (*c*) $\frac{1}{2}$ (*d*) none

21. If $\vec{a}=5\hat{i}-3\hat{j}$ and $\vec{b}=4\hat{i}+\hat{k}$, then the length of the vector $-\vec{a}\times\vec{b}$ is
(*a*) $\sqrt{176}$ (*b*) $\sqrt{188}$ (*c*) $\sqrt{430}$ (*d*) none

22. If $\vec{a}$ and $\vec{b}$ represent adjacent sides of a quadrilateral with $(\vec{a}+\vec{b})\perp(\vec{a}-\vec{b})$, then the quadrilateral is
(*a*) parallelogram (*b*) trapezium (*c*) rhombus (*d*) rectangle

23. A unit vector perpendicular to the plane determined by the vectors $\hat{i}-4\hat{j}+8\hat{k}$ and $2\hat{i}+3\hat{j}-6\hat{k}$ is
(*a*) $(4\hat{i}-3\hat{j}+2\hat{k})/\sqrt{29}$ (*b*) $(-4\hat{i}-3\hat{j}+2\hat{k})/\sqrt{29}$ (*c*) $(\hat{j}+2\hat{k})/\sqrt{5}$
(*d*) none

24. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, then the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is (*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

25. If $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+2\hat{k}$, then $\vec{a}.(\vec{b}\times\vec{c})$ is
(*a*) 9 (*b*) 12 (*c*) –6 (*d*) 0

26. The scalar triple product of $\hat{i}+2\hat{j}$, $\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{k}$ is
(*a*) 8 (*b*) 6 (*c*) 2 (*d*) none

* 27. The vectors $2\hat{i}-6\hat{j}-3\hat{k}$ and $4\hat{i}+3\hat{j}-\hat{k}$ are inclined to each other at an angle (*a*) $\cos^{-1}\frac{1}{2}$ (*b*) $\sin^{-1}\sqrt{\frac{25}{26}}$ (*c*) $\pi/4$ (*d*) none

28. The area of the parallelogram with adjacent sides represented by the vectors $3\hat{i}-\hat{j}-3\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$ is
(*a*) $7\sqrt{2}$ (*b*) $7\sqrt{2}/2$ (*c*) $\sqrt{123}$ (*d*) none

*29. If $|\vec{a}|=10$, $|\vec{b}|=2$ and $|\vec{a}\times\vec{b}|=12$, then $|\vec{a}\,.\,\vec{b}|$ is
(*a*) meaningless (*b*) 8 (*c*) 16 (*d*) 24

30. Which of the following is a unit vector?
(*a*) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (*b*) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ (*c*) $(\sin\theta, \cos\theta, 1)$
(*d*) $(\sin\alpha, \cos\alpha\cos\beta, \cos\alpha\sin\beta)$

31. The volume of the tetrahedron ABCD where A=(0, 1, 2), B=(3, 0, 1), C=(4, 3, 6) and D=(2, 3, 2) is
(*a*) 1 (*b*) 6 (*c*) 36 (*d*) none

*32. The vector $\vec{a}\times(\vec{b}\times\vec{a})$ is
(*a*) $\vec{0}$ (*b*) $\perp\vec{a}$ (*c*) $||\vec{a}$ (*d*) none

33. The cosine of the angle between the lines AB and AC, where A, B, C are the points (1, 2, –1), (2, 0, 3) and (3, –1, 2) respectively, is
(*a*) $\frac{20}{\sqrt{21}\sqrt{22}}$ (*b*) $\frac{20}{\sqrt{22}\sqrt{23}}$ (*c*) $\frac{19}{\sqrt{21}\sqrt{23}}$ (*d*) none

34. The volume of the parallelopiped with coterminal edges represented by the vectors $2\hat{i}-4\hat{j}+5\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}-5\hat{j}+2\hat{k}$ is
(*a*) 16 (*b*) 20 (*c*) 8 (*d*) none

35. The cosine of the angle between the vectors $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ is (*a*) 1 (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{6}$

36. If $\vec{a}$ and $\vec{b}$ are unit vectors, then $\vec{a}+\vec{b}$ is
(*a*) a unit vector (*b*) a unit vector when $\vec{a} \parallel \vec{b}$
(*c*) a unit vector when $\vec{a} \perp \vec{b}$ (*d*) none

37. If the sum of three unit vectors is $\vec{0}$, then the angle between any two of them is
(*a*) 0° (*b*) 60° (*c*) 120° (*d*) none

38. If the lengths of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are equal, then
(*a*) $|\vec{a}|=|\vec{b}|$ (*b*) $\vec{a} \parallel \vec{b}$ (*c*) $\vec{a} \perp \vec{b}$ (*d*) none

39. If $\vec{a}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{b}=3\hat{i}-4\hat{j}+8\hat{k}$, then $(\vec{a}+\vec{b})\times(\vec{a}+\vec{b})=$
(*a*) $\vec{0}$ (*b*) $13\hat{i}-4\hat{j}+18\hat{k}$ (*c*) $23\hat{i}-8\hat{j}+\hat{k}$ (*d*) none

40. If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$, then $\vec{a}$ is perpendicular to
(*a*) $\vec{b}$ (*b*) $\vec{c}$ (*c*) $\vec{b}$ and $\vec{c}$ (*d*) $\vec{b}-\vec{c}$

$\vec{a}.\vec{b}-\vec{a}.\vec{c}=0$
$\vec{a}.(\vec{b}-\vec{c})=0$
Dot produ
$\therefore \vec{a} \perp^{r} (\vec{b}-\vec{c})$

41. The scalar triple product of the vectors $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$ and $\hat{k}+\hat{i}$ is
(*a*) 1 (*b*) 2 (*c*) 4 (*d*) none

42. If $\vec{OA}=\hat{i}-\hat{j}$ and $\vec{OB}=\hat{j}-\hat{k}$, then a unit vector perpendicalar to the plane AOB is
(*a*) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (*b*) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (*c*) $(\hat{i}+\hat{j}+\hat{k})$ (*d*) none

43. $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{k}\times\hat{i})+\hat{k}.(\hat{j}\times\hat{i})=$
(*a*) 0 (*b*) 1 (*c*) 3 (*d*) none

44. $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=$
(*a*) $\vec{0}$ (*b*) $|\vec{a}|^2-|\vec{b}|^2$ (*c*) $2(\vec{a}\times\vec{b})$ (*d*) none

45. If $\vec{a}$ and $\vec{b}$ are unit vector ⊥ each other, then $|\vec{a}\times\vec{b}|$ is
(*a*) 0 (*b*) 1 (*c*) can't say for sure (*d*) none

46. If $\vec{a}+\vec{b}+\vec{c}=0$, then $\vec{a}\times\vec{b}=$

(*a*) $\vec{0}$ (*b*) can't find (*c*) $\vec{c}\times\vec{a}$ (*d*) $\vec{a}\times\vec{c}$

47. If $\vec{a}$ and $\vec{b}$ are unit vectors inclided at an angle 120°, then

$|\vec{a}+\vec{b}|=$ (*a*) 1 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) none

48. If $\vec{a}=(1, 1, -2)$, $\vec{b}=(-1, 2, 1)$ and $\vec{c}=(1, -2, 2)$, then a unit vector parallel to $\vec{a}+\vec{b}+\vec{c}$ is

(*a*) $\frac{1}{\sqrt{6}}(2, 1, 1)$ (*b*) $\frac{1}{\sqrt{3}}(1, 1, 1)$

(*c*) $\frac{1}{\sqrt{6}}(1, -2, 1)$ (*d*) $\frac{1}{\sqrt{3}}(1, -1, 1)$

49. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=5$, $|\vec{b}|=12$ and $|\vec{c}|=13$, then the anlge between $\vec{a}$ and $\vec{b}$ is

(*a*) 0 (*b*) $\pi/2$ (*c*) $\cos^{-1} 2/\sqrt{5}$ (*d*) none

50. If the volume of the parallelopiped formed by the vectors (1, –2, 3), (2, 1, –1) and (0, x, 1) is 12 units, then $x=$

(*a*) 0 (*b*) 3 (*c*) 1 (*d*) none.

51. A unit vector ⊥ to both $3\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}-2\hat{j}+4\hat{k}$ is

(*a*) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (*b*) $\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ (*c*) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ (*d*) none

52. If $\vec{a}=(1, 2, -1)$ and $\vec{b}=(2, 3, 1)$, the projection of $\vec{a}$ on $\vec{b}$ is

(*a*) $\sqrt{14}/2$ (*b*) $7/\sqrt{6}$ (*c*) $\frac{1}{2}$ (*d*) none

53. If the scalar product of the two vectors $(x, 2, 3)$ and $(2, -9, x)$ is 12, then x is

(*a*) 18/5 (*b*) 6 (*c*) 12 (*d*) none

54. The sine of the angle between the vectors $3\hat{i}+4\hat{j}$ and $\hat{i}+\hat{j}+\hat{k}$ is

(*a*) $\frac{\sqrt{26}}{5\sqrt{3}}$ (*b*) $\frac{\sqrt{74}}{\sqrt{15}}$ (*c*) $\frac{\sqrt{71}}{\sqrt{5}\sqrt{3}}$ (*d*) none

55. The scalar triple product of the vectors (2, –3, 1), (1, –1, 2) and (2, 1, –1) is

(*a*) –20 (*b*) –14 (*c*) 0 (*d*) none

Answers

1. (*b*) 2. (*c*) 3. (*b*) 4. (*c*) 5. (*d*) 6. (*d*) 7. (*c*) 8. (*a*) 9. (*c*) 10. (*c*) 11. (*c*)
12. (*a*) 13. (*c*) 14. (*a*) 15. (*d*) 16. (*c*) 17. (*c*) 18. (*d*) 19. (*c*) 20. (*b*)
21. (*d*) 22. (*c*) 23. (*d*) 24. (*d*) 25. (*a*) 26. (*b*) 27. (*b*) 28. (*a*) 29. (*c*)
30. (*d*) 31. (*b*) 32. (*b*) 33. (*a*) 34. (*c*) 35. (*d*) 36. (*d*) 37. (*c*) 38. (*c*)
39. (*a*) 40. (*d*) 41. (*b*) 42. (*b*) 43. (*b*) 44. (*c*) 45. (*b*) 46. (*c*) 47. (*a*)
48. (*b*) 49. (*b*) 50. (*c*) 51. (*b*) 52. (*a*) 53. (*b*) 54. (*a*) 55. (*b*)

5.2 TRIGONOMETRY

5.21 Properties of triangles and solution of triangles

(The problems are related to a triangle ABC)

1. The ratio $(a+b) : c$ is

(*a*) $\cos\frac{(A+B)}{2} : \sin\frac{C}{2}$ [✓ (*b*) $\cos\frac{A-B}{2} : \sin\frac{C}{2}$]

(*c*) $\sin\frac{A+B}{2} : \sin\frac{C}{2}$ (*d*) none

2. $\frac{b^2-c^2}{a^2}$ is equal to

[✓ (*a*) $\sin(B-C) / \sin(B+C)$] (*b*) $\frac{\sin(B-C)}{\cos A}$ (*c*) $\frac{\cos(B-C)}{\sin A}$

(*d*) none

* 3. $\frac{\cos 2B}{b^2} - \frac{\cos 2C}{c^2} =$

(*a*) b^2-c^2 (*b*) $\frac{1}{c^2} - \frac{1}{b^2}$ [✓ (*c*) $\frac{1}{b^2} - \frac{1}{c^2}$] (*d*) none

4. $\Sigma \frac{\cos A}{a} = \frac{a^2+b^2+c^2}{k\,abc}$, then $k=$

(*a*) 1 [✓ (*b*) 2] (*c*) 3 (*d*) none

5. If the sines of the angles of a triangle are in the ratio 1:2:4, then the sides are in the ratio

(*a*) 1:4:16 [✓ (*b*) 1:2:4] (*c*) 1:4:8 (*d*) none

6. If the sides are 2, 3, $\sqrt{7}$, then the cosine of the greatest angle is

[✓ (*a*) $\sqrt{7}/14$] (*b*) $1/\sqrt{7}$ (*c*) 1/2 (*d*) none

7. If $a=8$, $b=6$, $c=4$, then $\sin A=$

(*a*) $\sqrt{3}/4$ (*b*) $\sqrt{17}/4$ [✓ (*c*) $\sqrt{15}/4$] (*d*) none

8. **If $a=\sqrt{3}$, $b=\sqrt{2}$ and $c=(\sqrt{6}+\sqrt{2})/2$, then tan A=**
(*a*) $1/\sqrt{3}$ (*b*) $\sqrt{3}$ (*c*) -1 (*d*) $-\sqrt{3}$

9. **The circumradius of the triangle ABC, if $b=\sqrt{3}$, $c=1$ and A=30° is** (*a*) 1 (*b*) 2 (*c*) 1/2 (*d*) $\sqrt{3}$

* 10. **The circumdiameter of $\triangle$ABC where A=60°, B=45° and $b=5$ is** (*a*) $5/\sqrt{2}$ (*b*) $5\sqrt{2}$ (*c*) $10\sqrt{2}$ (*d*) none

11. **In $\triangle$ABC, if $a=\sqrt{2}$, $c=\sqrt{3}-1$ and B=135°, then b=**
(*a*) 1 (*b*) 2 (*c*) $2\sqrt{3}$ (*d*) 4

12. **The greatest angle of the triangle with sides x, y and $\sqrt{x^2+y^2+xy}$ is** (*a*) 120° (*b*) 135° (*c*) 150° (*d*) 105°

13. **If b cos B$=c$ cos C, then the triangle is**
(*a*) isosceles (*b*) right angled (*c*) either (*a*) or (*b*)
(*d*) both (*a*) and (*b*)

14. **If $b^2+a^2>c^2$, then C is**
(*a*) acute (*b*) obtuse (*c*) right angle

15. **$\Sigma(b+c)$ cos A=**
(*a*) 0 (*b*) a+b+c (*c*) 2 (a+b+c) (*d*) none

16. **If the ratio of two sides of a triangle is $2+\sqrt{3}$ and the included angle is 60°, then the difference of the other two angles is**
(*a*) 90° (*b*) 120° (*c*) 45° (*d*) 30°

17. **If A=120°, b=2, C=30°, then a=**
(*a*) $2\sqrt{3}$ (*b*) 2 (*c*) $\sqrt{3}/2$ (*d*) 1/2

* 18. **If b=30, c=20 and B=45°, the number of possible $\triangle$ABC is**
(*a*) nil (*b*) 1 (*c*) 2 (*d*) none

19. **If a, b and A are given, the number of possible $\triangle$ABC is**
(*a*) nil (*b*) 1 (*c*) 2 (*d*) any one of (*a*), (*b*) or (*c*)

* 20. **If b=50, c=140 and C=135°, then the number of possible triangle ABC is** (*a*) nil (*b*) 1 (*c*) 2 (*d*) none

* 21. **If b=25, c=30 and B=120°, then the number of possible $\triangle$ABC is** (*a*) nil (*b*) 1 (*c*) 2 (*d*) none

22. **If a=7, b=5 and c=8, then A=**
(*a*) 30° (*b*) 120° (*c*) 60° (*d*) 45°

23. **The greatest angle of the triangle with sides 3, 5 and 7 is**
(*a*) 90° (*b*) 105° (*c*) 120° (*d*) 150°

24. If $a=3$, $b=2\sqrt{3}$, $c=\sqrt{3}$, then A=
(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 75°

25. The smallest angle of a triangle with sides 2, $\sqrt{6}$ and $\sqrt{3}-1$ is
(*a*) 15° (*b*) 30° (*c*) 75° (*d*) 60°

26. The greatest angle of a triangle with sides $\sqrt{6}-\sqrt{2}$, $2\sqrt{2}$ and $2\sqrt{3}$ is (*a*) 120° (*b*) 150° (*c*) 90° (*d*) 105°

27. The greatest side of $\triangle$ ABC where $a=2$, B=120° and C=30° is
(*a*) 2 (*b*) $2\sqrt{3}$ (*c*) $3\sqrt{2}$ (*d*) none

28. The smallest side of $\triangle$ABC where A=30°, B=135° and $c=\sqrt{3}-1$ is (*a*) a (*b*) b (*c*) c

29. The smallest side of $\triangle$ABC where B=60°, C=15° and $b=\sqrt{6}$ is
(*a*) $\sqrt{3}+1$ (*b*) $\sqrt{3}$ (*c*) $2\sqrt{3}$ (*d*) $\sqrt{3}-1$

30. If $a=\dfrac{1}{\sqrt{6}-\sqrt{2}}$, $b=\dfrac{1}{\sqrt{6}+\sqrt{2}}$ and C=60°, then $c=$
(*a*) $\sqrt{3}$ (*b*) $\sqrt{3}/2$ (*c*) $2\sqrt{3}$ (*d*) none

31. If $a=2$, $b=\sqrt{6}$ and $c=\sqrt{3}+1$, then cosec $\dfrac{B}{2}$ is
(*a*) 2 (*b*) $\sqrt{2}$ (*c*) $2/\sqrt{3}$ (*d*) 3

32. If the angles of a triangle are in A.P., then one of the angles must be (*a*) $\pi/6$ (*b*) $\pi/4$ (*c*) $\pi/3$ (*d*) $\pi/2$

33. If $a=6$, A=45°, B=60°, then c is
(*a*) $3\sqrt{6}$ (*b*) $\sqrt{6}$ (*c*) $3(\sqrt{3}+1)$ (*d*) none

34. If $b=2$, B=30° and $c=2\sqrt{3}$, then C=
(*a*) 15° or 165° (*b*) 30° or 150° (*c*) 75° or 105° (*d*) 60° or 120°

35. If $a=2$, $b=\sqrt{6}+\sqrt{2}$ and B=150°, then c is *
(*a*) $\sqrt{3}+1$ (*b*) $\sqrt{2}$ (*c*) $\sqrt{3}$ (*d*) 2

36. If $a=2$, $b=\sqrt{2}$, $c=\sqrt{3}-1$, then C=
(*a*) 30° (*b*) 60° (*c*) 45° (*d*) 15°

37. If the sides of a triangle are in the ratio $1:\sqrt{3}:2$, then its angles are in the ratio ☆
(*a*) $1:\sqrt{3}:2$ (*b*) 1:3:4 (*c*) 1:2:3 (*d*) $1:3:2\sqrt{3}$

38. The angles of a triangle are in the ratio 2:1:3, then the ratio of the smallest side to the greatest is ☆
(*a*) 1:2 (*b*) $2:\sqrt{3}$ (*c*) $\sqrt{2}:\sqrt{3}$ (*d*) none

39. If $a=25$, $b=52$, $c=63$, then $\tan\frac{C}{2}$ is

(*a*) 9/4 (*b*) 9/5 (*c*) 9/7 (*d*) 9/8

40. If $A=30°$, $B=135°$ and $c=\sqrt{3}-1$, then $b=$

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$ (*c*) $2\sqrt{2}$ (*d*) 2

Answers

1. (*b*) 2. (*a*) 3. (*c*) 4. (*b*) 5. (*b*) 6. (*a*) 7. (*c*) 8. (*b*) 9. (*a*) 10. (*b*)
11. (*b*) 12. (*a*) 13. (*c*) 14. (*a*) 15. (*b*) 16 (*a*) 17. (*a*) 18. (*b*) 19. (*d*)
20. (*b*) 21. (*a*) 22. (*c*) 23. (*c*) 24. (*c*) 25. (*a*) 26. (*a*) 27. (*b*) 28. (*c*)
29. (*d*) 30. (*b*) 31. (*a*) 32. (*c*) 33. (*c*) 34. (*d*) 35. (*d*) 36. (*d*) 37. (*c*)
38. (*a*) 39. (*c*) 40. (*d*)

5.22 Inverse trigonometric functions

1. $\sin^{-1}(-1)$ is

(*a*) $\pi/2$ (*b*) 0 (*c*) $-\pi/2$ (*d*) none

2. $\cos^{-1}(-1) - \cos^{-1} 0 =$

(*a*) $3\pi/2$ (*b*) $\pi/2$ (*c*) $-\pi/2$ (*d*) none

3. $\sec^{-1} 2 + \operatorname{cosec}^{-1}(-\sqrt{2}) =$

(*a*) $\frac{\pi}{12}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{-\pi}{12}$ (*d*) none

4. $2\tan^{-1}(-1) + \cot^{-1}(0) =$

(*a*) 0 (*b*) π (*c*) $-\pi$ (*d*) $-\pi/2$

5. $\cos^{-1}\{\cos(-\pi/3)\} =$

(*a*) $-\pi/3$ (*b*) $\pi/3$ (*c*) $\frac{2\pi}{3}$ (*d*) none

6. $\tan^{-1}(\tan 700°) =$

(*a*) 20° (*b*) −20° (*c*) 160° (*d*) none

7. $\sin^{-1}\left(\tan\frac{3\pi}{4}\right) =$

(*a*) $\pi/2$ (*b*) π (*c*) $-\pi/2$ (*d*) none

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

8. $\cos^{-1}\left[\cot\left(-\frac{5\pi}{4}\right)\right] =$

(*a*) 0 (*b*) $\pi/2$ (*c*) $-\pi/2$ (*d*) none

9. $\cos(2\sin^{-1} 1) =$

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) not defined

10. $\sin(\tan^{-1} \frac{3}{5}) =$

(*a*) 4/5 (*b*) 5/4 (*c*) $3/\sqrt{34}$ (*d*) $5/\sqrt{34}$

11. $\cos(\text{cosec}^{-1} \frac{5}{3}) =$

(*a*) 4/5 (*b*) 3/5 (*c*) $\frac{3}{4}$ (*d*) none

12. $\tan[\sin^{-1}(-3/4)] =$

(*a*) $3/\sqrt{7}$ (*b*) $-3\sqrt{7}/7$ (*c*) $-2/\sqrt{7}$ (*d*) none

13. $\sin[\cos^{-1}(-2/3)] =$

(*a*) $\sqrt{3}/5$ (*b*) $\sqrt{5}/3$ (*c*) $-\sqrt{3}/5$ (*d*) $-\sqrt{5}/3$

14. $\sin[\sin^{-1} \frac{12}{13} - \cos^{-1} \frac{3}{5}] =$

(*a*) $\frac{56}{65}$ (*b*) $\frac{63}{65}$ (*c*) $\frac{16}{65}$ (*d*) none

15. $\cos^{-1}(\sin 220°)$

(*a*) 130° (*b*) 50° (*c*) 40° (*d*) none

16. $\cot(\cos^{-1} \frac{7}{25}) =$

(*a*) 24/25 (*b*) 25/24 (*c*) 25/7 (*d*) none

17. $\cos\left(2\sin^{-1}\frac{1}{\sqrt{5}}\right) =$

(*a*) –3/5 (*b*) 1 (*c*) $3/\sqrt{5}$ (*d*) 3/5

18. If $\tan^{-1}\left(\frac{8x}{1-15x^2}\right) = \tan^{-1} a + \tan^{-1} b$, then $(a, b) =$

(*a*) $(6x, 2x)$ (*b*) $(5x, 3x)$ (*c*) (5, 3) (*d*) none

19. $\sin^{-1} \cos(-105°) =$

(*a*) 15° (*b*) –15° (*c*) 165° (*d*) none

20. $\sec[\text{cosc}^{-1}(-\frac{13}{12})] =$

(*a*) $-\frac{13}{5}$ (*b*) $\frac{13}{5}$ (*c*) $\frac{12}{5}$ (*d*) none

21. $2\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} =$

(*a*) $\pi/2$ (*b*) $\tan^{-1} \frac{3}{4}$ (*c*) $\tan^{-1} \frac{20}{21}$ (*d*) $\frac{\pi}{4}$

22. $\cos^{-1} \frac{3}{5} + \cot^{-1} \frac{4}{3} =$

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{2}$ (*c*) $\cot^{-1} 2$ (*d*) none

23. $\sin(\tan^{-1} x)$ when $x < 0$

(*a*) $-x/\sqrt{1+x^2}$ (*b*) $1/\sqrt{1+x^2}$ (*c*) $\sqrt{1+x^2}$ (*d*) $x/\sqrt{1+x^2}$

24. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=$
(*a*) $\pi/2$ (*b*) $\pi/4$ (*c*) $\tan^{-1}\frac{1}{6}$ (*d*) $\tan^{-1}\frac{5}{6}$

25. $\sin\left(2\cos^{-1}\frac{\sqrt{5}}{3}\right)$ is equal to
(*a*) $4\sqrt{5}/9$ (*b*) $4\sqrt{5}/3$ (*c*) $4\sqrt{5}$ (*d*) $12/\sqrt{5}$

26. $\cos\left(-2\sin^{-1}\frac{3}{5}\right)=$
(*a*) $\frac{7}{25}$ (*b*) $\frac{24}{25}$ (*c*) $\frac{18}{25}$ (*d*) $-\frac{7}{25}$

27. If $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}x=\frac{\pi}{4}$, then $x=$
(*a*) $\frac{1}{7}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{7}{9}$

28. A value of x satisfying $\sin^{-1}x=2\tan^{-1}x$ is
(*a*) $1/\sqrt{2}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) $\sqrt{3}/2$

29. Which of the following doesn't exist?
(*a*) $\tan^{-1}(-180)$ (*b*) $\sin^{-1}(\sqrt{2}/\sqrt{3})$
(*c*) $\sec^{-1}\sqrt{2}$ (*d*) $\cos^{-1}(-2/\sqrt{2})$

30. $\cos^{-1}(-\cos 3\pi/4)=$
(*a*) $\frac{3\pi}{4}$ (*b*) $-\frac{\pi}{4}$ (*c*) $\frac{\pi}{4}$ (*d*) none

31. If $\cos^{-1}(-\frac{1}{2})=\sin^{-1}x$, then $x=$
(*a*) $-\sqrt{3}/2$ (*b*) $\sqrt{3}/2$ (*c*) $\frac{1}{2}$ (*d*) doesn't exist

32. $\sin^{-1}x+\cos^{-1}x=\pi/2$ is true for
(*a*) all x (*b*) $x>0$ (*c*) $x\neq 0$ (*d*) none

33. $\sin(\sin^{-1}x)=x$ is true for
(*a*) all x (*b*) $-1\leqslant x\leqslant 1$ (*c*) $-\pi/2\leqslant x\leqslant \pi/2$ (*d*) none

34. $\tan(\tan^{-1}x)=x$ is true for
(*a*) all x (*b*) $-\pi/2<x<\pi/2$ (*c*) $x\geqslant 0$ (*d*) none

35. If $2\cos^{-1}\frac{4}{5}=\tan^{-1}x$, then $x=$
(*a*) $\frac{3}{4}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{4}{27}$ (*d*) none

36. If $\cot^{-1}\frac{8}{15}+\cos^{-1}\frac{15}{17}=\cot^{-1}x$, then $x=$
(*a*) 0 (*b*) 1 (*c*) infinity (*d*) none

37. $\tan^{-1}\frac{1}{3}+\cot^{-1}4=$
(*a*) $\tan^{-1}\frac{7}{11}$ (*b*) $\tan^{-1}\frac{11}{7}$ (*c*) $\tan^{-1}\frac{11}{2}$ (*d*) $\tan^{-1}\frac{8}{11}$

38. $\cos\left(2\sin^{-1}\frac{1}{2}\right)=$
(*a*) $\sqrt{3}/2$ (*b*) $1/\sqrt{3}$ (*c*) $1/\sqrt{2}$ (*d*) $\frac{1}{2}$

39. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)=$

(*a*) $-\dfrac{1}{\sqrt{10}}$ (*b*) $\dfrac{1}{\sqrt{10}}$ (*c*) $\dfrac{1}{\sqrt{5}}$ (*d*) none

40. $\cos^{-1}\left(\sin\dfrac{3\pi}{7}\right)=$

(*a*) $\pi/14$ (*b*) $2\pi/7$ (*c*) $\pi/7$ (*d*) none

41. $\tan^2(\sin^{-1}x)=$

(*a*) $\dfrac{x^2}{\sqrt{1-x^2}}$ (*b*) $\dfrac{x^2}{1+x^2}$ (*c*) $\dfrac{x^2}{x^2-1}$ (*d*) $\dfrac{x^2}{1-x^2}$

42. If $\sin^{-1}\dfrac{1}{\sqrt{5}}+\sin^{-1}\dfrac{2}{\sqrt{5}}=\cot^{-1}x$, then x is

(*a*) infinity (*b*) 0 (*c*) $3/\sqrt{5}$ (*d*) $1/5$

43. The solution set of the equation $\tan^{-1}x+\tan^{-1}(1-x)=\pi/4$ is

(*a*) $\{0, 1\}$ (*b*) $\{0, -1\}$ (*c*) $\{1\}$ (*d*) none

44. If $\sin^{-1}x-\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{29}$, then $x=$

(*a*) $4/5$ (*b*) $3/5$ (*c*) $3/4$ (*d*) none

45. If $\sin^{-1}4/5+\cos^{-1}2/\sqrt{5}=\tan^{-1}\alpha$, then $\alpha=$

(*a*) $11/2$ (*b*) $11/4$ (*c*) $11/3$ (*d*) none

46. Which one of the following is true?

(*a*) $\tan^{-1}(-3/4)=\cos^{-1}(-3/5)$ (*b*) $\sin^{-1}(-4/5)=\cos^{-1}(3/5)$

(*c*) $\cos^{-1}(-3/5)=\pi-\sin^{-1}3/5$ (*d*) none

47. $\cos^{-1}\cos 350^\circ-\sin^{-1}\sin 350^\circ=$

(*a*) 0° (*b*) -20° (*c*) 110° (*d*) none

48. $\sin^{-1}x$ is equal to

(*a*) $\dfrac{\pi}{2}+\cos^{-1}x$ (*b*) $\cos^{-1}\sqrt{1-x^2}$ (*c*) $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$ (*d*) none

49. $\cot^{-1}x-\cot^{-1}y$, when $x, y \geqslant 0$, is equal to

(*a*) $\tan^{-1}\dfrac{x-y}{1+xy}$ (*b*) $\cot^{-1}\left(\dfrac{y-x}{1+xy}\right)$

(*c*) $\cot^{-1}\left(\dfrac{x-y}{1+xy}\right)$ (*d*) $\tan^{-1}\left(\dfrac{y-x}{1+xy}\right)$

50. $\tan\left[\sin^{-1}(-1/2)-\cos^{-1}\sqrt{3}/2\right]$ is equal to

(*a*) $\sqrt{3}$ (*b*) $-\sqrt{3}$ (*c*) $1/\sqrt{3}$ (*d*) 0

51. The solution of $\cos^{-1} x+\tan^{-1} x=\pi/2$ is

(*a*) 0 (*b*) 0, 2/3 (*c*) 0, 1/3 (*d*) none

52. The values of x satisfying $\cos^{-1}(2x^2-1)=2\cos^{-1} 1/2$ are

(*a*) $1/2, \sqrt{3}/2$ (*b*) $\pm 1/2, \pm\sqrt{3}/2$ (*c*) $\pm 1/2$ (*d*) none

53. The solution of $\cos^{-1} 2x=\sin^{-1} x$ is

(*a*) $\frac{1}{\sqrt{5}}$ (*b*) $\sqrt{5}/4$ (*c*) $2/\sqrt{5}$ (*d*) none

54. The values of x satisfying $\tan^{-1} x+2\cot^{-1} x=2\pi/3$ is

(*a*) $\pm\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}, \sqrt{3}$ (*c*) $\sqrt{3}$ (*d*) none

55. If $x+\frac{1}{x}=2$, then $\tan^{-1} x=$

(*a*) $\pm\pi/4$ (*b*) $\pi/4$ (*c*) $-\pi/4$ (*d*) none

56. The principal value $\sin^{-1} x$ where $x-1/x=0$ is

(*a*) $\pi/2$ (*b*) $\pi/2; 3\pi/2$ (*c*) $\pm\pi/2$ (*d*) none

57. $\operatorname{cosec}^{-1} 5/3+\cot^{-1} 7=$

(*a*) $\pi/2$ (*b*) $3\pi/4$ (*c*) π (*d*) $\pi/4$

Answers

1. (*c*) 2. (*b*) 3. (*a*) 4. (*a*) 5. (*b*) 6. (*b*) 7. (*c*) 8. (*d*) 9. (*b*) 10. (*c*) 11. (*a*) 12. (*b*) 13. (*b*) 14. (*c*) 15. (*a*) 16. (*d*) 17. (*d*) 18. (*b*) 19. (*b*) 20. (*b*) 21. (*d*) 22. (*b*) 23. (*d*) 24. (*b*) 25. (*a*) 26. (*a*) 27. (*c*) 28. (*c*) 29. (*d*) 30. (*c*) 31. (*d*) 32. (*d*) 33. (*b*) 34. (*a*) 35. (*d*) 36. (*a*) 37. (*a*) 38. (*d*) 39. (*b*) 40. (*a*) 41. (*d*) 42. (*b*) 43. (*a*) 44. (*b*) 45. (*a*) 46. (*c*) 47. (*d*) 48. (*c*) 49. (*d*) 50. (*b*) 51. (*a*) 52. (*c*) 53. (*a*) 54. (*c*) 55. (*b*) 56. (*c*) 57. (*d*)

5.23 General solutions of trigonometric equations

1. The general solution of $2\cos x+1=0$ is

(*a*) $2n\pi\pm\pi/3$ (*b*) $2n\pi\pm 2\pi/3$ (*c*) $2n\pi\pm\pi/6$ (*d*) none

2. The general solution of $2\cos\frac{x}{2}=\sqrt{3}$ is

(*a*) $2n\pi\pm\pi/6$ (*b*) $4n\pi\pm\pi/3$ (*c*) $4n\pi\pm 2\pi/3$ (*d*) none

3. The solution of $\cot\frac{x}{3}=-1/\sqrt{3}$ such that $0\leq x<2\pi$ is

(*a*) $\pi/3$ (*b*) $2\pi/3$ (*c*) no solution (*d*) none

4. **The number of solutions of the equation $(\tan x-1)(4\sin^2 x-3)=0$ such that $0\leqslant x\leqslant 2\pi$ is**
(*a*) 4 (*b*) 6 (*c*) 5 (*d*) 3

5. **The solution of $2\sin x-\operatorname{cosec} x=1$ such that $0\leqslant x\leqslant 2\pi$ is**
(*a*) $\pi/2, 7\pi/6$ (*b*) $\pi/2, \pi/6, 7\pi/6$
(*c*) $\pi/2, 7\pi/6, 11\pi/6$ (*d*) none

6. **The general solution of $\tan x+\tan 2x+\sqrt{3}\tan x\tan 2x=\sqrt{3}$ is**
(*a*) $n\pi/3+\pi/2$ (*b*) $n\pi/3+\pi/6$ (*c*) $n\pi/3+\pi/3$ (*d*) $n\pi/3+\pi/9$

7. **The general solution of $\sin 2x+\cos x=0$ is**
(*a*) $2n\pi\pm\pi/2;\ n\pi+(-1)^n\pi/6$ (*b*) $n\pi+(-1)^n\pi/2;\ n\pi\pm\pi/2$
(*c*) $2n\pi\pm\pi/2;\ n\pi-(-1)^n\pi/6$ (*d*) none

8. **The general solution of $2\tan x\sin x=\tan x$ is**
(*a*) $n\pi\pm(-1)^n\pi/6$ (*b*) $n\pi;\ n\pi\pm\pi/6$
(*c*) $n\pi;\ n\pi+(-1)^n\pi/6$ (*d*) none

9. **The solution of $\sin x+1=\cos x$ such that $0\leqslant x\leqslant 2\pi$ is**
(*a*) $0, \pi$ (*b*) $0, \pi/2$ (*c*) $3\pi/2$ (*d*) none

10. **The general solution of $\sin 3x+\sin x=0$ is**
(*a*) $n\pi/2$ (*b*) $n\pi$ (*c*) $2n\pi$ (*d*) none

11. **The general solution of $\tan x-\tan(\pi/2+x)=2$ is**
(*a*) $n\pi+\pi/4$ (*b*) $n\pi-\pi/4$ (*c*) $n\pi\pm\pi/4$ (*d*) $2n\pi+\pi/4$

12. **The general solution of $\cos 5x+\sin 3x=0$ is**
(*a*) $\frac{1}{2}(n\pi+\pi/2);\ \frac{1}{4}(2n\pi-\pi/2)$ (*b*) $n\pi+\pi/4;\ n\pi/2-\pi/8$
(*c*) $\frac{1}{2}(2n\pi+\pi/2);\ \frac{1}{8}(2n\pi-\pi/2)$ (*d*) none

13. **The general solution of $\cos\theta-\cos 7\theta=\sin 4\theta$ is**
(*a*) $n\pi/4;\ n\pi/3+(-1)^n\pi/12$ (*b*) $n\pi/2;\ n\pi/3+(-1)^n\pi/18$
(*c*) $n\pi/4;\ n\pi/3+(-1)^n\pi/18$ (*d*) none

14. **The general solution of $\tan 4x\tan 6x=1$ is**
(*d*) $(2n\pm1)\pi/20$ (*b*) $(4n\pm1)\pi/20$ (*c*) $(2n\pm1)\pi/4$ (*d*) none

15. **The common general solution of the equations $\tan\theta=-1/\sqrt{3}$ and $\sin\theta=-\frac{1}{2}$ is**
(*a*) $2n\pi\pm\pi/6$ (*b*) $n\pi+\pi/6$ (*c*) $n\pi-\pi/6$ (*d*) $2n\pi-\pi/6$

16. **The general solution of $\sqrt{3}\cos x+\sin x=3$ is**
(*a*) $2n\pi\pm\pi/6$ (*b*) $2n\pi\pm\pi/3$ (*c*) no solution (*d*) none

17. **The general solution of $\sin 5x-\sin 3x=0$ is**
(*a*) $n\pi/2;\ 2n\pi\pm\pi/2$ (*b*) $n\pi/4:\ 2n\pi\pm\pi/4$
(*c*) $n\pi/4;\ 2n\pi\pm\pi/2$ (*d*) none

18. The general solution of $4\sin^2 x=1$ is
(a) $n\pi+(-1)^n\pi/6$ (b) $n\pi-(-1)^n\pi/6$ (c) $n\pi\pm\pi/6$ (d) none

19. The general solution of $\sin x+\cos x=\sqrt{2}$ is
(a) $2n\pi+\pi/4$ (b) $n\pi+(-1)^n\pi/4$ (c) $2n\pi\pm\pi/4$ (d) none

20. The general solution of $2\sec x=\tan x+\cot x$ is
(a) $n\pi+\pi/6$ (b) $2n\pi\pm\pi/6$ (c) $n\pi+(-1)^n\pi/6$ (d) none

21. The general solution of $\cos 2x+\cos x+1=0$ is
(a) $2n\pi\pm\pi/2;\ 2n\pi\pm2\pi/3$ (b) $n\pi;\ n\pi+(-1)^n\pi/3$
(c) $2n\pi\pm\pi/2;\ 2n\pi\pm5\pi/6$ (d) none

22. The number of solutions between 0 and 2π of the equation $\cos^2\frac{x}{2}=\frac{1}{2}\cos^2 x$ is
(a) 0 (b) 2 (c) 1 (d) none

23. The general solution of $\sin 2x=\cos 2x$ is
[✓ (a) $n\pi+\pi/4$] (b) $2n\pi+\pi/4$ (c) $n\pi/2+\pi/4$ (d) $n\pi/2+\pi/8$

24. The general solution of $\tan 4x=\cot 5x$ is
(a) $n\pi+\frac{\pi}{10}$ (b) $\frac{n\pi}{9}+\frac{\pi}{2}$ [✓ (c) $\frac{n\pi}{9}+\frac{\pi}{18}$] (d) none

25. The general solution of $\sec^2 x=2(1-\tan^2 x)$ is
[✓ (a) $n\pi+\pi/6$] (b) $2n\pi\pm\pi/6$ [✓ (c) $n\pi\pm\pi/6$] (d) none

26. The general solution of $\sin^2 x+\sin x-2=0$ is
[✓ (a) $n\pi+(-1)^n\pi/2$] (b) $2n\pi\pm\pi/2$ (c) $n\pi+\pi/2$ (d) none

27. The general solution of $\sin\theta+\sin 5\theta=\sin 3\theta$ is
(a) $n\pi/3;\ n\pi+\pi/6$ (b) $n\pi\pm\pi/6$
(c) $n\pi/3;\ n\pi\pm\pi/6$ (d) none

28. The general solution of $\cos 2\theta=\cos^2\theta$ is
(a) $n\pi/2$ (b) $2n\pi$ (c) $n\pi$ (d) none

29. The general solution of $\tan^2 x+\cot^2 x=2$ is
(a) $n\pi/4$ [✓ (b) $n\pi\pm\pi/4$] (c) $2n\pi\pm\pi/4$ (d) none

30. The general solution of $\cos(3x+2)=\frac{1}{2}$ is
(a) $2n\pi\pm\frac{\pi-6}{3}$ (b) $2n\pi\pm\frac{\pi-6}{9}$ (c) no solution [✓ (d) none]

31. The general solution of $\cot(2x+\pi/6)=1$ is
(a) $n\pi/2\pm\pi/12$ (b) $n\pi/2+\pi/12$
(c) $n\pi/2\pm\pi/24$ [✓ (d) $n\pi/2+\pi/24$]

32. The general solution of $\sec^3 x=-1$ is
[✓ (a) $2n\pi\pm\pi$] (b) $n\pi\pm\pi/2$ (c) $2n\pi$ (d) none

33. The general solution of $\operatorname{cosec}^2 x = 2$ is
(*a*) $n\pi \pm \pi/4$ (*b*) $n\pi + (-1)^n (-\pi/4)$ (*c*) no solution (*d*) none

34. The general solution of $\sec x + \tan x = 0$ is
(*a*) $n\pi + (-1)^n \pi/2$ (*b*) $n\pi$ (*c*) no solution (*d*) none

35. The general solution of $\sec x - 1 = \tan x$ is
(*a*) $n\pi$ (*b*) $n\pi/2$ (*c*) $2n\pi$ (*d*) none

Answers

1. (*b*) 2. (*b*) 3. (*c*) 4. (*b*) 5. (*c*) 6. (*d*) 7. (*c*) 8. (*c*) 9. (*d*) 10. (*a*) 11. (*a*) 12. (*c*) 13. (*c*) 14. (*b*) 15. (*d*) 16. (*c*) 17. (*c*) 18. (*c*) 19. (*a*) 20. (*c*) 21. (*a*) 22. (*a*) 23. (*d*) 24. (*c*) 25. (*c*) 26. (*a*) 27. (*c*) 28. (*c*) 29. (*b*) 30. (*d*) 31. (*d*) 32. (*a*) 33. (*a*) 34. (*c*) 35. (*c*)

5.24 Complex numbers

1. The conjugate of $\dfrac{2+3i}{1+2i}$ is
(*a*) $\dfrac{8+i}{5}$ (*b*) $\dfrac{8-i}{5}$ (*c*) $\dfrac{2+3i}{5}$ (*d*) $\dfrac{2-3i}{5}$

2. The multiplicative inverse of $\dfrac{1+i}{1-i}$ is
(*a*) $1+i$ (*b*) $1-i$ (*c*) i (*d*) $-i$

3. The imaginary part of the conjugate of $4-i$ is
(*a*) i (*b*) $-i$ (*c*) 1 (*d*) -1

4. If $4i-6=(a+ib)(1-i)$, then (a, b) is
(*a*) (5, 1) (*b*) (5, −1) (*c*) (−5, 1) (*d*) (−5, −1)

5. The amplitude of $12\sqrt{3}+12i =$
(*a*) $\pi/3$ (*b*) $\pi/6$ (*c*) $2\pi/3$ (*d*) none

6. The amplitude of $-81 =$
(*a*) 0 (*b*) π (*c*) $-\pi$ (*d*) none

7. The amplitude of $-3i =$
(*a*) $\pi/2$ (*b*) $3\pi/2$ (*c*) $-\pi/2$ (*d*) none

8. The amplitude of $-4-3i$
(*a*) $\tan^{-1}\frac{3}{4}$ (*b*) $-\tan^{-1}\frac{3}{4}$ (*c*) $\tan^{-1}\frac{3}{4} - \pi$ (*d*) $-\pi - \tan^{-1}\frac{3}{4}$

9. The polar form of the complex number $\dfrac{4(\cos 230^\circ + i \sin 230^\circ)}{2(\cos 75^\circ + i \sin 75^\circ)} =$
(*a*) $\cos 150^\circ + i \sin 150^\circ$ (*b*) $2(\cos 150^\circ - i \sin 150^\circ)$
(*c*) $2(\cos 120^\circ + i \sin 120^\circ)$ (*d*) none

10. The modulus of the complex number $(2+i)^2\,(3-2i)^2=$
(*a*) 63 (*b*) 65 (*c*) 75 (*d*) none

11. The modulus and amplitude of the complex number $(2+i)^2-4(2+i)+5$ are
(*a*) 0; 0 (*b*) 3; 0 (*c*) 0; $\pi/2$ (*d*) none

12. The modulus and amplitude of $\left(\dfrac{1-i}{1+i}\right)^{100}$ is
(*a*) $1, -50\pi$ (*b*) $1, -100\pi$ (*c*) $1, 0$ (*d*) none

13. The amplitude of $\sin\alpha + i\,(1+\cos\alpha)$ where $0<\alpha<\pi$ is
(*a*) $\alpha/2$ (*b*) $-\alpha/2$ (*c*) $\pi/2+\alpha/2$ (*d*) $\pi/2-\alpha/2$

14. The modulus of $1+\cos\theta+i\sin\theta$ is
(*a*) $\left|2\cos\dfrac{\theta}{2}\right|$ (*b*) $\sin\dfrac{\theta}{2}$ (*c*) $2\sin\dfrac{\theta}{2}$ (*d*) $2\cos\dfrac{\theta}{2}$

15. The amplitude of the complex number $1+\cos\dfrac{\pi}{5}+i\sin\dfrac{\pi}{5}$ is
(*a*) $\pi/5$ (*b*) $\pi/10$ (*c*) $\pi/20$ (*d*) none

16. $\left(\cos\dfrac{5\pi}{3}-i\sin\dfrac{5\pi}{3}\right)^{21}$ is equal to
(*a*) 1 (*b*) -1 (*c*) i (*d*) $-i$

17. If $2\cos\alpha=x+\dfrac{1}{x}$ and $2\cos\beta=y+\dfrac{1}{y}$, then $2\cos(\alpha+\beta)$ is
(*a*) $\left(x+\dfrac{1}{x}\right)\left(y+\dfrac{1}{y}\right)$ (*b*) $xy-\dfrac{1}{xy}$ (*c*) $\dfrac{1}{xy}$ (*d*) $xy+\dfrac{1}{xy}$

18. The amplitude of $\dfrac{1+i\sqrt{3}}{\sqrt{3}+i}=$ ✓
(*a*) $\pi/3$ (*b*) $\pi/6$ (*c*) $\pi/4$ (*d*) none

19. $2\,(\cos 210^\circ+i\sin 210^\circ)=$
(*a*) $\sqrt{3}+i$ (*b*) $\sqrt{3}-i$ (*c*) $-\sqrt{3}-i$ (*d*) $-\sqrt{3}+i$

20. If $z_1=12\,(\cos 300^\circ+i\sin 300^\circ)$ and $z_2=6\,[\cos(-150^\circ)+i\sin(-150^\circ)]$, then $z_1/z_2=$
(*a*) $-2i$ (*b*) $2i$ (*c*) 2 (*d*) none

21. The point in the complex plane corresponding to $2-2i$ lies in the
(*a*) first quadrant (*b*) second quadrant
(*c*) third quadrant (*d*) fourth quadrant

22. The amplitude of $(-1+i\sqrt{3})(\sqrt{3}+i)$ is

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{2\pi}{3}$ (*c*) $\frac{5\pi}{6}$ (*d*) none

23. The amplitude of $(1+i\sqrt{3})^2$ is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{2\pi}{3}$ (*c*) $\frac{5\pi}{3}$ (*d*) none

24. The amplitude of $\frac{1-i}{1+i}$ is

(*a*) $\pi/2$ (*b*) π (*c*) $-\pi$ (*d*) $-\pi/2$

25. The modulus of $\frac{3-2i}{2+3i}$ is

(*a*) 0 (*b*) 1 (*c*) $\sqrt{13}$ (*d*) 13

26. The amplitude of $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^9$ is

(*a*) $\pi/2$ (*b*) $-\pi/2$ (*c*) $9\pi/2$ (*d*) none

27. The modulus of $\frac{(1+i)(\sqrt{3}+i)^3}{(1-i\sqrt{3})^4}$ is

(*a*) $\sqrt{2}$ (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{2}}$

28. If $z^4=i$, then one of the values of z is

(*a*) i (*b*) $\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$ (*c*) $\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$ (*d*) none

29. $(\sqrt{3}-i)^5=$

(*a*) $16\sqrt{3}+16i$ (*b*) $-16\sqrt{3}-16i$ (*c*) $-16\sqrt{3}+16i$ (*d*) none

30. $(\sin\theta+i\cos\theta)^{17}=$

(*a*) $\sin 17\theta+i\cos 17\theta$ (*b*) $\cos 17\theta+i\sin 17\theta$

(*c*) $\sin 17\theta-i\cos 17\theta$ (*d*) $\cos 17\theta-i\sin 17\theta$

*

31. $(\cos\theta+i\sin\theta)^4(\sin\theta-i\cos\theta)^5=$

(*a*) $\sin 9\theta-i\cos 9\theta$ (*b*) $\sin\theta+i\cos\theta$

(*c*) $\cos 9\theta+i\sin 9\theta$ (*d*) $\cos\theta-i\sin\theta$

32. Which of the following is a cube root of $\cos\frac{\pi}{15}+i\sin\frac{\pi}{15}$?

(*a*) $\cos\frac{\pi}{45}-i\sin\frac{\pi}{45}$ (*b*) $\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}$

(*c*) $\cos\frac{\pi}{15}-i\sin\frac{\pi}{15}$ (*d*) none

33. **If ω is an imaginary cube root of unity, then $(1+\omega)^3-(1+\omega^2)^3$ is**
(*a*) 2 (*b*) −2 (*c*) 0 (*d*) none

34. **The amplitude and modulus of $(1+i)^4$ are respectively**
(*a*) $-\pi, 4$ (*b*) $\pi, 2$ (*c*) $\pi, 4$ (*d*) $\pi/2, 4$

35. **The amplitude of one of the values of $(-\sqrt{3}+i)^{\frac{1}{5}}$ is**
(*a*) 25° (*b*) 50° (*c*) 15° (*d*) none 30°

36. **If $z^6=-1$, then one of the values of z is**
(*a*) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (*b*) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$
(*c*) $\cos 2\pi+i\sin 2\pi$ (*d*) none

37. **One of the fourth roots of $-16i$ is**
(*a*) $-2\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)$ (*b*) $-2\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$
(*c*) $2\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)$ (*d*) none

38. **If a complex number has its modulus 1, then it lies in the Argand diagram on**
(*a*) the real axis (*b*) the imaginary axis
(*c*) the unit circle (*d*) none

39. **The sum of the fourth roots of unity is**
(*a*) 4 (*b*) −4 (*c*) i (*d*) 0

40. **The amplitude of the conjugate of the complex number $(\sqrt{3}-\sqrt{3}i)\ i$ is**
(*a*) $-\frac{\pi}{4}$ (*b*) $\frac{3\pi}{4}$ (*c*) $\frac{\pi}{4}$ (*d*) none

41. **$(1+i)^8+(1-i)^8=$**
(*a*) $2^8\cos\pi/8$ (*b*) 2^8 (*c*) 2^5 (*d*) 2^4

42. **$\dfrac{(\cos\theta+i\sin\theta)^7}{(\cos\theta-i\sin\theta)^4}=$**
(*a*) $\cos 3\theta+i\sin 3\theta$ (*b*) $\cos 3\theta-i\sin 3\theta$
(*c*) $\cos 11\theta-i\sin 11\theta$ (*d*) none

43. **If $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$, then $z^{20}=$**
(*a*) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (*b*) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (*c*) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (*d*) none

44. $\left|\frac{i(2+i)\ (4-3i)}{1+2i}\right| =$

(*a*) 25 (*b*) 5 (*c*) 125 (*d*) none

45. The argument of $\frac{\sqrt{3}+i}{\sqrt{3}-i}$ is

(*a*) 0 (*b*) $\frac{\pi}{6}$ (*c*) $\frac{\pi}{3}$ (*d*) none

46. The real part of $(1+\cos\theta+i\sin\theta)^{-1}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{2}\sec(\theta/2)$ (*c*) $\frac{1}{2}\cos(\theta/2)$ (*d*) none

47. The amplitude of $(1+i)^3\,(1-i)^4$ is

(*a*) $\pi/4$ (*b*) $-\pi/4$ (*c*) $3\pi/4$ (*d*) none

48. The amplitude of $\left(\frac{1-i^3}{1+i^3}\right)^2$ is

(*a*) 0 (*b*) π (*c*) $\pi/2$ (*d*) none

49. If ω is a cube root of 1, then

(*a*) $1+\omega+\omega^2=0$ (*b*) $\omega^2=\frac{1}{\omega}$ (*c*) $\omega=1$ (*d*) all of them

50. The amplitude of $(-i)^5$ is

(*a*) $5\pi/2$ (*b*) $-5\pi/2$ (*c*) $\pi/2$ (*d*) $-\pi/2$

51. The cube roots of i are $(n=0, 1, 2)$

(*a*) $\cos\frac{2n+1}{6}\pi+i\sin\frac{2n+1}{6}\pi$ (*b*) $\cos\frac{4n+1}{6}\pi+i\sin\frac{4n+1}{6}\pi$

(*c*) $\cos\frac{4n+1}{3}\pi+i\sin\frac{4n+1}{3}\pi$ (*d*) none

52. One of the cube roots of $\frac{1+2i}{1-(1-i)^2}$ is

(*a*) -1 (*b*) 1 (*c*) i (*d*) $-i$

*53. If $(1+i)^n+(1-i)^n=x+iy$, then $y=$ — Do all the steps

(*a*) $\cos\frac{n\pi}{4}$ (*b*) $2^{n+1}\cos\frac{n\pi}{4}$ (*c*) $2^{n/2+1}\cos\frac{n\pi}{4}$ (*d*) 0

54. If $x+\frac{1}{x}=2\cos 4\theta$, then $x^2-\frac{1}{x^2}$ is equal to

(*a*) $\pm 2i\cos 4\theta$ (*b*) $\pm 2i\sin 4\theta$ (*c*) $\pm 2i\sin 8\theta$ (*d*) $2i\sin 8\theta$

55. The value of $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ for any integer n is

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) i

56. If $\frac{2}{i}+\frac{4}{i^2}+\frac{2}{i^3}+\frac{4}{i^4}=x+iy$, then $(x, y)=$

(*a*) $(1, 1)$ (*b*) $(-1, 1)$ (*c*) $(1, 0)$ (*d*) $(0, 0)$

57. $(\sin\theta+i\cos\theta)^n=\sin n\theta+i\cos n\theta$ holds when

(*a*) n is odd (*b*) n is even (*c*) $n\equiv 0 \pmod 4$ (*d*) $n\equiv 1 \pmod 4$

58. If ω is an imaginary cube root of unity, then the roots of -8 are

(*a*) $-2, -2\omega, 2\omega^2$ (*b*) $2, -2\omega, 2\omega^2$ (*c*) $-2, -2\omega, -2\omega^2$ (*d*) none

59. If ω is an imaginary cube root of unity, then $[(1+\omega)(1+\omega^2)]^5$ is

(*a*) ω (*b*) 1 (*c*) ω^2 (*d*) 0

60. The modulus of the complex number $1+\cos\theta+i\sin\theta$, $2\pi<\theta<3\pi$, is

(*a*) $-2\cos\frac{\theta}{2}$ (*b*) $\sqrt{2}\cos\frac{\theta}{2}$ (*c*) $2\cos\frac{\theta}{2}$ (*d*) none

61. If $(1+i)^n-(1-i)^n=0$, then n is a multiple of

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

62. The modulus of the complex number $\frac{(2+i)^3(i-1)^3}{(i-2)^2(1-i)^3}$ is

(*a*) $\sqrt{5}$ (*b*) $\sqrt{45}$ (*c*) $\sqrt{125}$ (*d*) none

63. If z_1, z_2 and z_3 are the three cube roots of the complex number $\cos 2\pi+i\sin 2\pi$, then $z_1 z_2 z_3=$

(*a*) $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$ (*b*) $\cos 0+i\sin 0$

(*c*) $\cos\pi+i\sin\pi$ (*d*) none

64. The modulus of the complex number $\frac{2+3i}{2-3i}+\frac{2-3i}{2+3i}$ is

(*a*) $\frac{10}{13}$ (*b*) $\sqrt{\frac{10}{13}}$ (*c*) $\frac{100}{169}$ (*d*) none

65. If 1, ω_1 and ω_2 are the cube roots of unity, $\omega_1\omega_2^2+\omega_2\omega_1^2$ is

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) none

66. If P is the point corresponding to $2+2i$ in the complex plane and OP is rotated about 0 in the anti-clockwise direction through an angle $\frac{\pi}{2}$, then the new position of P is in

(*a*) I quadrant (*b*) II quadrant (*c*) III quadrant (*d*) IV quadrant

67. If $x=\sin\theta+i\cos\theta$, then $x^5+\dfrac{1}{x^5}$ is

(*a*) $2\sin 5\theta$ (*b*) $2\cos 5\theta$ (*c*) $2i\sin 5\theta$ (*d*) $2i\cos 5\theta$

68. If $a+ib=\dfrac{(1-i)\,(2-i)}{3-i}$, then a^2+b^2 is

(*a*) 1 (*b*) $\frac{7}{25}$ (*c*) 25 (*d*) none

69. The smallest positive integer for which $\left(\dfrac{1-i}{1+i}\right)^n=i$ is

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) none

70. If ω is an imaginary cube root of unity, then
$(1+\omega)\,(1+\omega^2)\,(1+\omega^3)\,(1+\omega^4)\,(1+\omega^5)=$

(*a*) 1 (*b*) 2 (*c*) 2ω (*d*) none

71. $$\left(\frac{1+\sin\frac{\pi}{4}+i\cos\frac{\pi}{4}}{1+\sin\frac{\pi}{4}-i\cos\frac{\pi}{4}}\right)^4=$$

(*a*) 1 (*b*) -1 (*c*) i (*d*) $-i$

Answers

1. (*b*) 2. (*d*) 3. (*c*) 4. (*d*) 5. (*b*) 6. (*b*) 7. (*c*) 8. (*c*) 9 (*d*) 10 (*b*)
11. (*a*) 12 (*c*) 13. (*d*) 14. (*a*) 15. (*b*) 16. (*b*) 17. (*d*) 18. (*b*) 19. (*c*)
20. (*b*) 21. (*d*) 22. (*c*) 23. (*b*) 24. (*d*) 25. (*b*) 26. (*b*) 27. (*d*) 28. (*c*)
29. (*b*) 30. (*a*) 31. (*a*) 32. (*d*) 33. (*c*) 34. (*c*) 35. (*d*) 36. (*a*) 37. (*c*)
38. (*c*) 39. (*d*) 40. (*a*) 41. (*c*) 42. (*d*) 43. (*a*) 44. (*b*) 45. (*c*) 46. (*a*)
47. (*b*) 48. (*b*) 49. (*b*) 50. (*d*) 51. (*b*) 52. (*b*) 53. (*d*) 54. (*c*) 55. (*a*)
56. (*d*) 57. (*d*) 58. (*c*) 59. (*b*) 60. (*a*) 61. (*b*) 62. (*a*) 63. (*b*) 64. (*a*)
65. (*c*) 66. (*b*) 67. (*a*) 68. (*a*) 69. (*a*) 70. (*b*) 71. (*b*).

5.3 ANALYTICAL GEOMETRY

5.31 Pair of straight lines

1. If the acute angle between the pair of lines $ax^2-4xy+y^2=0$ is $\dfrac{\pi}{3}$, then one of the values of a is

(*a*) $\sqrt{3}$ (*b*) 3 [✓ (*c*) 1] (*d*) none

2. The separate equations of the lines $6x^2-5xy+y^2=0$ are

   (a) $3x+y=0$ and $2x-y=0$ (b) $3x-y=0$ and $2x+y=0$

   [✓ (c) $3x-y=0$ and $2x-y=0$] (d) none

3. The condition for the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of parallel lines is

   (a) $h^2=ab$ (b) $h^2=ab$ and $ag^2=bh^2$

   (c) $h^2=ab$ and $af=bg$ (d) none

4. The point of intersection of the pair of lines $2x^2+7xy+3y^2+18x+19y+28=0$ is

   (a) $(-1, 2)$ (b) $(1, -2)$ [✓ (c) $(-1, -2)$] (d) $(1, 2)$

5. The distance between the parallel lines $16x^2-8xy+y^2+36x-9y+18=0$ is

   (a) $\frac{2}{\sqrt{17}}$ [✓ (b) $\frac{3}{\sqrt{17}}$] (c) $\frac{1}{\sqrt{17}}$ (d) none

6. If one of the lines of $6x^2+kxy+y^2=0$ is $2x+y=0$, then $k=$

   (a) 5 (b) 4 (c) 3 (d) none

7. The value of k for which the equation $3x^2+2xy-8y^2-8x+14y+k=0$ represents a pair of lines is

   (a) -4 (b) -3 (c) -2 (d) none

8. The equation of the pair of lines through origin perpendicular to the pair of lines $x^2+3xy+2y^2=0$ is

   (a) $x^2+3xy-y^2=0$ (b) $x^2+3xy-2y^2=0$

   (c) $2x^2-3xy-y^2=0$ (d) $2x^2-3xy+y^2=0$

9. The equation $4x^2-4y^2+1=0$ represents a

   (a) pair of ⊥ lines (b) pair of lines (c) circle (d) hyperbola

10. The acute angle between the pair of lines $x^2+4xy-y^2=0$ is

    (a) 0° (b) 45° (c) 60° (d) 90°

11. The point of intersection of the pair of lines $x^2-y^2+2x+2y=0$ is

    (a) $(0, 0)$ (b) $(1, 1)$ (c) $(-1, 1)$ (d) all of them

12. The equation of the pair of lines passing through origin and inclined at an angle 30° and 60° to x-axis is

    (a) $\sqrt{3}\,(x^2+y^2)-2xy=0$ (b) $\sqrt{3}\,(x^2+y^2)-4xy=0$

    (c) $\sqrt{3}\,(x^2+y^2)-8xy=0$ (d) none

13. The point of intersection of the lines $xy+x-y-1=0$ is

    (a) $(-1, -1)$ (b) $(2, -1)$ (c) $(1, -1)$ (d) none

14. The point of intersection of the pair of lines $2x^2-3xy+y^2=0$ is
(*a*) (1, 1) (*b*) (1, –1) (*c*) (2, –1) (*d*) (0, 0)

15. The angle between the lines $\sin\alpha\,(x^2+y^2)=2xy, 0<\alpha<\pi/2$, is
(*a*) α (*b*) $\pi/2-\alpha$ (*c*) $\pi-\alpha$ (*d*) none

16. If the sum of the slopes of the lines $ax^2+hxy+by^2=0$ is equal to the product of the slopes, then
(*a*) $h+a=0$ (*b*) $2h-a=0$ (*c*) $2h+a=0$ (*d*) none

17. If the slopes of the lines $x^2-hxy+4y^2=0$ are equal, then $h=$
(*a*) 4 (*b*) ±4 (*c*) 8 (*d*) none

18. The condition for the equation $a^2x+2hxy+by^2=0$ to represent a pair of distinct real lines is
(*a*) $h^2=ab$ (*b*) $h^2>ab$ (*c*) $h^2<ab$ (*d*) $a+b=0$

19. The sum and product of the slopes of the pair of lines $y^2+xy-2x^2=0$ are respectively
(*a*) 1 ; 2 (*b*) 1 ; –2 (*c*) $\frac{1}{2}$; $\frac{1}{2}$ (*d*) –1 ; –2

20. The angle between the pair of lines $\sqrt{3}\,(x^2+y^2)-4xy=0$ is
(*a*) 30° (*b*) 60° (*c*) $\tan^{-1}(\sqrt{3}/2)$ (*d*) $\tan^{-1}(2/\sqrt{3})$

21. Which of the following equations represents a pair of coincident lines?
(*a*) $4x^2-4xy+y^2=0$ (*b*) $x^2-y^2=0$
(*c*) $4x^2-2xy+y^2=0$ (*d*) $9x^2-6xy+4y^2=0$

22. Which of the following equations represents a pair of ⊥ lines?
(*a*) $x^2-2y^2=0$ (*b*) $x^2=y^2$ (*c*) $5x^2-4xy+5y^2=0$ (*d*) $x^2+y^2=xy$

23. If $x^2+ky^2+x-y=0$ represents a pair of straight lines, then $k=$
(*a*) 1 (*b*) –1 (*c*) $\frac{1}{2}$ (*d*) $-\frac{1}{2}$

24. If one of the equations of the pair of lines $4x^2+4xy+y^2-6x-3y-4=0$ is $2x+y-4=0$, then the other line is
(*a*) $2x+y+1=0$ (*b*) $2x+y-1=0$ (*c*) $x+y+1=0$ (*d*) $x+y-1=0$

25. The point of intersection of the pair of lines $x^2-5xy+4y^2+x+2y-2=0$ is
(*a*) (1, 2) (*b*) (1, –1) (*c*) (2, 1) (*d*) (2, 2)

26. The angle between the pair of lines $2x^2+3xy-2y^2+3x+y+1=0$ is (*a*) $\pi/3$ [✓ (*b*) $\pi/2$] (*c*) $\pi/4$ (*d*) 0

27. The angle between the pair of lines $x^2+2\sqrt{3}\,xy+3y^2-3x-3\sqrt{3}y-4=0$ is
(*a*) 90° (*b*) $\tan^{-1}\frac{1}{2}$ (*c*) $\tan^{-1} 2$ (*d*) 0°

28. The angle between the pair of lines $2x^2+3xy+y^2-4x-3y+2=0$ is
(*a*) $\tan^{-1}\left(\frac{2}{3}\right)$ (*b*) $\tan^{-1}\frac{1}{6}$ (*c*) $\tan^{-1}\frac{1}{3}$ (*d*) $\tan^{-1} 3$

29. The point of intersection of the pair of lines $(x-1)^2+2(x-1)y-y^2=0$ is (*a*) (1, 1) (*b*) (0, 1) (*c*) (1, 0) (*d*) (0, 0)

30. The point of intersection of the pair of lines $x^2+y^2=2xy$ is
(*a*) (1, 1) (*b*) (–1, –1) (*c*) $(\sqrt{2}, \sqrt{2})$ (*d*) (0, 0)

31. The distance between the pair of parallel lines $9x^2-24xy+16y^2-12x+16y-12=0$ is (*a*) 8/5 (*b*) 4/5 (*c*) 2/5 (*d*) 1/5

32. If $32x^2-16xy+ky^2+20x-5y+3=0$ represents a pair of parallel lines, then $k=$ (*a*) –2 (*b*) ±2 (*c*) 2 (*d*) 1

Answers

1. (*c*) 2. (*c*) 3. (*d*) 4. (*c*) 5. (*b*) 6. (*a*) 7. (*b*) 8. (*d*) 9. (*d*) 10. (*d*) 11. (*c*) 12. (*b*) 13. (*c*) 14. (*d*) 15. (*b*) 16. (*a*) 17. (*b*) 18. (*b*) 19. (*d*) 20. (*a*) 21. (*a*) 22. (*b*) 23. (*b*) 24. (*a*) 25. (*c*) 26. (*b*) 27. (*d*) 28. (*c*) 29. (*c*) 30. (*d*) 31. (*a*) 32. (*c*)

5.32 Circle

1. If $4x^2+axy-by^2+4x-3y-1=0$ represents a circle, then
(*a*) $a=0$ (*b*) $b=4$ (*c*) $a=0$ and $b=4$ (*d*) $a=0$ and $b=-4$

2. The equation of the circle with centre at (4, –2) and radius 8 is
(*a*) $x^2+y^2-4x+2y-44=0$ (*b*) $x^2+y^2-8x+4y-44=0$
(*c*) $x^2+y^2+8x-4y-44=0$ (*d*) none

3. The radius of the circle having (3, 4) as centre and $2x-y+8=0$ as a tangent is
(*a*) 10 (*b*) $2\sqrt{5}$ (*c*) 2 (*d*) none

4. The equation of the circle with centre at (–4, –2) and passing through the point (1, 3) is
(*a*) $(x+4)^2+(y+2)^2=50$ (*b*) $(x-4)^2+(y-2)^2=50$
(*c*) $(x+4)^2+(y+2)^2=10$ (*d*) none

5. The radius of the circle $4x^2+4y^2+80x+12y+265=0$ is
(*a*) 6 (*b*) $\sqrt{13}/2$ (*c*) 12 (*d*) none

6. The centre of the circle $3x^2+3y^2-6x+27y-2=0$ is
(*a*) (3, –27/2) (*b*) (1, 9/2) (*c*) (–1, 9/2) (*d*) (1, –9/2)

7. The length of the tangent from (–1, 1) to the circle $x^2+y^2-5x+4y+5=0$ is
(*a*) 4 (*b*) $2\sqrt{2}$ (*c*) 3 (*d*) $\sqrt{11}$

8. The radius of the circle $16x^2+16y^2-8x+32y-239=0$ is
(*a*) 6 (*b*) 4 (*c*) 9 (*d*) none

9. The equation of the circle with centre at (1, 1) and cutting orthogonally the circle $x^2+y^2-4x+6y-2=0$ is
(*a*) $x^2+x^2-2x-2y-8=0$ (*b*) $x^2+y^2+2x+2y-6=0$
(*c*) $x^2+y^2-2x-2y=0$ (*d*) $x^2+y^2-2x-2y+6=0$

10. The equation of the diameter of the circle $x^2+y^2-14x+6y+9=0$ through a point (2, 1) is
(*a*) $3x-2y=1$ (*b*) $4x+5y-13=0$ (*c*) $2x+3y-7=0$ (*d*) none

11. Two circles touch each other at (3, –3). The equation of their radical axis is $x+y=0$. What is the equation of the line of centres?
(*a*) $y-x=0$ (*b*) $x-y+6=0$ (*c*) $x-y=6$ (*d*) none

12. The equation of the circle with centre at (2, 1) and touching the line $4x-3y+5=0$ is
(*a*) $(x+2)^2+(y+1)^2=16$ (*b*) $(x-2)^2+(y-1)^2=2$
(*c*) $(x-2)^2+(y-1)^2=4$ (*d*) none

13. When will the equation $x^2+y^2+2gx+2fy+c=0$ represent a pair of real lines?
(*a*) $g^2+f^2=c$ (*b*) $g^2+f^2-c>0$ (*c*) never (*d*) none

14. Two circles $x^2+y^2+2x-2y+1=0$ and $x^2+y^2-2x+2y+1=0$ intersect at P and Q. The equation of the line PQ is
(*a*) $4x-3y=0$ (*b*) $2x+3y=5$ (*c*) $x=y$ (*d*) $x+y=0$

15. The value of k for which the circles $x^2+y^2-x+22y+3=0$ and $2x^2+2y^2+14x+12y+k=0$ are orthogonal is
(*a*) –119 (*b*) 119 (*c*) –119/2 (*d*) 119/2

16. The equation of the tangent to the circle $x^2+y^2-2x+8y-23=0$ at (3, –10) on it is
(*a*) $x-3y-33=0$ (*b*) $x+3y+27=0$ (*c*) $3x+y+1=0$ (*d*) none

17. The radical axis of the circles $x^2+y^2+4x=1$ and $4x^2+4y^2=9$ is
(*a*) $16x=5$ (*b*) $16x+5=0$ (*c*) $x=2$ (*d*) $x+2=0$

18. The equation $x^2+y^2-6x-8y+25=0$ represents
(*a*) a real circle (*b*) a point circle (*c*) an imaginary circle
(*d*) none

19. The centre of a circle of radius 5 and touching both the axes is
(*a*) (5, 5) (*b*) (–5, 5) (*c*) (–5. –5) (*d*) all of them

20. The radius of the circle circumscribing the right angled triangle with vertices (3, 4), (–1, –4) and (5, –2) is
(a) $\sqrt{17}$ (b) $\sqrt{20}$ (c) $\sqrt{23}$ (d) none

21. The radical centre of the three circles $x^2+y^2+5x-2y+4=0$, $x^2+y^2+2x+4y+1=0$ and $x^2+y^2+4x+4y+k=0$ is (1, 1). Then $k=$ (a) –1 (b) 1 (c) 5 (d) none

22. The equation of the circle passing through $(a, 0)$, $(0, a)$ and $(-a, 0)$ is
(a) $(x-a)^2+y^2=a^2$ (b) $x^2+(y-a)^2=a^2$
(c) $(x+a)^2+y^2=a^2$ (d) $x^2+y^2=a^2$

23. The circumcentre of the triangle with vertices (0, 0), (5, 0) and (0, 10) is
(a) (5/2, 5) (b) (–5/2, –5) (c) (5, 10) (d) none

24. If the line $y=mx+c$ is a tangent to the circle $x^2+y^2=a^2$, then
(a) $a^2=c^2(1+m^2)$ (b) $c^2=m^2(1+a^2)$
(c) $c^2=a^2(1+m^2)$ (d) none

25. The centre of the circle inscribed in the square with vertices (0, 0), (2, 0), (2, 2) and (0, 2) is
(a) (1, 0) (b) $(\frac{1}{2}, \frac{1}{2})$ (c) (1, 1) (d) none

26. The radius of the circle $x^2+y^2-2\sec\theta x+2y+\tan^2\theta=0$ is
(a) 1 (b) $\sqrt{2}$ (c) 0 (d) none

27. The radical axis of the circles $x^2+y^2-x+4y+1=0$ and $2x^2+2y^2-x+2y=0$ is
(a) $x^2+y^2-2y-1=0$ (b) $x-6y-2=0$ (c) $x-6y+2=0$ (d) $2y+1=0$

28. Through which of the following points, there exists NO tangent line to the circle $x^2+y^2-4x+6y-12=0$
(a) (4, –1) (b) (–2, 1) (c) (3, 4) (d) (0, 5)

29. The equation of the tangent to the circle $x^2+y^2-4x+6y-37=0$ at (3, 4) is
(a) $2x+7y-34=0$ (b) $x+7y-31=0$ (c) $x-7y+25=0$ (d) none

30. The equation of a circle is $x^2+y^2-36=0$. The equation of the chord whose mid-point is (3, 2) is
(a) $3x-2y-5=0$ (b) $2x+3y-12=0$ (c) $3x+2y-13=0$ (d) none

31. If the x-coordinates of the points of intersection of the circles $x^2+y^2-6x-8y=0$ and $x^2+y^2-4x+6y-12=0$ are 6 and –1, then their y-coordinates are
(a) 8 and 1 (b) 8 and 7 (c) 0 and 1 (d) 0 and 5

32. The length of the tangent to the circle $4x^2+4y^2-2x+5y-8=0$, from the point $(-4, 4)$ is
(a) $\sqrt{37}$ (b) $2\sqrt{37}$ (c) 37 (d) none

33. At which point the line $y=\sqrt{3}x+4$ touches the circle $x^2+y^2=4$
(a) $(\sqrt{3}, 7)$ (b) $(-2\sqrt{3}, -2)$ (c) $(-\sqrt{3}, 1)$ (d) $(2\sqrt{3}, 10)$

34. If the two circles $x^2+y^2+6x+8y-24=0$ and $x^2+y^2=4$ touch each other, the equation of the common tangent is
(a) $3x+4y-14=0$ (b) $6x+8y-25=0$
(c) $3x+4y+10=0$ (d) $3x+4y=10$

35. If the equation of the circle having x-axis as a tangent is $x^2+y^2+8x-2y+c=0$, then $c=$
(a) 1 (b) -10 (c) 14 (d) 16

36. The equation of the tangent to the circle $x^2+y^2-2x-4y-5=0$ at $(4, 3)$ is
(a) $3x-y-9=0$ (b) $3x+y-15=0$ (c) $3x+2y=18$ (d) $x-y=1$

37. The radius of the circle with centre at $(1, 2)$ and cutting orthogonally the circle $x^2+y^2+4x+16y-30=0$ is
(a) $\sqrt{41}$ (b) $\sqrt{15}$ (c) $\sqrt{10}$ (d) $\sqrt{11}$

38. A circle passes through the point $(4, 1)$. If two of its diameters are $x+y=6$ and $x+2y=4$, then its centre and radius are respectively
(a) $(8, -2)$; $\sqrt{5}$ (b) $(-8, 2)$; $\sqrt{5}$ (c) $(8, -2)$; 25 (d) $(8, -2)$; 5

39. The radical centre of the circles $x^2+y^2-6x-8y+2=0$, $x^2+y^2-4x-y+2=0$ and $x^2+y^2-4x-4y+2=0$ is
(a) $(1, -1)$ (b) $(2, 2)$ (c) $(0, 0)$ (d) none

40. The distance between the centres of the circles $x^2+y^2-6x-2y+1=0$ and $x^2+y^2+2x-8y+13=0$ is
(a) $\sqrt{19}$ (b) $3\sqrt{2}$ (c) 25 (d) 5

41. If the two circles $x^2+y^2+2x-4y+k=0$ and $x^2+y^2-6x+4y-19=0$ cut orthogonally then the equation of their radical axis is
(a) $8x-8y+19=0$ (b) $x-y+3=0$ (c) $x=y$ (d) none

42. If the radical axis of the circles $x^2+y^2=a^2$ and $3x^2+3y^2-2x+y-1=0$ is $2x-y-11=0$, then $a=$
(a) ± 2 (b) 4 (c) ± 4 (d) 2

43. For what values of k will the line $4x-3y=k$ touch the circle $x^2+y^2=25$?
(a) ± 5 (b) ± 25 (c) $\pm\sqrt{5}$ (d) none

44. The circle $x^2+y^2-8x+4y+4=0$ touches
(*a*) x–axis (*b*) y–axis (*c*) both the axes (*d*) none

45. If one end of a diameter of the circle $x^2+y^2+2x+4y-15=0$ is (4, 1), what is the other end?
(*a*) (–2, –7) (*b*) (5, 4) (*c*) (2, 1) (*d*) (–6, –5)

46. The power of the point (–1, 1) w.r.t. the circle $x^2+y^2+4x+6y+9=0$ is
(*a*) 13 (*b*) 12 (*c*) $\sqrt{11}$ (*d*) none

47. Where do the points (2, 1) and (5, 1) lie w.r.t. the circle $(x-5)^2+(y-1)^2=5$
(*a*) outside; on (*b*) inside; on
(*c*) outside; inside (*d*) inside; outside

48. Which of the following is NOT true?
(*a*) There are infinitely many circles cutting orthogonally a given circle
(*b*) There are infinitely many circles cutting orthogonally two intersecting circles
(*c*) There is only one circle cutting orthogonally three intersecting circles (*d*) none

49. The radius of the circle $x^2+y^2+4x-6y+20=0$ is
(*a*) $\sqrt{11}$ (*b*) $\sqrt{-11}$ (*c*) $\sqrt{33}$ (*d*) none

50. The area of the circle $x^2+y^2-10x+8y+5=0$ is
(*a*) 36π (*b*) 46π (*c*) 6π (*d*) 12π

51. If the line $x \cos \alpha + y \sin \alpha = p$ touches the circle $x^2+y^2=r^2$, then (*a*) $p^2+r^2=0$ (*b*) $p^2=r^2$ (*c*) $p=r$ (*d*) $p=r^2$

52. The length of the chord cut off by the y-axis on the circle $x^2+y^2+8x-6y+8=0$ is
(*a*) 2 (*b*) 4 (*c*) 6 (*d*) none

53. The centre of the circle $4x^2+4y^2+8x-16y+19=0$ is
(*a*) (4, –8) (*b*) (–4, 8) (*c*) (1, –2) (*d*) (–1, 2)

Answers

1. (*d*) 2. (*b*) 3. (*b*) 4. (*a*) 5. (*a*) 6. (*d*) 7. (*a*) 8. (*b*) 9. (*c*) 10. (*b*) 11. (*c*) 12. (*c*) 13. (*c*) 14. (*c*) 15. (*b*) 16. (*a*) 17. (*b*) 18. (*b*) 19. (*d*) 20. (*b*) 21. (*a*) 22. (*d*) 23. (*a*) 24. (*c*) 25. (*c*) 26. (*b*) 27. (*b*) 28. (*a*) 29. (*b*) 30. (*c*) 31. (*c*) 32. (*a*) 33. (*c*) 34. (*d*) 35. (*d*) 36. (*b*) 37. (*d*) 38. (*d*) 39. (*c*) 40. (*d*) 41. (*b*) 42. (*a*) 43. (*b*) 44. (*b*) 45. (*d*) 46. (*a*) 47. (*c*) 48. (*d*) 49. (*d*) 50. (*a*) 51. (*b*) 52. (*a*) 53. (*d*)

5.33 Conic section

1. **The conics with eccentricities $\sqrt{3}/2$ and $4/\sqrt{15}$ are respectively**
   (*a*) ellipse, hyperbola (*b*) hyperbola, ellipse
   (*c*) ellipse, ellipse (*d*) none
2. **The end points of the latus rectum of $y^2-8x=0$ are**
   (*a*) $(-2, \pm 4)$ (*b*) $(2, \pm 4)$ (*c*) $(\pm 4, 2)$ (*d*) none
3. **The length of latus rectum of the parabola $x=t$, $y=t^2$ is**
   (*a*) 1 (*b*) 4 (*c*) $\frac{1}{4}$ (*d*) none
4. **$x=5\cos\theta$, $y=5\sin\theta$ are the parametric equations of**
   (*a*) an ellipse (*b*) a hyperbola (*c*) a circle (*d*) a parabola
5. **The vertices of the hyperbola $16y^2-25x^2=400$ are**
   (*a*) $(\pm 4, 0)$ (*b*) $(0, \pm 5)$ (*c*) $(\pm 4, 0)$ and $(0, \pm 5)$ (*d*) none
6. **The equation $x^2/9-y^2/4=0$ represents**
   (*a*) a hyperbola (*b*) a rectangular hyperbola
   (*c*) a pair of lines (*d*) none
7. **The centre of the hyperbola $144x^2-25y^2-576x+200y+3776=0$ is** (*a*) $(-2, 4)$ (*b*) $(2, 2)$ (*c*) $(2, -4)$ (*d*) $(2, 4)$
8. **The distance between the foci of the ellipse $x^2+4y^2=16$ is**
   (*a*) $2\sqrt{3}$ (*b*) $4\sqrt{3}$ (*c*) $\sqrt{3}$ (*d*) none
9. **The extremities of the minor axis of the ellipse $25x^2+9y^2=25$ are**
   (*a*) $(0, \pm 1)$ (*b*) $(\pm 1, 0)$ (*c*) $(\pm 5/3, 0)$ (*d*) $(0, \pm 5/3)$
10. **The length of a latus rectum of the ellipse $x^2+9y^2+4x-18y-23=0$ is** (*a*) 8/3 (*b*) 4/3 (*c*) 36 (*d*) none
11. **The eccentricity of the ellipse $x^2/144+y^2/44=1$ is**
    (*a*) 5/12 (*b*) 5/6 (*c*) $\frac{1}{2}$ (*d*) none
12. **The equation of the ellipse with centre at (0, 0), a vertex at (0, −13) and one focus at (0, −12) is**
    (*a*) $x^2/169+y^2/25=1$ (*b*) $x^2/25+y^2/169=1$
    (*c*) $x^2/25+y^2/144=1$ (*d*) none
13. **Two vertices of an ellipse are (3, −10) and (3, 2). If one focus is (3, 6), the equation of the major axis is**
    (*a*) $x-2=0$ (*b*) $x-3=0$ (*c*) $y=2$ (*d*) $y=6$
14. **The tangent at (−2, −2) on the parabola $x^2+2y=0$ is**
    (*a*) $2x+y+6=0$ (*b*) $y=2x+2$ (*c*) $x+2y+6=0$ (*d*) $2y=x-2$

15. The focus of the parabola $y^2-8x-32=0$ is
(*a*) $(2, 0)$ (*b*) $(-2, 0)$ (*c*) $(0, 2)$ (*d*) $(6, 0)$

16. The end points of latus rectum of the parabola $x^2-2x-12y+37=0$ are
(*a*) $(7, 6), (-5, 6)$ (*b*) $(7, -6), (6, 6)$ (*c*) $(7, \pm 6)$ (*d*) none

17. The eccentricity of the hyperbola $x^2-4y^2+6x+16y-11=0$ is
(*a*) $\sqrt{5}/3$ (*b*) $\sqrt{5}/2$ (*c*) $5/3$ (*d*) $\sqrt{5/2}$

18. The locus of a point whose distance from $(5, 2)$ is equal to its distance from $3x+y=4$ is
(*a*) a straight line (*b*) a pair of lines
(*c*) a parabola (*d*) a rectangular hyperbola

19. The extremities of the major axis of an ellipse are $(3, -10)$ and $(3, 2)$. If the length of a latus rectum $=4$, what is the length of minor axis?
(*a*) 24 (*b*) 12 (*c*) $2\sqrt{3}$ (*d*) $4\sqrt{3}$

20. The distance between the directrices of the ellipse $(x+1)^2/9+(y-2)^2/25=1$ is
(*a*) $25/4$ (*b*) 25 (*c*) $25/2$ (*d*) none

21. The foci of the ellipse $4x^2+9y^2=36$ are
(*a*) $(\pm 5, 0)$ (*b*) $(\pm\sqrt{5}, 0)$ (*c*) $(0, \pm\sqrt{3})$ (*d*) none

22. The equation of the directrices of the ellipse $25x^2+16y^2=400$ are (*a*) $3y=\pm 25$ (*b*) $2y=\pm 5$ (*c*) $3x=\pm 25$ (*d*) none

23. The centre of the ellipse $16x^2+9y^2+32x-36y-92=0$ is
(*a*) $(1, 2)$ (*b*) $(1, -2)$ (*c*) $(2, -1)$ (*d*) $(-1, 2)$

24. The eccentricity of the ellipse $x^2+4y^2-6x+32y+69=0$ is
(*a*) $1/\sqrt{3}$ (*b*) $1/\sqrt{2}$ (*c*) $\sqrt{3}/2$ (*d*) none

25. The equation of the minor axis of the ellipse $x^2+2y^2+4x-12y+20=0$ is
(*a*) $x=2$ (*b*) $x+2=0$ (*c*) $y=3$ (*d*) $y+3=0$

26. The focus of the parabola $x^2+12y=0$ is
(*a*) $(3, 0)$ (*b*) $(-3, 0)$ (*c*) $(0, 3)$ (*d*) $(0, -3)$

27. The vertex of the parabola $y^2-8x-4y+4=0$ is
(*a*) $(2, -3)$ (*b*) $(-3, -2)$ (*c*) $(-2, 3)$ (*d*) $(0, 2)$

28. The length of a latus rectum of the hyperbola $16y^2-9x^2=144$ is
(*a*) $\frac{32}{3}$ (*b*) $\frac{16}{3}$ (*c*) $\frac{9}{2}$ (*d*) $\frac{9}{4}$

29. The locus of a point, the sum of whose distances from $(0, 4)$ and $(0, -4)$ is 20, is
(*a*) a circle (*b*) an ellipse (*c*) a hyperbola (*d*) none

30. The equation of the directrix of the parabola $x^2+9y=0$ is
(*a*) $4x-9=0$ (*b*) $4x+9=0$ (*c*) $4y-9=0$ (*d*) $4y+9=0$

31. The focus of the parabola $x^2-2x-12y+25=0$ is
(*a*) $(1, 5)$ (*b*) $(-1, 4)$ (*c*) $(-1, 5)$ (*d*) none

32. The length of the latus rectum of the parabola $2y^2+4y+20x+1=0$ is
(*a*) 20 (*b*) 10 (*c*) 5 (*d*) 40

33. If the vertex and focus of a parabola are $(0, -2)$ and $(-5, -2)$ respectively, then the directrix is
(*a*) $y=-2$ (*b*) $x=5$ (*b*) $y-2=0$ (*d*) $x+5=0$

34. If the end points of latus rectum of a parabola are $(-3, 6)$ and $(-3, 2)$, the equation of its axis is
(*a*) $x+3=0$ (*b*) $y=4$ (*c*) $x-3=0$ (*d*) $y+4=0$

35. The equation of the parabola with vertex at $(0, 0)$, axis $y=0$ and passing through the point $(4, 5)$ is
(*a*) $4y^2=25x$ (*b*) $16y^2=5x$ (*c*) $5x^2=20y$ (*d*) none

36. A parabola has focus at $(2, 3)$ and its directrix is $y+1=0$. Then its vertex is
(*a*) $(2, 2)$ (*b*) $(2, -1)$ (*c*) $(2, 1)$ (*d*) none

37. The eccentricity of the ellipse $36x^2+11y^2-216x+110y+203=0$ is (*a*) 5/6 (*b*) 2/3 (*c*) 1/3 (*d*) none

38. The foci of the hyperbola $9y^2-4x^2=36$ are
(*a*) $(\pm 3, 0)$ (*b*) $(0, \pm 3)$ (*c*) $(\pm\sqrt{13}, 0)$ (*d*) $(0, \pm\sqrt{13})$

39. The equation of the normal to $y^2=20x$ at $(5, 10)$ is
(*a*) $x-y+5=0$ (*b*) $x+y-15=0$ (*c*) $x+y-5=0$ (*d*) $x-y+15=0$

40. The length of the conjugate axis of a hyperbola with vertices $(\pm 12, 0)$ and latus rectum$=32/3$ is
(*a*) 64 (*b*) 16 (*c*) 32 (*d*) none

41. The distance between a focus and an end point of the minor axis of the ellipse $4x^2+9y^2=36$ is
(*a*) 3 (*b*) 6 (*c*) 2 (*d*) 4

42. The equation of the hyperbola with centre at $(0, 0)$, a vertex at $(4, 0)$ and a focus at $(5, 0)$ is
(*a*) $\frac{x^2}{16}-\frac{y^2}{25}=1$ (*b*) $\frac{x^2}{16}-\frac{y^2}{4}=1$ (*c*) $\frac{x^2}{16}-\frac{y^2}{9}=1$ (*d*) none

43. The equation of the hyperbola with centre at origin, a focus at $(-4, 0)$ and $e=2$ is

(*a*) $\frac{x^2}{4}-\frac{y^2}{12}=1$ (*b*) $\frac{x^2}{9}-\frac{y^2}{12}=1$

(*c*) $\frac{y^2}{4}-\frac{x^2}{12}=1$ (*d*) $\frac{x^2}{12}-\frac{y^2}{4}=1$

44. Which of the following is an equilateral hyperbola?

(*a*) $x^2-y^2=1$ (*b*) $y^2-x^2=1$ (*c*) $xy=1$ (*d*) all of them

45. The equation of the transverse axis of $9x^2-4y^2-36x+32y+8=0$ is

(*a*) $x=2$ (*b*) $y=2$ (*c*) $x=4$ (*d*) $y=4$

46. A focus of the ellipse $25x^2+169y^2=4225$ is

(*a*) $(12, 0)$ (*b*) $(13, 0)$ (*c*) $(5, 0)$ (*d*) none

47. The eccentricity of the ellipse $3x^2+2y^2-30x-4y+23=0$ is

(*a*) $1/3$ (*b*) $\sqrt{3}/3$ (*c*) $\sqrt{2}/2$ (*d*) $\sqrt{2}/3$

48. The centre of the ellipse $9x^2+25y^2-36x-150y+36=0$ is

(*a*) $(7, 3)$ (*b*) $(6, 3)$ (*c*) $(4, 3)$ (*d*) $(2, 3)$

49. The equation of an ellipse with centre at $(0, 0)$, length of the major axis as 10 and a focus at $(0, 4)$ is

(*a*) $9x^2+25y^2=225$ (*b*) $25x^2+9y^2=225$

(*c*) $25x^2+16y^2=400$ (*d*) none

50. The centre of the hyperbola $9x^2-16y^2-18x-32y=151$ is

(*a*) $(1, -1)$ (*b*) $(-1, 1)$ (*c*) $(1, 1)$ (*d*) $(-1, -1)$

51. The equation of the tangent to the parabola $y^2=16x$ at $(4, 8)$ is

(*a*) $4x+y=20$ (*b*) $4x-y=8$ (*c*) $y+4x=22$ (*d*) $x-y+4=0$

52. The equation of the conjugate axis of $9x^2-4y^2+54x+16y-79=0$ is

(*a*) $y=2$ (*b*) $y=-3$ (*c*) $x=3$ (*d*) $x=-3$

53. A vertex of the hyperbola $(x+5)^2/36-(y-4)^2/64=1$ is

(*a*) $(-5, 4)$ (*b*) $(11, -4)$ (*c*) $(-11, 4)$ (*d*) none

54. The locus of a point which is twice as from the point $(3, 1)$ as from the line $x+y=5$ is

(*a*) an ellipse (*b*) a parabola (*c*) a circle (*d*) a hyperbola

55. The focus of the parabola $x^2-6x+8y+25=0$ is

(*a*) $(-3, 4)$ (*b*) $(-3, -4)$ (*c*) $(3, -4)$ (*d*) none

56. The equation of the axis of $y^2-16x+2y+49=0$ is
(*a*) $y+1=0$ (*b*) $y=1$ (*c*) $x=1$ (*d*) $x=-1$

57. The equation of directrix of the parabola $x^2-2x-6y-53=0$ is
(*a*) $2y+11=0$ (*b*) $y+21=0$ (*c*) $2y+21=0$ (*d*) none

58. The vertex of the parabola $y^2+20x+4y-60=0$ is
(*a*) $(16, -2)$ (*b*) $(16/5, -2)$ (*c*) $(-9/5, -2)$ (*d*) none

59. If d_1 and d_2 are respectively the distances of any point on an ellipse from a focus and the corresponding directrix, then
(*a*) $d_1>d_2$ (*b*) $d_1=d_2$ (*c*) $d_1<d_2$ (*d*) $d_1 \geqslant d_2$

60. The end points of the latus rectum of the parabola $y=4x^2$ are
(*a*) $(\pm 2, 1)$ (*b*) $(\pm\frac{1}{2}, \frac{1}{2})$ (*c*) $(\pm 1/8, \frac{1}{16})$ (*d*) $(\frac{1}{16}, \pm 1/8)$

61. The focus of the parabola $x^2+10x+6y+19=0$ is
(*a*) $(-5, 1)$ (*b*) $(-5, -1/2)$ (*c*) $(0, -3/2)$ (*d*) $(-13/2, 1)$

62. The vertex of the parabola $y^2+6x+8y+1=0$ is
(*a*) $(-3, -4)$ (*b*) $(-5/2, 4)$ (*c*) $(-5/2, -4)$ (*d*) $(5/2, -4)$

63. The equation of the normal to the ellipse $x^2+4y^2=20$ at $(4, -1)$ is
(*a*) $x-2y=6$ (*b*) $x+2y=2$ (*c*) $x=y+5$ (*d*) $x+y=3$

64. If the length of the latus rectum of an ellipse is half of its minor axis, then its eccentricity is
(*a*) $1/\sqrt{2}$ (*b*) $\sqrt{3}/\sqrt{2}$ (*c*) $\sqrt{3}/2$ (*d*) $1/2$

65. The focus of a parabola with end points of latus rectum at $(2, -1)$ and $(2, 5)$ is
(*a*) $(2, 3)$ (*b*) $(2, 2)$ (*c*) $(0, -3)$ (*d*) $(0, 3)$

66. The focus of the parabola $y^2+8x-6y+1=0$ is
(*a*) $(1, 3)$ (*b*) $(3, -1)$ (*c*) $(-1, 3)$ (*d*) none

67. The equation of directrix of the parabola $y^2-6y-12x-3=0$ is
(*a*) $x=4$ (*b*) $x=-4$ (*c*) $y=4$ (*d*) $y+4=0$

68. The equation of the directrix of the parabola $(y+k)^2=4a(x+h)$ is
(*a*) $x=h-a$ (*b*) $y=-k+a$ (*c*) $x=-h-a$ (*d*) $y=k+a$

69. The length of the latus rectum of the parabola $2y^2+3x=0$ is
(*a*) 3 (*b*) 2/3 (*c*) 3 (*d*) 3/2

70. The equation of the axis of symmetry of the parabola $x^2=2-y$ is
(*a*) x–axis (*b*) y–axis (*c*) $y=2$ (*d*) $y+2=0$

71. The coordinates of the focus of the parabola $x=4t^2$, $y=8t$ are
(*a*) (1, 0) (*b*) (2, 0) (*c*) (4, 0) (*d*) (8, 0)

72. A focus of the ellipse $9x^2+25y^2=225$ is
(*a*) (20, 0) (*b*) (4, 0) (*c*) (5, 0) (*d*) none

73. The equation of a directrix of the ellipse $9x^2+5y^2=45$ is
(*a*) $2y=9$ (*b*) $2y=3$ (*c*) $2x=1$ (*d*) $2x=9$

74. An end point of a latus rectum of $x^2/25+y^2/16=1$ is
(*a*) (4, 16/5) (*b*) (3, 16/5) (*c*) (–3, –25/4) (*d*) all of them

75. The vertices of the ellipse $4x^2+9y^2=1$ are
(*a*) ($\pm1/2$, $\pm1/3$) (*b*) ($\pm\frac{1}{2}$, 0); (0, $\pm1/3$)
(*c*) ($\pm\frac{1}{3}$, 0), (0, $\pm1/2$) (*d*) none

76. The equation of the directrix of the parabola $y^2+8x=0$ is
(*a*) $y=2$ (*b*) $y+2=0$ (*c*) $x+2=0$ (*d*) $x=2$

77. The centre of the ellipse $9x^2+2y^2-36x-50y-164=0$ is
(*a*) (2, 5) (*b*) (1, –2) (*c*) (–2, 5) (*d*) none

78. The slope of the normal to the parabola $y^2-4y-6x+10=0$ at (7, –4) is (*a*) 2 (*b*) –2 (*c*) 1/2 (*d*) none

79. The locus of a point, the difference of whose distances from the points (2, 1) and (3, 1) is 10, is
(*a*) a circle (*b*) a parabola (*c*) an ellipse (*d*) a hyperbola

80. The points on the parabola $y^2=4x$ at which the ordinate is double the abscissa are
(*a*) (2, 4), (1, 2) (*b*) (0, 0) (*c*) (–1, –2), (0, 0) (*d*) none

81. The length of a latus rectum of the ellipse $(x-2)^2/49+(y-5)^2/64=1$ is
(*a*) 64/7 (*b*) 49/8 (*c*) 128/7 (*d*) 49/4

82. The lengths of the major and minor axes of $\dfrac{(x+1)^2}{4}+\dfrac{(y-1)^2}{9}=16$ are
(*a*) 9, 4 (*b*) 6, 4 (*c*) 24, 16 (*d*) none

83. The equation of the minor axis of $100\,(x+1)^2+250\,(y-3)^2=8333$ is (*a*) $x+1=0$ (*b*) $y-3=0$ (*c*) $x=0$ (*d*) $y=0$

84. The distance between the foci of the ellipse $x=3\cos\theta$, $y=4\sin\theta$ is (*a*) $2\sqrt{17}$ (*b*) 5 (*c*) 10 (*d*) $2\sqrt{7}$

85. The eccentricity of the hyperbola $\sqrt{3}x^2/25-\sqrt{3}y^2/25=3000$ is
(*a*) $2\sqrt{3}$ (*b*) $2\sqrt{2}$ (*c*) $\sqrt{2}$ (*d*) $\sqrt{3}$

86. The length of the conjugate axis of the hyperbola $16x^2-9y^2-64x-18y+199=0$ is
(*a*) 6 (*b*) 8 (*c*) 3 (*d*) 4

87. A focus of the hyperbola $9x^2-16y^2=144$ is at
(*a*) (2, 0) (*b*) (4, 0) (*c*) (5, 0) (*d*) (0, 3)

88. The centre of the hyperbola, whose foci are two vertices of the ellipse $2x^2+3y^2=1$ and whose vertices are the foci of the ellipse, is
(*a*) (2, 0) (*b*) (3, 0) (*c*) (2, 3) (*d*) (0, 0)

89. If $y=2x+c$ is a tangent to the parabola $y^2=16x$, then $c=$
(*a*) 4 (*b*) 2 (*c*) 1 (*d*) none

90. The eccentricity of the ellipse $25x^2+9y^2-150x-90y+225=0$ is
(*a*) 4/5 (*b*) 5/4 (*c*) 1/2 (*d*) none

91. The parabola $x^2+ax+by+c=0$ opens
(*a*) upward (*b*) downward (*c*) right (*d*) either (*a*) or (*b*)

92. The eccentricity of an ellipse is 1/2. If the length of the minor axis is 6, what is the length of the semi-major axis?
(*a*) $\sqrt{3}$ (*b*) $2\sqrt{3}$ (*c*) $4\sqrt{3}$ (*d*) none

93. The length of a focal chord of the ellipse $x^2/25+y^2/16=1$ perpendicular to major axis is
(*a*) 3.2 (*b*) 6.4 (*c*) 1.8 (*d*) none

94. The equation of the parabola with vertex at (0, 0), axis as y-axis and passing through the point (–2, –4) is
(*a*) $y=x^2$ (*b*) $y=-x^2$ (*c*) $y^2=8x$ (*d*) $y^2=-8x$

95. What is the equation of the axis of the parabola having (4, 6) and (–2, 6) as the end points of latus rectum?
(*a*) $x=1$ (*b*) $x=0$ (*c*) x–axis (*d*) $x=2$

96. The equation of the parabola with directrix $x=2$ and focus (1, 0) is
(*a*) $y^2=-2x+3$ (*b*) $y^2=2x-3$ (*c*) $x^2-2y=3$ (*d*) $x^2=-2y+3$

97. The equation of the parabola with vertex at (2, –3) and focus at (2, 0) is
(*a*) $(x-2)^2=-12(y+3)$ (*b*) $(x-2)^2=12(y+3)$
(*c*) $(x-2)^2=3(y+3)$ (*d*) none

98. The equation of the parabola with directrix $x+2=0$ and having $(2, 1\pm4)$ as the end points of latus rectum is
(*a*) $y^2=4(x-2)$ (*b*) $(y+1)^2=4(x+2)$ (*c*) $(y-1)^2=8x$ (*d*) none

99. An ellipse has foci at $(-8, 2)$ and $(4, 2)$. If $e=2/3$, what is the length of minor axis?
(*a*) 18 (*b*) $6\sqrt{3}$ (*c*) $6\sqrt{5}$ (*d*) none

100. The eccentricity of the ellipse $x^2/16+y^2/25=1$ is
(*a*) 0.4 (*b*) 0.5 (*c*) 0.6 (*d*) 0.75

101. The eccentricity of the hyperbola $9x^2-16y^2=144$ is
(*a*) 5/4 (*b*) 4/3 (*c*) 4/5 (*d*) none

102. In an ellipse $e=1/\sqrt{2}$ and length of a latus rectum$=9/4$. What is the length of major axis?
(*a*) 9 (*b*) 18 (*c*) 9/2 (*d*) none

103. In an ellipse $x^2/a^2+y^2/b^2=1$, $a=1$ and the equation of a directrix is $x=\sqrt{3}$. What is the length of minor axis?
(*a*) 1/3 (*b*) 1/6 (*c*) $\frac{1}{\sqrt{3}}$ (*d*) $2\sqrt{2}/\sqrt{3}$

104. If the distance between foci$=2$ and the distance between the directrices$=8$, then the length of major axis is
(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 8

105. The equation of an ellipse the length of whose axes are 8 and 6, is
(*a*) $x^2/16+y^2/9=1$ (*b*) $x^2/9+y^2/16=1$
(*c*) $(x-1)^2/9+(y-1)^2/16=1$ (*d*) all of them

106. The vertices of a hyperbola are $(-5, -3)$ and $(-5, -1)$. If $e=\sqrt{5}$, the length of latus rectum$=$
(*a*) 4 (*b*) 8 (*c*) 2 (*d*) 16

107. The equation of the hyperbola having the ends of its conjugate axis as $(0, \pm2)$ and $e=3/\sqrt{5}$ is
(*a*) $x^2/15-y^2/4=1$ (*b*) $x^2/4-y^2/5=1$
(*c*) $x^2/5-y^2/4=1$ (*d*) all of them

108. If the distance between foci of a hyperbola of eccentricity $\sqrt{3}$ is 6, then the length of semi-conjugate axis is
(*a*) $\sqrt{6}$ (*b*) $2\sqrt{6}$ (*c*) $4\sqrt{3}$ (*d*) none

109. The number of hyperbolas with centre at $(0, 0)$, a focus at $(6, 0)$ and length of transverse axis 20 is
(*a*) infinite (*b*) two (*c*) one (*d*) none

110. The equation of the normal to the parabola $y^2=12x$ at (3, 6) is
(*a*) $x-y+3=0$ (*b*) $x+y=9$ (*c*) $2x+y-12=0$ (*d*) none

111. The equation of the tangent to the parabola $y^2=4x+4y$ at (–1, 2) is
(*a*) $y=2$ (*b*) $x+y=1$ (*c*) $2x+y=0$ (*d*) $x+1=0$

112. The point of contact of the tangent to the parabola $y^2=16x$ which is parallel to the line $x-y+1=0$ is
(*a*) (8, 2) (*b*) (4, 8) (*c*) (–4, 8) (*d*) none

113. The equation of the tangent to the ellipse $4x^2+9y^2=72$ at (3, 2) is
(*a*) $2x+3y-6=0$ (*b*) $2x+3y-12=0$ (*c*) $2x-3y=0$ (*d*) $x+y=5$

114. The equation of the normal to the ellipse $x^2+6y^2=154$ at (2, –5) is
(*a*) $15x+2y-20=0$ (*b*) $15x+y+25=0$
(*c*) $15x+y-25=0$ (*d*) none

115. If the line $y=mx+c$ is a tangent to $3x^2+4y^2=12$, then $c^2=$
(*a*) $3m^2+4$ (*b*) $4m^2+3$ (*c*) m^2+3 (*d*) $3m^2-4$

116. The equation of the tangent to the hyperbola $2x^2-y^2-8x-6=0$ at (–1, 2) is
(*a*) $3x+y+1=0$ (*b*) $6x-2y+1=0$ (*c*) $6x+2y+5=0$ (*d*) none

117. The equation of the normal to the hyperbola $x^2-y^2=4$ at (2, 0) is
(*a*) $x=2$ (*b*) $y=0$ (*c*) $x+y=2$ (*d*) none

118. The condition for the line $lx+my+n=0$ to be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is
(*a*) $n^2=a^2l^2+b^2m^2$ (*b*) $m^2=a^2l^2-b^2n^2$
(*c*) $n^2=a^2l^2-b^2m^2$ (*d*) none

119. The end points of the latus rectum of the parabola $(y+3)^2=-4(x+2)$ are
(*a*) (–3, –2), (–3, –1) (*b*) (–3, –5), (–3, –1)
(*c*) (–3, –5), (–1, –5) (*d*) none

120. The eccentricity of the ellipse $4x^2+9y^2+16x-18y-11=0$ is
(*a*) $\sqrt{3}/4$ (*b*) $\sqrt{5}/2$ (*c*) $\sqrt{5}/3$ (*d*) $1/2$

121. The equation of the parabola with focus at (2, –3) and vertex at (2, –1) is
(*a*) $(x-2)^2=4(y+1)$ (*b*) $(x-2)^2=-8(y+1)$
(*c*) $(y-1)^2=8(x-2)$ (*d*) none

122. **The distance between foci of the hyperbola $36x^2-25y^2=900$ is**
(*a*) $2\sqrt{71}$ (*b*) $2\sqrt{61}$ (*c*) $2\sqrt{41}$ (*d*) none

123. **The equation of the hyperbola $x^2/a^2-y^2/b^2=1$ with focus at (4, 0) and $e=2$ is.**
(*a*) $x^2/4-y^2/8=1$ (*b*) $x^2/8-y^2/16=1$ (*c*) $x^2/4-y^2/12=1$ (*d*) none

124. **The equation of the tangent to the parabola $y^2=8x$ which is perpendicular to $x+y+4=0$ is**
(*a*) $4x-y-3=0$ (*b*) $y=x+2$ (*c*) $x-y+11=0$ (*d*) $2y=2x+3$

125. **The equation of the normal to the parabola $y^2=16x$ at (4, 8) is**
(*a*) $x-y+4=0$ (*b*) $x+y-12=0$ (*c*) $2x+y-16=0$ (*d*) none

126. **The equation of the axis of the parabola $y^2+x-y+2=0$ is**
(*a*) $y=1$ (*b*) $y+1=0$ (*c*) $2x=1$ (*d*) $2y=1$

127. **The equations of the tangents to the ellipse $x^2/16+y^2/4=1$ which are perpendicular to $x+2y=5$ are**
(*a*) $y=2x\pm5$ (*b*) $y=2x\pm68$ (*c*) $y=2x\pm70$ (*d*) none

128. **The equation of the axis of the parabola having (–1, 1) and (-1, 3) as the end points of latus rectum is**
(*a*) $x+1=0$ (*b*) $y=2$ (*c*) $x+2=0$ (*d*) $y=4$

129. **The length of the latus rectum of $x^2-y^2=16$ is k times the length of the transeverse axis, then $k=$**
(*a*) 2 (*b*) 1 (*c*) 4 (*d*) none

130. **A focus of an ellipse in standard position is (4, 0) and the corresponding directrix is $x=25/4$. What is its equation?**
(*a*) $\frac{x^2}{25}+\frac{y^2}{8}=1$ (*b*) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (*c*) $\frac{x^2}{49}+\frac{y^2}{25}=1$ (*d*) none

131. **The equation $x^2/(1-a)-y^2/(1+a)=1$ when $a>1$ represents**
(*a*) an ellipse (*b*) a hyperbola (*c*) a circle (*d*) none

132. **The equations of the axes of the hyperbola $16x^2-64x-y^2-4y+44=0$ are**
(*a*) $x=2,\ y=-2$ (*b*) $x=-2,\ y=2$
(*c*) $x=2,\ y=2$ (*d*) $x=-2,\ y=-2$

133. **The equation of the directrix of the parabola having its focus at (2, 3) and vertex at (–2, 3) is**
(*a*) $x+6=0$ (*b*) $x=0$ (*c*) $y=6$ (*d*) $x=-4$

134. **The equation of directrix of the parabola $x^2+4y+4=0$ is**
(*a*) $x=0$ (*b*) $y=-1$ (*c*) $y=0$ (*d*) none

135. One focus of the ellipse $x^2/80+y^2/64=1$ is at
(*a*) (4, 0) (*b*) $(\sqrt{34}, 0)$ (*c*) (0, –4) (*d*) (0, 4)

136. The equation of the tangent to the ellipse $7x^2+3y^2=75$ at (–3, –2) is
(*a*) $x-9y-15=0$ (*b*) $x+9y+21=0$
(*c*) $7x+2y+25=0$ (*d*) none

137. The length of the latus rectum of the parabola with vertex at (–3, 2) and directrix $y=4$ is
(*a*) 4 (*b*) 8 (*c*) 16 (*d*) none

138. The length of the latus rectum of the ellipse $3x^2+4y^2+6x-8y-5=0$ is
(*a*) 3 (*b*) 4 (*c*) 6 (*d*) 9

139. What is the eccentricity of the ellipse if its length of latus rectum is equal to the length of its semi-major axis?
(*a*) $1/\sqrt{2}$ (*b*) $\sqrt{3}/2$ (*c*) 1/2 (*d*) 1/4

140. The equations of the directrices of the hyperbola $y^2/16-x^2/25=1$ are
(*a*) $x=\pm16/\sqrt{41}$ (*b*) $y=\pm16/\sqrt{31}$
(*c*) $x=\pm16/\sqrt{31}$ (*d*) $y=\pm16/\sqrt{41}$

Answers

1. (*a*) 2. (*b*) 3. (*a*) 4. (*c*) 5. (*b*) 6. (*c*) 7. (*d*) 8. (*b*) 9. (*b*) 10. (*b*)
11. (*b*) 12 (*b*) 13. (*b*) 14. (*b*) 15. (*b*) 16. (*a*) 17. (*b*) 18 (*c*) 19. (*d*)
20. (*c*) 21. (*b*) 22. (*a*) 23. (*d*) 24. (*c*) 25. (*b*) 26. (*d*) 27. (*d*) 28. (*a*)
29. (*b*) 30. (*c*) 31. (*a*) 32. (*b*) 33. (*b*) 34. (*b*) 35. (*a*) 36. (*c*) 37. (*a*)
38. (*d*) 39. (*b*) 40. (*b*) 41. (*a*) 42. (*c*) 43. (*a*) 44. (*d*) 45. (*a*) 46. (*a*)
47. (*b*) 48. (*d*) 49. (*b*) 50. (*a*) 51. (*d*) 52. (*d*) 53. (*c*) 54. (*d*) 55. (*c*)
56. (*a*) 57. (*c*) 58. (*b*) 59. (*c*) 60. (*c*) 61. (*b*) 62. (*d*) 63. (*d*) 64. (*c*)
65. (*b*) 66. (*c*) 67. (*b*) 68. (*c*) 69. (*d*) 70. (*b*) 71. (*c*) 72. (*b*) 73. (*a*)
74 (*b*) 75. (*b*) 76. (*d*) 77. (*a*) 78. (*a*) 79. (*d*) 80. (*d*) 81. (*c*) 82. (*c*)
83. (*a*) 84. (*d*) 85. (*c*) 86. (*a*) 87. (*c*) 88. (*d*) 89. (*b*) 90. (*a*) 91. (*d*)
92. (*b*) 93. (*b*) 94. (*b*) 95. (*a*) 96. (*a*) 97. (*b*) 98. (*c*) 99. (*c*) 100. (*c*)
101. (*a*) 102. (*c*) 103. (*d*) 104. (*c*) 105. (*d*) 106. (*b*) 107. (*c*)
108. (*a*) 109. (*d*) 110. (*b*) 111. (*d*) 112. (*b*) 113. (*b*) 114. (*c*)
115. (*b*) 116. (*a*) 117. (*b*) 118. (*c*) 119. (*b*) 120. (*c*) 121. (*b*)
122. (*b*) 123. (*c*) 124. (*b*) 125. (*b*) 126. (*d*) 127. (*d*) 128. (*b*)
129. (*b*) 130. (*b*) 131. (*d*) 132. (*d*) 133. (*a*) 134. (*c*) 135. (*a*)
136. (*c*) 137. (*b*) 138. (*a*) 139. (*a*) 140. (*d*)

5.4 CALCULUS

5.41 Limits and Continuity

*1. $\underset{x \to -\pi}{\text{Lt}} \left[\sin\left(\frac{\pi}{2}+x\right)+\cos(\pi+x)\right]/x^2=$

(*a*) 0 (*b*) $2/\pi^2$ (*c*) $1/\pi^2$ (*d*) none

2. $\underset{x \to \pi/4}{\text{Lt}} \frac{1-\sec^2 x}{1-\operatorname{cosec}^2 x}=$

(*a*) 2 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) none

3. $\underset{x \to -5}{\text{Lt}} \frac{x^2+3x-10}{x^3+125}=$

(*a*) 7/25 (*b*) 7/125 (*c*) –7/75 (*d*) none

4. $\underset{x \to 3}{\text{Lt}} \frac{x^{-3}-3^{-3}}{x^{-2}-3^{-2}}=$

(*a*) 1 (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) none

5. $\underset{x \to 3}{\text{Lt}} \left[\frac{x}{x-3}-\frac{9}{x^2-3x}\right]=$

(*a*) 0 (*b*) 3 (*c*) 9 (*d*) 2

6. $\underset{x \to 2}{\text{Lt}} \frac{2-x}{\sqrt{3-x}-1}=$

(*a*) $2/\sqrt{3}$ (*b*) 1 (*c*) 2 (*d*) none

7. $\underset{x \to \infty}{\text{Lt}} (\sqrt{x^2+3x}-x)=$

(*a*) 3 (*b*) $\sqrt{3}$ (*c*) $\sqrt{3}/2$ (*d*) 3/2

$= \frac{3}{2}$

8. $\underset{x \to 1}{\text{Lt}} \frac{2x^2-x-1}{x^2+x-2}=$

$\frac{(x-1)(2x+1)}{(x-1)(x+2)}$ $\frac{3}{3} = 1$

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 0 (*d*) 4

9. $\underset{x \to 0}{\text{Lt}} \frac{\operatorname{cosec} x-\cot x}{x}=$

$\frac{1-\cos x}{\sin x}$ $= \frac{1}{2}$

(*a*) $\frac{1}{2}$ (*b*) 0 (*c*) 1 (*d*) none

10. $\underset{x \to 0}{\text{Lt}} \frac{x}{\sqrt{1-\cos x}}=$

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) $\sqrt{2}$ (*d*) none

11. $\lim\limits_{x\to 3} \dfrac{x^3-27}{\sqrt{x^2+7}-4} =$

(*a*) 6 (*b*) 36 (*c*) 0 (*d*) none

$\dfrac{(x-3)(x^2+3x+9)(\sqrt{x^2+7}+4)}{x^2+7-16}$

$(x^2+3x+9)(\sqrt{x^2+7}+4)$

12. $\lim\limits_{x\to 4} \dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}} =$

(*a*) $\frac{1}{3}$ (*b*) $-\frac{1}{3}$ (*c*) 1 (*d*) none

$\left(\dfrac{4+x}{4+x}\right)\left(\dfrac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right) = -\dfrac{4}{4}\cdot\dfrac{2}{6} = -\dfrac{1}{3}$

$\dfrac{x+3}{\ldots} = 36$

13. $\lim\limits_{n\to\infty} \dfrac{1+2+3+\ldots+n}{n^2+1} =$

(*a*) 1 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) none

$\dfrac{n^2+n}{2n^2+2} = \dfrac{1}{2}$

14. $\lim\limits_{x\to 0} \dfrac{x\sin x}{1-\cos x} =$

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) none

$\dfrac{x^2 \cdot \frac{\sin x}{x}}{1-\cos x} \quad 2 \ldots = 2$

15. $\lim\limits_{x\to 0} \dfrac{x^2+x}{x+\sin x} =$

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) none

$\dfrac{x(x+1)}{x\left(1+\frac{\sin x}{x}\right)} \quad \dfrac{x+1}{2} = \dfrac{1}{2}$

16. $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+1}\right)^x =$

(*a*) 1 (*b*) *e* (*c*) 1/*e* (*d*) none

$\left(\dfrac{x}{x\left(1+\frac{1}{x}\right)}\right)^x \quad \dfrac{1}{\left(1+\frac{1}{x}\right)^x} = \dfrac{1}{e}$

17. $\lim\limits_{x\to\pi} \dfrac{1+\cos x}{\tan^2 x} =$

(*a*) 2 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) none

$\dfrac{(1+\cos x)\cos^2 x}{(1+\cos x)(1-\cos x)} = \dfrac{\cos^2 x}{1-\cos x} = \dfrac{1}{2}$

18. $\lim\limits_{x\to -1} \dfrac{1+x^{\frac{1}{7}}}{1+x^{\frac{1}{5}}} =$

(*a*) 1 (*b*) 7/5 (*c*) 5/7 (*d*) none

19. $\lim\limits_{x\to \pi/4} \dfrac{1-\tan x}{\pi/4-x} =$

(*a*) 2 (*b*) -2 (*c*) 1 (*d*) -1

$\dfrac{1-\tan\left(\frac{\pi}{4}-\theta\right)}{\theta}$

$1-\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\theta}$

$\dfrac{1+\tan\theta-1+\tan\theta}{1+\tan\theta}$

$\dfrac{2\tan\theta}{\theta} \cdot \dfrac{1}{1+\tan\theta}$

$2\cdot\dfrac{1}{1+0}$

20. $\lim\limits_{x\to 0} \dfrac{\sin x-\sin 3x}{\sin 3x-\sin 4x} = \dfrac{-2\cos 2x\sin x}{-2\cos\left(\frac{7x}{2}\right)\sin\left(\frac{x}{2}\right)}$

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) none

$\dfrac{\sin x}{x} \ldots = 2$

21. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{2x+1} =$

(*a*) $\frac{1}{2}$ (*b*) 2 (*c*) $\sqrt{2}$ (*d*) 0

$\dfrac{1+x-1+x}{\ldots} = \dfrac{2x}{x\left(1+\frac{1}{2x}\right)\left(\sqrt{1+x}+\sqrt{1-x}\right)}$

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22. $\underset{x \to 2}{\text{Lt}} \dfrac{x^2+3x-10}{3x^2-5x-2} =$

(*a*) $\frac{1}{3}$ (*b*) 5 (*c*) 1 (*d*) none

23. $\underset{x \to 0}{\text{Lt}} \dfrac{x \sin 3x}{1-\cos x} =$

(*a*) 3 (*b*) $\frac{3}{2}$ (*c*) 6 (*d*) none

24. $\underset{x \to 0}{\text{Lt}} \dfrac{x \sin 2x}{\tan^2 3x} =$

(*a*) $\frac{2}{3}$ (*b*) $\frac{4}{3}$ (*c*) $\frac{1}{6}$ (*d*) $\underline{\frac{2}{9}}$

*25. $\underset{x \to 0}{\text{Lt}} \; x \cot x =$

(*a*) 0 ✓(*b*) 1 (*c*) dosen't exist (*d*) none

26. $\underset{x \to 0}{\text{Lt}} \dfrac{1-\cos nx}{1-\cos mx} =$

(*a*) m/n (*b*) n/m (*c*) m^2/n^2 (*d*) n^2/m^2

27. $\underset{x \to 2}{\text{Lt}} \dfrac{\sqrt{x^2+x-3}-\sqrt{x+1}}{x-2} =$

(*a*) $\sqrt{3}$ (*b*) $\sqrt{3}/2$ ✓(*c*) $2/\sqrt{3}$ (*d*) none

28. $\underset{x \to \infty}{\text{Lt}} \dfrac{2x+3}{\sqrt{3x^2-1}} =$

(*a*) $\sqrt{3}$ (*b*) $1/\sqrt{3}$ (*c*) 0 (*d*) none.

29. $\underset{x \to 0}{\text{Lt}} \dfrac{\tan x - x}{2x - \sin x} =$

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) 0

*30. $\underset{x \to 0}{\text{Lt}} \dfrac{\sin 3x - \sin 5x}{x} =$

(*a*) 0 (*b*) 2 (*c*) −2 (*d*) none

31. $\underset{n \to \infty}{\text{Lt}} \; n\left(\sqrt{n^2+8}-n\right) =$

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

*32. $\underset{x \to -2}{\text{Lt}} \dfrac{x^3+6x^2+11x+6}{x^2+3x+2} =$

(*a*) 3 (*b*) −2 (*c*) 1 (*d*) none

33. $\underset{x \to 0}{\text{Lt}} \dfrac{\sec x - 1}{\tan^2 x} =$

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) $\frac{1}{2}$

34. $\underset{x \to 0}{\text{Lt}} \tan 5x \cot 3x =$

(*a*) 15 (*b*) $\frac{1}{15}$ (*c*) $\frac{5}{3}$ (*d*) $\frac{3}{5}$

35. $\underset{x \to \pi/4}{\text{Lt}} \dfrac{\cos x - \sin x}{\pi/4 - x} =$

(*a*) $2\sqrt{2}$ (*b*) $-\sqrt{2}$ (*c*) $\sqrt{2}$ (*d*) none

36. $\underset{x \to 0}{\text{Lt}} \dfrac{\cos 3x - \cos x}{x^2} =$

(*a*) 4 (*b*) –4 (*c*) 2 (*d*) none

37. $\underset{x \to a}{\text{Lt}} \dfrac{\cos x - \cos a}{x - a} =$

(*a*) sin *a* (*b*) –sin *a* (*c*) $\frac{1}{2}$ sin *a* (*d*) none

38. $\underset{x \to 0}{\text{Lt}} \dfrac{\sin^2 \frac{x}{4}}{x^2} =$

(*a*) 4 (*b*) $\frac{1}{4}$ (*c*) 16 (*d*) $\frac{1}{16}$

39. $\underset{x \to \infty}{\text{Lt}} \dfrac{(x+1)(x+2)}{x^2(x-3)} =$

(*a*) 0 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) 1

40. $\underset{x \to 0}{\text{Lt}} \dfrac{\cos 2x - \cos 3x}{\cos 3x - \cos 4x} =$

(*a*) 5/7 (*b*) 1 (*c*) –1 (*d*) none

41. If $f(x) = 3x + x^2 + 7$, $\underset{h \to 0}{\text{Lt}} \dfrac{f(x+h) - f(x)}{h} =$

(*a*) 3 (*b*) $2x+3$ (*c*) 0 (*d*) none

42. The function $\dfrac{x^2 - x - 6}{x - 3}$ is continuous at

(*a*) $x > 0$ (*b*) $x = 3$ (*c*) $x \neq 3$ (*d*) any real value of *x*

43. The function $\sqrt{x-1}$ is continuous at

(*a*) $x = 0$ (*b*) $x = -2$ (*c*) $x = 1$ (*d*) all of them

44. $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x} =$

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) 1

45. $\underset{x \to \infty}{\text{Lt}} (\sqrt{n^2+1} - n) =$ n

(*a*) $\frac{1}{2}$ (*b*) $1/\sqrt{2}$ (*c*) $\sqrt{2}$ (*d*) 0

46. $\underset{x \to 0}{\text{Lt}} \dfrac{\sin 3x \sin 4x}{\sin 5x} =$

(*a*) $\frac{5}{12}$ (*b*) $\frac{12}{5}$ (*c*) 0 (*d*) none

47. $\underset{x \to 1}{\text{Lt}} \dfrac{x^m - 1}{x^n - 1} =$

(*a*) $\dfrac{m}{n}$ (*b*) $\dfrac{n}{m}$ (*c*) 1 (*d*) 0

48. $\underset{x \to 1}{\text{Lt}} \dfrac{f(x) - f(1)}{x - 1}$, where $f(x) = \sqrt{x^2+1}$, is

(*a*) 2 (*b*) $\sqrt{2}$ (*c*) $1/\sqrt{2}$ (*d*) none

49. The function which is not continuous at $x=0$ is

(*a*) $\dfrac{\sin x}{x}$ (*b*) $\dfrac{|x|}{x}$ (*c*) $\dfrac{x^2+x}{x^2-x}$ (*d*) all of them

50. $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{1+x} - 1}{x} =$

[✓ (*a*) $\frac{1}{2}$] (*b*) 2 (*c*) $\sqrt{2}$ (*d*) 0

51. The function $|x|$ is continuous for

(*a*) all x (*b*) $x \neq 0$ (*c*) $x > 0$ (*d*) $x < 0$

***52.** Which of the following functions is NOT continuous for $x<0$?

(*a*) $\log|x|$ (*b*) e^x (*c*) $\tan x$ (*d*) none

53. $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{x} =$

[✓ (*a*) 1] (*b*) $\sqrt{2}$ (*c*) 0 (*d*) $\frac{1}{2}$

54. $\underset{x \to 0}{\text{Lt}} \dfrac{x}{2 - \sqrt{4-x}} =$

(*a*) 2 (*b*) 0 [✓ (*c*) 4] (*d*) none

55. $\underset{h \to 0}{\text{Lt}} \dfrac{(x+h)^2 - x^2}{h} =$

(*a*) 1 (*b*) x (*c*) 0 [✓ (*d*) $2x$]

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56. $$\underset{n\to\infty}{\text{Lt}} \frac{\sqrt{2n^2+3n}-1}{n+1}=$$

(a) 0 (b) 2 (c) ∞ (d) $\sqrt{2}$

57. $$\underset{n\to\infty}{\text{Lt}} \left(1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}\right)=$$

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

58. $$\underset{x\to\pi}{\text{Lt}} \frac{\cos 3x+1}{\cos 2x+1}=$$

(a) 1 (b) $\frac{9}{4}$ (c) $-\frac{9}{4}$ (d) -1

59. $$\underset{x\to 0}{\text{Lt}} \frac{(1+\cos x)(1-\cos 2x)}{x^2}=$$

(a) 2 (b) 4 (c) 1 (d) none

60. $$\underset{x\to\infty}{\text{Lt}} \left(1-\frac{1}{x}\right)^{-x}=$$

(a) e^{-1} (b) e (c) e^{-x} (d) e^x

61. $$\underset{\theta\to 0}{\text{Lt}} \frac{\sin^2 2\theta}{\tan^2 3\theta}=$$

(a) $\frac{2}{3}$ (b) $\frac{8}{2}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

62. $$\underset{n\to\infty}{\text{Lt}} \frac{1^3+2^3+3^3+\ldots+n^3}{4n^3+3n^2+7n+10}$$

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{1}{12}$ (d) none

63. $$\underset{x\to a}{\text{Lt}} \frac{\sin x-\sin a}{\sqrt[3]{x}-\sqrt[3]{a}}=$$

(a) $2\cos a$ (b) $3a^{2/3}\cos a$ (c) $\frac{1}{3}a^{1/3}\cos a$ (d) none

64. $$\underset{h\to 0}{\text{Lt}} (1+h)^{2/h}=$$

(a) e (b) 1 (c) e^2 (d) $\sqrt{e}$

65. $$\underset{x\to 0}{\text{Lt}} \frac{5^x-4^x}{x}=$$

(a) 1 (b) $\log 4-\log 5$ (c) $\log 5-\log 4$ (d) none

66. $$\underset{x\to 3}{\text{Lt}} \frac{x-3}{\sqrt{x-2}-\sqrt{4x-11}}=$$

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{4}{3}$ (d) none

67. $\underset{x\to a}{\text{Lt}} \dfrac{\tan x-\tan a}{x-a}=$

(*a*) $\dfrac{\tan a}{a}$ (*b*) 0 (*c*) $\sec a$ (*d*) $\sec^2 a$

68. $\underset{h\to 0}{\text{Lt}} \dfrac{(1+h)^{-1}-1}{h}=$

(*a*) -1 (*b*) 1 (*c*) 0 (*d*) none

69. $\underset{x\to \frac{1}{2}}{\text{Lt}} \dfrac{16x^4-1}{8x^3-1}=$

(*a*) $\frac{1}{2}$ (*b*) 2 (*c*) 0 (*d*) none

70. $\underset{x\to -a}{\text{Lt}} \dfrac{x^4+a^4}{x+a}=$

(*a*) $4a^3$ (*b*) 4 (*c*) 0 (*d*) none

Answers

1. (*a*) 2. (*b*) 3. (*c*) 4. (*c*) 5. (*d*) 6. (*c*) 7. (*d*) 8. (*b*) 9. (*a*) 10. (*c*) 11. (*b*) 12. (*b*) 13. (*c*) 14. (*c*) 15. (*a*) 16. (*c*) 17. (*d*) 18. (*c*) 19. (*a*) 20. (*a*) 21. (*d*) 22. (*c*) 23. (*c*) 24. (*d*) 25. (*b*) 26. (*d*) 27. (*c*) 28. (*d*) 29. (*d*) 30. (*c*) 31. (*b*) 32. (*c*) 33. (*d*) 34. (*c*) 35. (*c*) 36. (*b*) 37. (*b*) 38. (*d*) 39. (*c*) 40. (*a*) 41. (*b*) 42. (*c*) 43. (*c*) 44. (*a*) 45. (*d*) 46. (*c*) 47. (*a*) 48. (*c*) 49. (*d*) 50. (*a*) 51. (*a*) 52. (*c*) 53. (*a*) 54. (*c*) 55. (*d*) 56. (*d*) 57. (*c*) 58. (*d*) 59. (*b*) 60. (*b*) 61. (*c*) 62. (*d*) 63. (*b*) 64. (*c*) 65. (c) 66. (*a*) 67. (*d*) 68. (*a*) 69. (*d*) 70. (*d*).

5.42 Differentiation

1. If $f(x)=2\cos x-3\sin x+x^2-2x+5$, then $f'(0)=$

(*a*) 0 (*b*) -5 (*c*) -4 (*d*) none

2. The derivative of $\sin^{-1}(e^x)+\cos^{-1}(e^x)$ w.r.t. x is

(*a*) $\dfrac{2}{\sqrt{1-e^{2x}}}$ (*b*) $\dfrac{2}{\sqrt{1-e^{x}}}$ (*c*) 0 (*d*) $\pi/2$

3. The derivative of $3^{-x}+x^{-3}+3^3$ w.r.t. x is

(*a*) $3^{-x}\log 3+3x^2$ (*b*) $3^x \log 3-3x^{-4}$

(*c*) $3^{-x}\log 3-3x^{-2}$ (*d*) $-3^{-x}\log 3-3x^{-4}$

4. $\dfrac{d}{dx}\left[\cot\left(\cot^{-1}\dfrac{2}{x}\right)\right]=$

(*a*) $\dfrac{2}{x}$ (*b*) $\operatorname{cosec}^2\left(\dfrac{2}{x}\right)\cdot\dfrac{1}{\sqrt{1-4/x^2}}$ (*c*) $-2/x^2$ (*d*) none

5. If $y=4^{e^x}$, then $\dfrac{dy}{dx}=$

(*a*) $4^{e^x}\log 4$ (*b*) $4^{e^x}e^x$ (*c*) $4^{e^x}e^x\log x$ (*d*) $4^{e^x}e^x\log 4$

6. If $y=\log(\log\sin x)$, $\dfrac{dy}{dx}=$

(*a*) $\dfrac{1}{\sin x\log\sin x}$ (*b*) $\dfrac{\cos x}{\log\sin x}$ (*c*) $\dfrac{\tan x}{\log\sin x}$ (*d*) $\dfrac{\cot x}{\log\sin x}$

7. If $y=x^x$, then $\dfrac{dy}{dx}=$

(*a*) x^x (*b*) $x^x\log ex$ (*c*) $x^x\log(x)$ (*d*) none

8. If $x+e^x=y+e^y$, then $\dfrac{dy}{dx}$ when $x=0=y$ is

(*a*) 2 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 0

9. Which of the following functions $f(x)$ satisfies $f''(x)=f(x)$

(*a*) $\cos x$ (*b*) e^{-x} (*c*) e^{2x} (*d*) $e^{\log x}$

10. If $y=\log\cos\sqrt{x}$, $\dfrac{dy}{dx}=$

(*a*) $\dfrac{\tan\sqrt{x}}{2\sqrt{x}}$ (*b*) $-\dfrac{\tan\sqrt{x}}{2\sqrt{x}}$ (*c*) $\dfrac{1}{\cos\sqrt{x}}$ (*d*) $\tan\sqrt{x}$

11. If $y=x^2e^x$, then $y_2=$

(*a*) $(x^2+2x+2)e^x$ (*b*) $(x^2+4x+2)e^x$

(*c*) $(x^2+4x+4)e^x$ (*d*) none

12. If $\cot y=\sqrt{\dfrac{1-\cos x}{1+\cos x}}$, then $\dfrac{dy}{dx}=$

(*a*) -1 (*b*) $-\frac{1}{2}$ (*c*) $\dfrac{1}{1+x^2}$ (*d*) none

13. If $y=\dfrac{x^2+1}{x^2+2}$, then $(x^2+2)^2\dfrac{dy}{dx}=$

(*a*) x (*b*) $4x^2$ (*c*) $2x$ (*d*) none

14. If $y^2+3x=25$, then $\dfrac{d^2y}{dx^2}=$

(*a*) $\dfrac{-3}{y^3}$ (*b*) $-\dfrac{9}{4y^3}$ (*c*) $-\dfrac{3}{2y^3}$ (*d*) none

*15. If $y = e^{-5}$, $\frac{d^2y}{dx^2} =$

(*a*) $25e^{-5}$ (*b*) $-25e^{-5}$ (*c*) $5e^{-5}$ (*d*) none

16. If $y = \frac{x}{\sqrt{1-x}}$, $\frac{dy}{dx} =$

(*a*) $\frac{1-x}{2(1-x)^{3/2}}$ (*b*) $\frac{1+x}{(1-x)^{3/2}}$ (*c*) $\frac{2-x}{2(1-x)^{3/2}}$ (*d*) $\frac{2-x}{(1-x)^{3/2}}$

17. If $y = \log_x^e$ then $\frac{dy}{dx} =$

(*a*) $-\frac{\log_x^e}{(\log x)^2}$ (*b*) $-\frac{1}{(\log x)^2}$ (*c*) $-\frac{1}{x(\log x)^2}$ (*d*) none

*18. The second derivative of $x^2 \log x + x + 5$ w.r.t. x. is

(*a*) $3 + \log x$ (*b*) $2 + 3 \log x$ (*c*) $3 + 2 \log x$ (*d*) none

19. The derivative of $(x^2+1)/(x^2-1)$ w.r.t. x is

(*a*) $\frac{-2x}{(1-x^2)^2}$ (*b*) $\frac{4x}{(x^2-1)^2}$ (*c*) $\frac{-4x}{(1-x^2)^2}$ (*d*) none

*20. If $x^3 + y^3 = 3xy + 5$, then $\frac{dy}{dx}$ is

(*a*) $\frac{x^2-y}{x-y^2}$ (*b*) $\frac{y-x^2}{x-y^2}$ (*c*) $\frac{x-y^2}{x^2-y}$ (*d*) none

21. If $y = \sec^2(\sec^{-1} x)$, then $\frac{dy}{dx} =$

(*a*) $2 \sec(\sec^{-1}x) \frac{1}{x\sqrt{x^2-1}}$ (*b*) $2x$

(*c*) $2 \sec^2(\sec^{-1}x) \tan(\sec^{-1}x)$ (*d*) none

22. If $x = \cos^3 t$, $y = \sin^3 t$, then $\frac{dy}{dx}$ at $t = \pi/4$ is equal to

(*a*) -1 (*b*) 1 (*c*) $1/2$ (*d*) $-1/2$

*23. If $y = \cos^{-1}\sqrt{x}$, $\frac{dy}{dx} =$

(*a*) $\frac{-1}{\sqrt{1-x}}$ (*b*) $\frac{1}{2\sqrt{x}.(1-x^2)}$ (*c*) $-\frac{\sin^{-1}\sqrt{x}}{2\sqrt{x}}$ (*d*) none

24. If $f(x)=\dfrac{1}{4-x}$, then $(4-x)^3 f''(x)=$

(*a*) 8 (*b*) 2 (*c*) x (*d*) none

25. If $f(x)=\sqrt{x^2+a^2}$, then $[f(x)]^3 f''(x)=$

(*a*) x^2+a^2 (*b*) $a^2 x$ (*c*) a^2 (*d*) none

26. If $y=3x+\sin x$, then $\dfrac{d^2y}{dx^2}+y=$

(*a*) $3x$ (*b*) $3x-\sin x$ (*c*) 3 (*d*) none

27. If $y^2=19a^2x$, then $\dfrac{d^2y}{dx^2}$ is inversely proportional to

(*a*) y (*b*) y^2 (*c*) y^3 (*d*) none

28. The second derivative of $e^x \sin x$ w.r.t. x is

(*a*) $2e^x \sin x$ (*b*) $2e^x \cos x$ (*c*) $-2e^x \sin x$ (*d*) $2e^x(\sin x-\cos x)$

29. The second derivative of $\tan x$ w.r.t. $\sin x$ is

(*a*) $\sec^2 x \tan x$ (*b*) $3\sec^3 x \tan x$ (*c*) $\sec^3 x$ (*d*) $3\sec^4 x \tan x$

30. The derivative of $\cot^{-1}\sqrt{\dfrac{1-\cos 4x}{1+\cos 4x}}$ w.r.t. x is

(*a*) $-1/2$ (*b*) $\tan^2 2x$ (*c*) 2 (*d*) none

31. If $xy=4$, $\dfrac{dy}{dx}=$

(*a*) $-y/x$ (*b*) $-4/x^2$ (*c*) $-y^2/4$ (*d*) all of them

32. If $x=a(\cos\theta+\theta\sin\theta)$ and $y=a(\sin\theta-\theta\cos\theta)$, then $\dfrac{dy}{dx}$ at $\theta=3\pi/4$ is (*a*) $-a$ (*b*) a (*c*) 1 (*d*) -1

33. The derivative of $\cos(x^x)$ w.r.t. x is

(*a*) $-\cos(x^x).x^x(\log x+1)$ (*b*) $-\sin(x^x).(\log x+1)$

(*c*) $\sin(x^x)\,x^x.(\log x+1)$ (*d*) none

34. If $y=\dfrac{x+3}{(2-x)^2(5-x)^3}$ and $\dfrac{dy}{dx}=y\left[\dfrac{a}{x+3}+\dfrac{b}{2-x}+\dfrac{c}{5-x}\right]$ then

(*a*) $a=1=b=c$ (*b*) $a=1,\ b=2,\ c=3$

(*c*) $a=1,\ b=-2,\ c=-3$ (*d*) none

35. If $y=5^x$, then $\dfrac{dy}{dx}=$

(*a*) $x5^{x-1}$ (*b*) 5^{x-1} (*c*) $5^x \log 5$ (*d*) $x\log 5$

*36. If $\sin y = \sqrt{x}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2\sqrt{x-x^2}}$ (b) $\frac{1}{\sqrt{1-x}}$ (c) $\frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{x}}$ (d) none

✓37. If $y = \frac{1}{x-1} + \frac{1}{x+1}$ and $y'' = \frac{a}{(x-1)^3} + \frac{b}{(x+1)^3}$, then

(a) $a=1, b=2$ (b) $a=-2, b=2$ (c) $a=-1, b=-1$ (d) none

38. If $f(x) = e^{\log \sin 4x}$, then $f''(x) =$

(a) $e^{\log \sin 4x}(4 \sin 4x - 4 \cos 4x)$ (b) $16 \sin 4x$

(c) $-16 \sin 4x$ (d) none

39. If $3x^2 + 5xy + y^2 = 7$, then $\frac{dy}{dx}$ when $x = 0$ is

(a) $\frac{6}{5}$ (b) $-\frac{5}{2}$ (c) $\frac{5}{6}$ (d) none

*40. If $y = (\log x)^x$, then $\frac{1}{y}\frac{dy}{dx} =$

(a) $\log x + \frac{1}{\log x}$ (b) $(\log x)^x \left[\log(\log x) + \frac{1}{\log x}\right]$

(c) $\log(\log x) + \frac{1}{\log x}$ (d) none

*41. If $y = \sec^{-1}\left(\frac{1}{1-2x^2}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{2}{1+x^2}$ (b) $\frac{-2}{\sqrt{1-x^2}}$ (c) $\frac{2}{\sqrt{1-x^2}}$ (d) $\frac{1}{\sqrt{1-x^2}}$

42. If $y = \sin^{-1}(\tan x) + \tan^{-1}(\sin x)$, then $\frac{dy}{dx} =$

(a) $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}} + \frac{\cos x}{1+\sin^2 x}$ (b) 0 (c) 1 (d) none

43. If $y = \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2(1+x^2)}$ (b) $\frac{-1}{2(1+x^2)}$ (c) $\frac{1}{2\sqrt{1-x^2}}$ (d) $\frac{-1}{2\sqrt{1-x^2}}$

*44. If $y = \sqrt{\frac{2x+3}{3x+2}}$, $\frac{dy}{dx}$ at $x = 0$ is

(a) $\frac{-5}{4\sqrt{6}}$ (b) $\frac{5}{2\sqrt{6}}$ (c) $\frac{-5}{2\sqrt{6}}$ (d) none

45. If $y\sqrt{5-x^2}=x$, then $\frac{dy}{dx}$ at $x=2$ is

(*a*) 0 (*b*) $\sqrt{5}$ (*c*) 5 (*d*) none

46. If $y=x^3\sin 2x$, and $\frac{dy}{dx}=x^2(a\sin 2x+bx\cos x)$, then

(*a*) $a=3, b=2$ (*b*) $a=2, b=-3$ (*c*) $a=1, b=3$ (*d*) $a=2, b=3$

47. If $y=\tan^{-1}3x$, $\frac{dy}{dx}=$

(*a*) $\frac{1}{\sqrt{1-9x^2}}$ (*b*) $\frac{1}{1+9x^2}$ (*c*) $\frac{3}{\sqrt{1-9x^2}}$ (*d*) $\frac{3}{1+9x^2}$

48. If $y=2\sin 4x\cos x$, then $y''=$

(*a*) $25\sin 5x+9\sin 3x$ (*b*) $-25\sin 5x+9\sin 3x$

(*c*) $-25\sin 5x-9\sin 3x$ (*d*) none

49. If $y=x^2\sqrt{x^2+4}$, then $\sqrt{x^2+4}\,\frac{dy}{dx}=$

(*a*) $3x^2+8x$ (*b*) $3x^3+8$ (*c*) $3x^3+8x$ (*d*) none

50. If $y=\sqrt{e^x}$, then $\frac{d^2y}{dx^2}=$

(*a*) $\frac{1}{4}\sqrt{e^x}$ (*b*) $\frac{1}{4}e^x$ (*c*) $\frac{1}{2}e^{x/2}$ (*d*) none

51. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t^2}{1+t^2}$, then $\frac{d^2y}{dx^2}=$

(*a*) $\frac{1-t^2}{(1+t^2)^2}$ (*b*) $\frac{2t^2}{(1+t^2)^2}$ (*c*) 0 (*d*) -1

52. The derivative of $\frac{a^2}{2}\sin^{-1}\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x^2}$ w.r.t. x is

(*a*) $-\sqrt{a^2-x^2}$ (*b*) $\frac{1}{\sqrt{a^2-x^2}}$ (*c*) $\frac{-1}{\sqrt{a^2-x^2}}$ (*d*) $\sqrt{a^2-x^2}$

53. The derivative of $\tan^3(x^2)$ w.r.t. x is

(*a*) $3x\tan^2(x^2)\sec^2(x^2)$ (*b*) $6x\tan^2(x^2)\sec^2(x^2)$

(*c*) $6\tan^2(x^2)\sec^2(x^2)$ (*d*) none

54. If $e^{x+y}=e^x+e^y$, then $\frac{dy}{dx}=$

(*a*) $1-e^y$ (*b*) $-e^{y-x}$ (*c*) $\frac{e^x(e^y-1)}{e^y(1-e^x)}$ (*d*) all of them

55. If $y=\cos^{-1}x$ and $(1-x^2)y''+f(x)y'+g(x)y=0$, then
(*a*) $f(x)=x$, $g(x)=0$ (*b*) $f(x)=-x$, $g(x)=0$
(*c*) $f(x)=1=g(x)$ (*d*) none

56. If $y=\dfrac{1+\tan^2 x}{1-\tan^2 x}$, then $\dfrac{dy}{dx}=$
(*a*) $2\sec 2x\tan 2x$ (*b*) $2\cos 2x$ (*c*) $\sec 2x$ (*d*) none

57. If $f(x)=e^{e^{e^x}}$, then $f'(0)=$
(*a*) 1 (*b*) e^e (*c*) e^{e^e} (*d*) e^{e+1}

58. If If $y=\sin^{-1}[\cos(\sin x)]$, then $\dfrac{dy}{dx}=$
(*a*) $-\sin x$ (*b*) $\cos x$ (*c*) $-\cos x$ (*d*) none

59. If $y=\sin^2 x$, $\dfrac{d^2y}{dx^2}=$
(*a*) $\sin 2x$ (*b*) $-2\cos 2x$ (*c*) $2\cos 2x$ (*d*) $2\sin 2x$

60. If $y=x\log x$, then $y_2=$
(*a*) $1+1/x$ (*b*) $1/x$ (*c*) $1+\log x$ (*d*) none

61. If $f(x)=(x^3+3x^2+1)(x^2+2)$, then $f'(-1)=$
(*a*) -15 (*b*) 37 (*c*) 0 (*d*) none

62. If $y=\sec x+\tan x$, then $\dfrac{1}{y}\dfrac{dy}{dx}=$
(*a*) $\sec x$ (*b*) $\tan x$ (*c*) $\cos x$ (*d*) none

63. If $x=e^{2t}+e^{-2t}$ and $y=e^{2t}-e^{-2t}$, then $\dfrac{dy}{dx}=$
(*a*) x/y (*b*) y/x (*c*) $-x/y$ (*d*) none

64. If $y=\tan^{-1}(\sin x)+\tan^{-1}(\operatorname{cosec} x)$, then $\dfrac{dy}{dx}=$
(*a*) $-\dfrac{2}{1+\sin^2 x}$ (*b*) $\dfrac{4}{1+\sin^2 x}$ (*c*) 0 (*d*) none

65. If $y=\dfrac{\cos 2x}{\sin x}$, then $\sin^2 x\, y'=$
(*a*) $-2(\sin 2x\sin x+\cos 2x\cos x)$ (*b*) $2\sin 2x\sin x-\cos 2x\cos x$
(*c*) $-2\sin 2x\sin x+\cos 2x\cos x$ (*d*) none

66. If $y=\sqrt{x+\sqrt{x}}$, then $\frac{dy}{dx}$ at $x=1$ is

(a) $\frac{1}{4\sqrt{2}}$ (b) $\frac{3}{4\sqrt{2}}$ (c) 3/4 (d) none

67. If $y=\cos^{-1}\sqrt{\frac{1-x}{2}}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{2\sqrt{1-x^2}}$ (b) $\frac{-1}{2\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{-1}{\sqrt{1-x^2}}$

68. If $x=2at$ and $y=at^2$, then $\frac{d^2y}{dx^2}=$

(a) $\frac{-1}{2at^3}$ (b) 1 (c) $\frac{1}{2at^3}$ (d) none

69. If $y=\frac{\sin 2x}{1+\cos 2x}$, then $\frac{d^2y}{dx^2}=$

(a) $2\sec^3 x$ (b) $2\sec^2 x\tan x$ (c) $2\sec x\tan x$ (d) none

70. If $y=x\sin y$, $\frac{dy}{dx}=$

(a) $\frac{\sin y}{1-x\cos y}$ (b) $\frac{\cos y}{1-x\sin y}$ (c) $1+\cos y$ (d) none

71. If $f(x)=xe^x$, then $f''(0)=$

(a) 0 (b) 1 (c) 2 (d) none

72. If $y=(x-2)^2(x-3)^2$ and $\frac{dy}{dx}=(x-2)(x-3)(ax+b)$, then $(a, b)=$

(a) (4, –6) (b) (4, –10) (c) (2, 10) (d) none

73. If $y=x^{2x}$ and $\frac{dy}{dx}=yf(x)$, then $f(x)$

(a) $\log x+2$ (b) $2(x+\log x)$ (c) $2(1+\log x)$ (d) none

74. If $y=(2\cos^2 x-1)/(1-2\sin^2 x)$, then $\frac{dy}{dx}=$

(a) $2\sin 2x$ (b) 1 (c) 0 (d) none

75. If $y=\frac{1}{\sqrt{x+1}-\sqrt{x+2}}$, then $\frac{dy}{dx}=$

(a) $-\left[\frac{1}{\sqrt{x+1}}+\frac{1}{\sqrt{x+2}}\right]$ (b) $-\frac{1}{2}\left[\frac{1}{\sqrt{x+1}}+\frac{1}{\sqrt{x+2}}\right]$

(c) $-\frac{1}{2}\left[\frac{1}{\sqrt{x+1}}-\frac{1}{\sqrt{x+2}}\right]$ (d) none

76. If $x=a\cos\log t$, $y=a\sin\log t$, then $\dfrac{dy}{dx}=$

(a) x/y (b) $-x/y$ (c) y/x (d) $-y/x$

* **77.** If $y=\cos^{-1}[\sqrt{1-x^2}\tan(\sin^{-1}x)]$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{-1}{1-x^2}$ (b) $\dfrac{-1}{\sqrt{1-x^2}}$ (c) 0 (d) none

78. Which of the following is $\cosh x$?

(a) $\frac{1}{2}(e^{-x}+e^{x})$ (b) $\frac{1}{2}(e^{-x}-e^{x})$ (c) $\frac{1}{2}(e^{x}-e^{-x})$ (d) none

79. If $f(x)=\sinh x^2-\cosh\sqrt{x}$, then $f'(x)=$

(a) $-2x\cosh x^2+\sinh\sqrt{x}\,.\,1/2\sqrt{x}$ (b) $2x\cosh x^2-\sinh\sqrt{x}\,.1/2\sqrt{x}$ (c) $2x\cosh x^2+\sinh 2x.1/2\sqrt{x}$ (d) none

80. If $y=\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)$, then $\dfrac{dy}{dx}=$ ✓

(a) $\dfrac{3a}{a^2+x^2}$ (b) $\dfrac{3}{a^2+x^2}$ (c) $\dfrac{3a}{1+x^2}$ (d) none

* **81.** If $x=a\cos^3\theta$ and $y=a\sin^3\theta$, then $\dfrac{d^2y}{dx^2}$ at $\theta=\pi/4$ is

(a) $\dfrac{8\sqrt{2a}}{3}$ (b) $\dfrac{4\sqrt{2a}}{3}$ (c) $\dfrac{4\sqrt{2}}{3a}$ (d) none

82. If $x=a\sin^2 t$, $y=a\cos^2 t$, then $\dfrac{dy}{dx}=$

(a) $\tan t$ (b) $\cot t$ (c) 1 (d) -1

83. If $y=\dfrac{x}{\sqrt{x^2+1}}$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{1}{2\sqrt{x^2+1}}$ (b) $\dfrac{1}{(x^2+1)^{3/2}}$ (c) $\dfrac{-1}{(x^2+1)^{3/2}}$ (d) none

84. If $y=\dfrac{a\sin x+b}{a+b\sin x}$, then $(a+b\sin x)^2y'=$

(a) $(a^2+b^2)\cos x$ (b) a^2-b^2 (c) $(a^2-b^2)\cos x$ (d) none

85. If $y=(\log x)^{\log x}$, then $\dfrac{1}{y}\dfrac{dy}{dx}=$

(a) $(1+\log x)/x$ (b) $[1+\log(\log x)]/x$
(c) $(x+\log x)/x$ (d) none

86. If $y=(\sec^2x-\tan^2x)(\text{cosec}^2x-1)$, then $\frac{dy}{dx}=$

(a) $-\text{cosec}^4x$ (b) $2\cot^2x\,\text{cosec}^2x$ (c) $-2\cot x\,\text{cosec}^2x$ (d) none

87. If $y=\frac{x+a^2}{\sqrt{x^2+a^2}}$, then $(x^2+a^2)^{3/2}\,y'=$

(a) x^2-a^2 (b) a^2x+a^2 (c) a^2x-a^2 (d) a^2-a^2x

88. If $f(x)=\cosh^3(2x)$, then $f''(x)=$

(a) $12\cosh^3 2x+24\cosh 2x\sinh^2 2x$
(b) $12\cosh^2 2x-12\cosh 2x\sinh^2 2x$
(c) $12\cosh^2 2x+24\cosh 2x\sinh^2 2x$ (d) none

89. If $y=\sec^{-1}\left(\frac{\sqrt{4+x^2}}{2}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{x\sqrt{x^2-1}}$ (c) $\frac{2}{4+x^2}$ (d) none

90. If $y=xe^x/(1+x^2)$, then $\frac{dy}{dx}(1+x^2)^2=$

(a) $e^x(x^2+x-1)$ (b) $e^x(x^3-x^2+x+1)$
(c) $e^x(x^3+3x^2-6x+1)$ (d) none

91. If $y=e^x\sin x\log x$, then $\frac{dy}{dx}=$

(a) $e^x\left(\sin x\log x+\cos x\log x+\frac{\sin x}{x}\right)$

(b) $e^x\left(\log x+\frac{\sin x}{x}+\sin x\log x\right)$

(c) $e^x\left(-\sin x\log x+\cos x\log x+\frac{\sin x}{x}\right)$ (d) none

92. If $f(x)=\cos^7 7x$, then $f'(0)=$

(a) 49 (b) 7 (c) 0 (d) none

93. If $y=\sin^2 4x$, then $\frac{d^2y}{dx^2}=$

(a) $8\cos 8x$ (b) $32\sin 8x$ (c) $32\cos 8x$ (d) none

94. If $y=\frac{1}{6}\tan^{-1}\frac{2x}{3}$, then $\frac{d^2y}{dx^2}=$

(a) $\frac{1}{\underline{4x^2+9}}$ (b) $\frac{8x}{4x^2+9}$ (c) $\frac{8x}{(4x^2+9)^2}$ (d) $\frac{-8x}{(4x^2+9)^2}$

95. If $y=\frac{\sin^2 x}{\sin 2x}-\frac{\cos^2 x}{\sin 2x}$ then $\frac{dy}{dx}=$

(*a*) $\cot 2x$ (*b*) $-\text{cosec}^2 2x$ (*c*) $-2\,\text{cosec}^2 2x$ (*d*) $2\,\text{cosec}^2 2x$

96. If $y=\cos^{-1}x+\text{cosec}^{-1}\frac{1}{x}$, then $\frac{dy}{dx}$ when $x=1$ is

(*a*) 2 (*b*) 0 (*c*) -2 (*d*) none

97. If $\cos^{-1}2x+\cos^{-1}y=\pi$, then $\frac{d^2y}{dx^2}=$

(*a*) 2 (*b*) -2 (*c*) 0 (*d*) none

98. If $y=\cos^{-1}\left(\log\frac{1}{x}\right)$, then $\frac{dy}{dx}=$

(*a*) $\frac{-1}{x^2\sqrt{1-(\log x)^2}}$ (*b*) $\frac{1}{x^2\sqrt{1-(\log x)^2}}$

(*c*) $-\frac{1}{x\sqrt{1-(\log x)^2}}$ (*d*) $\frac{1}{x\sqrt{1-(\log x)^2}}$

99. If $y=\text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}$, then $\frac{dy}{dx}=$

(*a*) $-\frac{1}{x\sqrt{x^2-1}}$ (*b*) $\frac{2}{1+x^2}$ (*c*) $-\frac{1}{1+x^2}$ (*d*) none

100. Which of the following is NOT true?

(*a*) $\cosh^2 x-\sinh^2 x=1$ (*b*) $\tanh x\ \coth x=1$

(*c*) $\coth x=\frac{\cosh x}{\sinh x}$ (*d*) none

101. If $y=\tan\left(\frac{x+2}{x+1}\right)$, then $(1+y^2)\frac{dy}{dx}=$

(*a*) $-\frac{1}{(x+1)^2}$ (*b*) $\frac{1}{(x+1)^2}$ (*c*) $\sec^2\left(\frac{x+2}{x+1}\right)\cdot\frac{1}{1+x^2}$ (*d*) none

102. If $\frac{x}{y}+\frac{y}{x}+2=0$, then $\frac{d^2y}{dx^2}=$

(*a*) 1 (*b*) -1 (*c*) 0 (*d*) none

103. If $y=\sqrt{\sin x}$, $\frac{dy}{dx}=$

(*a*) $\frac{\cos x}{2\sqrt{\sin x}}$ (*b*) $\frac{\sin x}{2\sqrt{\sin x}}$ (*c*) $\frac{1}{2\sqrt{\sin x}}$ (*d*) $\sqrt{\cos x}$

104. The derivative of $e^{\sin x}$ w.r.t. $e^{\cos x}$ is

(*a*) $-\cot x\, e^{\sin x-\cos x}$ (*b*) $\tan x\, e^{\cos x-\sin x}$

(*c*) $\cot x\, e^{\cos x-\sin x}$ (*d*) none

105. If $y=x^y$, then $(1-y\log x)\dfrac{dy}{dx}=$

(*a*) y/x (*b*) y^2 (*c*) x/y (*d*) y^2/x

106. The derivative of $e^{-x}\sin x$ w.r.t. x is

(*a*) $e^{-x}(\sin x-\cos x)$ (*b*) $e^{-x}(\cos x-\sin x)$

(*c*) $e^{-x}(\sin x+\cos x)$ (*d*) none

107. If $y=2^x.e^x$, then $\dfrac{dy}{dx}=2^x.e^x f(x)$ where $f(x)$ is

(*a*) 1 (*b*) $\log(2e)$ (*c*) $\log 2$ (*d*) none

108. If $y=\cos^3(x^2)$, then $\dfrac{dy}{dx}$ at $x=\sqrt{\pi}$ is

(*a*) not defined (*b*) 0 (*c*) -1 (*d*) 1

109. If $y=x\cosh x-\sinh x+5$, $\dfrac{dy}{dx}=$

(*a*) $-x\cosh x$ (*b*) $x\cosh x$ (*c*) $-x\sinh x$ (*d*) $x\sinh x$

110. If $y=\operatorname{cosec}^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$, then $\dfrac{dy}{dx}=$

(*a*) $-\dfrac{1}{1+x^2}$ (*b*) $\dfrac{1}{1+x^2}$ (*c*) $-\dfrac{2}{1+x^2}$ (*d*) $\dfrac{2}{1+x^2}$

111. If $\sin^{-1}x+\sin^{-1}y=\pi/2$, then $\dfrac{dy}{dx}=$

(*a*) 1 (*b*) $\dfrac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (*c*) $\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ (*d*) none

112. If $y=\log\sqrt{\dfrac{1+\cos x}{1-\cos x}}$, then $\dfrac{dy}{dx}=$

(*a*) $-\operatorname{cosec} x$ (*b*) $\sin x$ (*c*) $-\sin x$ (*d*) none

113. If $y=\log\left(\dfrac{1}{x}+\dfrac{\sqrt{1+x^2}}{x}\right)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{1}{x}$ (*b*) $\dfrac{1}{\sqrt{1+x^2}}$ (*c*) $\dfrac{-1}{x\sqrt{1+x^2}}$ (*d*) none

114. If $y=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$, then $\dfrac{dy}{dx}=$

(*a*) $-\operatorname{cosech}^2 x$ (*b*) $\operatorname{cosech}^2 x$ (*c*) $\operatorname{sech}^2 x$ (*d*) none

115. If $y=\log \sinh\sqrt{x}$, $\dfrac{dy}{dx}=$

(*a*) $\coth\sqrt{x}$ (*b*) $\dfrac{1}{2\sqrt{x}}\tanh\sqrt{x}$ (*c*) $\dfrac{1}{2\sqrt{x}}\coth\sqrt{x}$ (*d*) none

116. If $y=\log\sqrt{\sin x}$, $\dfrac{dy}{dx}=$

(*a*) $\dfrac{\cos x}{\sqrt{\sin x}}$ (*b*) $\frac{1}{2}\cot x$ (*c*) $\frac{1}{2}\tan x$ (*d*) $\sqrt{\sin x\cos x}$

117. If $\tan y=\dfrac{x+1}{x+2}$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{3}{2x^2+6x+5}$ (*b*) $\dfrac{2x+3}{2x^2+6x+7}$ (*c*) $\dfrac{1}{2x^2+6x+5}$ (*d*) none

118. The derivative of $\sin x$ w.r.t. $\sin^{-1}x$ is

(*a*) x (*b*) 1 (*c*) $\cos x\cdot\sqrt{1-x^2}$ (*d*) none

119. If $y=\log_x^x+\log x$, then $\dfrac{dy}{dx}=$

(*a*) $1+\log x$ (*b*) $\dfrac{1}{x}$ (*c*) 0 (*d*) none

120. If $y=\dfrac{2^{3x}}{3^{2x}}$ and $\dfrac{dy}{dx}=y\,f(x)$, then $f(x)=$

(*a*) $\log(2/3)$ (*b*) $\log(8/9)$ (*c*) 1 (*d*) none

121. If $y=\dfrac{\cos x-\sin x}{\cos x+\sin x}$, then $\dfrac{dy}{dx}$ at $x=3\pi/4$ is

(*a*) 1 (*b*) 0 (*c*) -1 (*d*) none

122. If $y=\cosh^{-1}(\sec\theta)$, then $\dfrac{dy}{d\theta}=$

(*a*) $\sec\theta$ (*b*) $-\sec\theta$ (*c*) $\operatorname{cosec}\theta$ (*d*) $\tan\theta$

123. If $y=\tan^{-1}\left(\dfrac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)$, then $\dfrac{d^2y}{dx^2}=$

(*a*) $\dfrac{2ab\sin x\cos x}{(a\cos x+b\sin x)^2}$ (*b*) 1 (*c*) x (*d*) 0

124. If $x^y=e^x$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{x}{\log x}$ (*b*) $\dfrac{x-y}{x\log x}$ (*c*) $\dfrac{\log x+1}{(\log x)^2}$ (*d*) none

125. If $y=\log_2(3x^2-5x+8)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{6x-5}{3x^2-5x+8}$ (*b*) $\dfrac{\log^2{}_e\cdot(6x-5)}{3x^2-5x+8}$

(*c*) $\dfrac{\log_2{}^e\,.\,(6x-5)}{3x^2-5x+8}$ (*d*) none

126. If $y=\cosh^{-1}\sqrt{x}$, $\dfrac{dy}{dx}=$

(*a*) $\dfrac{1}{2\sqrt{x^2-x}}$ (*b*) $\dfrac{1}{2\sqrt{x^2+x}}$

(*c*) $-\dfrac{1}{2\sqrt{x}\sqrt{x-1}}$ (*d*) none

127. If $x=a\sin t$, $y=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$, then $\dfrac{dy}{dx}=$

(*a*) $\tan t$ (*b*) $\cot t$ (*c*) $-\cot t$ (*d*) $-\tan t$

128. If $y=\sin^{-1}(\log x)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{1}{x\sqrt{1-\log x}}$ (*b*) $\dfrac{1}{x\sqrt{1-(\log x)^2}}$

(*c*) $-\dfrac{1}{x\sqrt{1-(\log x)^2}}$ (*d*) none

129. If $y=\sin^{-1}\left(\dfrac{2t}{1+t^2}\right)$ and $x=\cos^{-1}\left(\dfrac{1-t^2}{1+t^2}\right)$, then $\dfrac{dy}{dx}=$

(*a*) $2t$ (*b*) 2 (*c*) 1 (*d*) none

130. If $y=\sin^{-1}(\tan x)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ (*b*) $-\dfrac{\sec^2 x}{\sqrt{\tan^2 x-1}}$ (*c*) $\sec x$ (*d*) none

131. If $y=\sqrt{4t^2-1}\cosh^{-1}2t$, then $\dfrac{dy}{dt}=$

(*a*) $2t-\dfrac{4t\cosh^{-1}2t}{\sqrt{4t^2-1}}$ (*b*) $2+\dfrac{4t\cosh^{-1}2t}{\sqrt{4t^2-1}}$

(*c*) $2+\dfrac{2t\cosh^{-1}2t}{\sqrt{4t^2-1}}$ (*d*) none

132. If $f(s)=(s^2-1)^5$, then $f''(s)=$

(a) $80s^2(s^2-1)^3+10(s^2-1)^4$ (b) $80\,s(s^2-1)^3+10\,s(s^2-1)^4$

(c) $80\,s(s^2-1)^4+10(s^2-1)^4$ (d) none

133. The derivative of $\tan^{-1}\dfrac{\sqrt{x}+\sqrt{a}}{1-\sqrt{ax}}$ w.r.t. $\sqrt{x}$ is

(a) $\dfrac{1}{2\sqrt{x}\,(1+x)}$ [✓ (b)] $\dfrac{1}{1+x}$ (c) $\dfrac{1}{1+x^2}$ (d) none

134. If $x=\log t+\sin t$ and $y=e^t+\cos t$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{t\,(e^t+\sin t)}{1+t\cos t}$ (b) $\dfrac{e^t-\sin t}{1+t\cos t}$ [✓ (c)] $\dfrac{t\,(e^t-\sin t)}{1+t\cos t}$ (d) none

135. The derivative of $x.\coth^{-1}(x^2+1)$ w.r.t. x is

(a) $\coth^{-1}(x^2+1)-\dfrac{2}{x^2+2}$ (b) $\coth^{-1}(x^2+1)+\dfrac{2}{x^2+2}$

(c) $\coth^{-1}(x^2+1)+\dfrac{2x}{x^2+2}$ (d) none

136. If $y=\tanh^{-1}(e^x)$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{e^x}{1-e^{2x}}$ (b) $\dfrac{e^x}{e^{2x}-1}$ (c) $\dfrac{e^x}{1+e^{2x}}$ (d) none

137. If $x^2+y^2=1$, then $\dfrac{d^2y}{dx^2}=$

(a) $1/y^3$ (b) $-1/y^3$ (c) $1/y^2$ (d) $-1/y^2$

138. If $y=\tan^{-1}\left(\dfrac{2\tan x}{1+\tan^2 x}\right)$, then $\dfrac{dy}{dx}=$

(a) $\tan 2x$ (b) 2 (c) $\dfrac{2x}{1+x^2}$ (d) none

139. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\ldots}}}$, $\dfrac{dy}{dx}=$

(a) $\dfrac{\cos x}{2y-1}$ (b) $\dfrac{\cos x}{2y+1}$ (c) $\dfrac{\sin x}{2y-1}$ (d) none

140. If $y=\cos(x+y)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{\sin(x+y)}{1-\sin(x+y)}$ (*b*) $\dfrac{\sin(x+y)}{1+\sin(x+y)}$

(*c*) $-\dfrac{\sin(x+y)}{1+\sin(x+y)}$ (*d*) $-\dfrac{\sin(x+y)}{1-\sin(x+y)}$

141. If $y=\cot^{-1}(2/x^2)$, then $\dfrac{dy}{dx}=$

(*a*) $\dfrac{4x}{x^4+16}$ (*b*) $-\dfrac{4x}{x^4+4}$ (*c*) $\dfrac{4}{x^2+16}$ (*d*) $\dfrac{4x}{x^4+4}$

142. If $y=\cot^2\sqrt{\theta}$, then $\dfrac{dy}{d\theta}=$

(*a*) $-\text{cosec}^2\,\theta$ (*b*) $-\dfrac{\cot\sqrt{\theta}\,\text{cosec}^2\sqrt{\theta}}{2\sqrt{\theta}}$

(*c*) $-\dfrac{\cot\sqrt{\theta}\,\text{cosec}^2\sqrt{\theta}}{\sqrt{\theta}}$ (*d*) none

143. If $y=\cos\{2\sin^{-1}(\cos x)\}$, then $\dfrac{dy}{dx}=$

(*a*) $\sin\{2\sin^{-1}(\cos x)\}$ (*b*) $-2\sin\{2\sin^{-1}(\cos x)\}$

(*c*) $2\sin\{2\sin^{-1}(\cos x)\}$ (*d*) none

144. If $y=\sinh x\cosh x$, then $\dfrac{dy}{dx}=$

(*a*) 1 (*b*) $\cosh^2x+\sinh^2x$ (*c*) $\sinh^2x-\cosh^2x$ (*d*) none

145. The second derivative of $\sin 3x\cos 5x$ w.r.t. x is

(*a*) $-32\sin 8x+2\sin 2x$ (*b*) $64\sin 8x-4\sin 2x$

(*c*) $32\sin 8x-2\sin 2x$ (*d*) none

146. If $x=2\cos t-\cos 2t$, $y=2\sin t-\sin 2t$, then $\dfrac{dy}{dx}$ when $=\pi/2$ is

(*a*) -2 (*b*) 0 (*c*) 1 (*d*) -1

147. If $y=\{(2x+3)^2+5\}^3$, then $\dfrac{dy}{dx}$ at $x=-1$ is

(*a*) 432 (*b*) 36 (*c*) 320 (*d*) none

148. If $y=\sqrt{\dfrac{1+x}{1-x}}$, then $y'/y=$

(*a*) $\dfrac{1}{(1-x)}$ (*b*) $\dfrac{1}{(1-x)\sqrt{1-x^2}}$ (*c*) $\dfrac{1}{1-x^2}$ (*d*) none

149. If $y=xe^{-x^2}$, then $\dfrac{dy}{dx}=$

(a) $e^{-x^2}(2x^2-1)$ (b) $e^{-x^2}(1-2x)$ (c) $e^{-x^2}(1-2x^2)$ (d) none

150. If $x=a(t-\sin t)$, $y=a(4-\cos t)$, then $\dfrac{dy}{dx}$ at $t=\pi/2$ is

(a) 1 (b) 0 (c) ∞ (d) -1

*151. If $x^2+y^2=x^2y^2$, then $\dfrac{dy}{dx}=$

(a) x^3/y^3 (b) $-y^3/x^3$ (c) y^3/x^3 (d) none

152. If $y=x+\tan^{-1}y$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{y^2}{1+y^2}$ (b) $y^2+\dfrac{1}{y^2}$ (c) $1+\dfrac{1}{y^2}$ (d) none

153. If $f(x)=\cos^{-1}(\sqrt{1-x^2})$, then $f'(0)=$

(a) not defined (b) 0 (c) 1 (d) -1

154. If $x=a\cos\theta$ and $y=b\sin\theta$, then $\dfrac{d^2y}{dx^2}=$

(a) $\dfrac{b}{a}\operatorname{cosec}^2\theta$ (b) $-\dfrac{b}{a^2}\operatorname{cosec}^3\theta$

(c) $-\dfrac{b}{a^2}\sin^3\theta$ (d) none

155. If $y=x\tanh 2x$, $\dfrac{dy}{dx}=$

(a) $\tanh 2x+2x\,\mathrm{sech}^2 2x$ (b) $\tanh 2x+2\,\mathrm{sech}^2 2x$

(c) $\tanh 2x-\mathrm{sech}^2 2x \,.\, 2x$ (d) none

156. If $y=\tan x+\cot x$, then $\dfrac{d^2y}{dx^2}=$

(a) $\sec^2x\tan x-\operatorname{cosec}^2x\cot x$ (b) $2(\sec^2x\tan x-\operatorname{cosec}^2x\cot x)$

(c) $2(\sec^2x\tan x+\operatorname{cosec}^2x\cot x)$ (d) none

157. If $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, then $\dfrac{d^2y}{dx^2}$ when $\theta=\pi/2$ is

(a) -1 (b) 1 (c) $1/a$ (d) $-1/a$

158. If $y=x\sin(1/x)$, then $\dfrac{dy}{dx}=$

(a) $-1/x\cos(1/x)+\sin(1/x)$ (b) $(1/x)\cos 1/x-\sin(1/x)$

(c) $1/x\cos(1/x)+\sin(1/x)$ (d) none

159. If $x^2 \sin y = y^2 \sin x$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y^2\cos x-2x\sin y}{x^2\cos y-2y\sin x}$ (b) $\dfrac{y^2\cos x+2x\sin y}{x^2\cos y-2y\sin x}$

(c) $\dfrac{y^2\sin x-2x\cos y}{x^2\cos y-2y\sin y}$ (d) none

160. If $x/y = \log(xy)$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y(x+y)}{x(x-y)}$ (b) $\dfrac{x(x-y)}{y(x+y)}$ (c) $\dfrac{y(x-y)}{x(x+y)}$ (d) none

Answers

1. (b) 2. (c) 3. (d) 4. (c) 5. (d) 6. (d) 7. (b) 8. (c) 9. (b) 10. (b)
11. (b) 12. (b) 13. (c) 14. (b) 15. (d) 16. (c) 17. (c) 18. (c) 19. (c)
20. (a) 21. (b) 22. (a) 23. (d) 24. (b) 25. (c) 26. (a) 27. (c) 28. (b)
29. (d) 30. (d) 31. (d) 32. (d) 33. (d) 34. (b) 35. (c) 36. (a) 37. (d)
38. (c) 39. (b) 40. (c) 41. (c) 42. (d) 43. (c) 44. (a) 45. (c) 46. (a)
47. (d) 48. (c) 49. (c) 50. (c) 51. (c) 52. (d) 53. (b) 54 (d) 55. (b)
56. (a) 57. (d) 58. (c) 59. (a) 60. (b) 61. (a) 62. (a) 63. (a) 64. (c)
65. (c) 66. (b) 67. (a) 68. (d) 69. (b) 70. (a) 71. (c) 72. (b) 73. (c)
74. (c) 75. (b) 76. (b) 77. (b) 78. (a) 79. (b) 80. (a) 81. (c) 82. (d)
83. (b) 84. (c) 85. (b) 86. (c) 87. (d) 88. (a) 89. (c) 90. (b) 91. (a)
92. (c) 93. (c) 94. (d) 95. (d) 96. (b) 97. (c) 98. (d) 99 (d) 100 (d).
101. (a) 102. (c) 103. (a) 104. (a) 105. (d) 106. (b) 107. (b) 108. (b)
109. (d) 110. (c) 111. (d) 112. (a) 113. (c) 114. (a) 115. (c)
116. (b) 117. (c) 118. (c) 119. (b) 120. (b) 121. (d) 122. (a)
123. (d) 124. (b) 125. (c) 126. (a) 127. (b) 128. (b) 129. (c)
130. (a) 131. (b) 132. (a) 133. (b) 134. (c) 135. (a) 136. (a)
137. (b) 138. (d) 139. (a) 140. (c) 141. (d) 142. (c) 143. (c)
144. (b) 145. (a) 146. (d) 147. (a) 148. (c) 149. (c) 150. (a)
151. (b) 152. (c) 153. (c) 154. (b) 155. (a) 156. (c) 157. (d)
158. (a) 159. (a) 160. (c)

5.4 Application of differentiation

1. The slope of the normal to the curve $y = \cos 2x$ at $x = \pi/6$ is
(a) $\sqrt{3}$ (b) $-1/\sqrt{3}$ (c) $2/\sqrt{3}$ (d) none

2. The gradient of the curve $y=x^3+3x^2+1$ at (1, 5) is
(*a*) 8 [✓ (*b*) 9] (*c*) 12 (*d*) none

3. The equation of the tangent to the curve $y=x^2-6x$ at $x=1$ is
[✓ (*a*) $4x+y+1=0$] (*b*) $x-4y-21=0$ (*c*) $4x-y-9=0$ (*d*) none

4. If the length of the subnormal at any point of the curve $y=a^{1-n}x^n$ is constant, then $n=$
(*a*) 3 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) none

5. The function x^3+4x+1 has
(*a*) a minimum value (*b*) neither maximum nor minimum
(*c*) minimum value but no maximum (*d*) none

6. The angle between the curves $xy=4$ and $x^2-y^2=15$ at (–4, –1) is (*a*) 60° [✓ (*b*) 90°] (*c*) $\tan^{-1}\frac{1}{2}$ (*d*) $\tan^{-1} 5/7$

7. The tangents to the curve $y=2x^3-3$ at $x=1$ and $x=-1$ are
[✓ (*a*) parallel] (*b*) perpendiculur
(*c*) inclined at 45° to each other (*d*) none

8. The point on the curve $y=3x-x^2$, where the gradient is –5, is (*a*) (4, 4) (*b*) (–5, –35) (*c*) (4, –4) (*d*) none

9. The x– coordinates of the points, at which the tangents to the curve $y=x^3-3x^2-8x+7$ have inclination π/4 with the positive direction of x–axis, are
(*a*) ±1 (*b*) ±3 (*c*) –1, 3 (*d*) none

10. The subtangent of the curve $y=3^{x+2}$ is
(*a*) proportional to abscissa (*b*) proportional to ordinate
(*c*) constant (*d*) none

11. The function $-2x^2+3x+5$ has
(*a*) a minimum value (*b*) no minimum value
(*c*) both minimum and maximum values (*d*) none

12. The minimum value of the function x^x is at $x=$
(*a*) 0 (*b*) 1 (*c*) e (*d*) $1/e$

13. The two numbers whose sum is 72 and having the maximum product are
(*a*) 40, 32 (*b*) 52, 12 (*c*) 38, 34 (*d*) none

14. If the velocity $v=3t^2+t$, the acceleration when $v=2$ is
(*a*) –5 (*b*) 5 (*c*) 7 (*d*) none

15. Find the rate of decrease of the surface area of a sphere, when its radius is 5 cm, if the radius decreases at 0.01 cm/sec.
(*a*) $2\pi/5$ cm²/sec (*b*) $4\pi/5$ cm²/sec
(*c*) $-2\pi/3$ cm²/sec (*d*) none

16. The rate of change of the function $3x^2-x^3$ w.r.t. x at $x=5$ is
(*a*) 15 (*b*) 0 (*c*) –15 (*d*) none

17. The length of the subnormal to the curve $x^2-3xy+y^2=11$ at $(1, -2)$ is (*a*) $\frac{16}{7}$ (*b*) $\frac{7}{4}$ (*c*) $\frac{32}{7}$ (*d*) none

18. The slope of the normal to the curve $y=x^2+3x+2$ at $(-2, 0)$ is (*a*) 1 (*b*) –1 (*c*) $\frac{1}{2}$ (*d*) $-\frac{1}{2}$.

19. At what point on the curve $4y=x^2+3x+2$, the tangent is parallel to x-axis?
(*a*) $(-\frac{3}{2}, -\frac{1}{16})$ (*b*) $(-1, 0)$ (*c*) $(-\frac{3}{2}, -\frac{1}{2})$ (*d*) none

20. The slope of the normal to the curve $x^3+y^3=2xy$ at $(1, 1)$ is
(*a*) –1 (*b*) 1 (*c*) –2 (*d*) $\frac{1}{2}$

21. The subnormal to the curve $y=\sqrt{3x+2}$ at any point is
(*a*) 1 (*b*) $\frac{3}{2}$ (*c*) $\frac{3}{2}x$ (*d*) none

22. If θ is the acute angle between the curves $y=2^x$ and $y=3^x$ at a point of intersection, then $\tan\theta=$
(*a*) 0 (*b*) $\dfrac{\pi}{4}$ (*c*) $\dfrac{\log 3-\log 2}{1+\log 3 \log 2}$ (*d*) none

23. The length of the subtangent to the parabola $y=4x^2$ at $(-2, 16)$ is
(*a*) 1 (*b*) 2 (*c*) 4 (*d*) none

24. The acute angle between the curves $y=x^2$ and $xy=27$ at $(3, 9)$ is
(*a*) $\tan^{-1}(4/17)$ (*b*) $\tan^{-1}(8/17)$ (*c*) $\tan^{-1}(9/17)$ (*d*) none

25. The function $4x^2+7x+8$ has
(*a*) no maximum (*b*) no minimum
(*c*) both (a) and (b) (*d*) none

26. The least perimeter of a rectangle of area 100 units is
(*a*) 10 (*b*) 20 (*c*) 80 (*d*) 40

27. If $s=t^n$ and velocity and acceleration are equal at $t=3$, then $n=$
(*a*) 2 (*b*) 4 (*c*) 8 (*d*) 1

28. If $s=\sqrt{t^2+1}$, then acceleration is
(*a*) $2/s^3$ (*b*) $1/s^3$ (*c*) $-1/s^3$ (*d*) none

29. If $s=\frac{1}{2}gt^2$ where $g=980$ cm/sec^2, then velocity is
(a) t [✓ (b) gt] (c) $\frac{1}{2}gt$ (d) none

30. The length of the subtangent to the curve $y=2x^2-3x+4$ at (1, 3) is [✓ (a) 3] (b) 1 (c) 9 (d) none

31. The coordinates of the points on the curve $y=x+4/x$, the tangents at which are parallel to x-axis, are
(a) $(\pm 2, \pm 4)$ (b) (2, 4), (–2, –4) (c) (2, –4), (–2, 4) (d) none

32. If the tangent to the curve $y=x^2+ax$ at origin is parallel to the line joining the points (3, 2) and (–1, 6), then $a=$
(a) 1 (b) 2 (c) –1 (d) –2

33. The slope of normal to the curve $x=e^t \sin t$, $y=e^t \cos t$ at at $t=0$ is (a) 1 [✓ (b) –1] (c) 0 (d) none

34. The length of the subtangent to the curve $x=y^n$ varies as
(a) ordinate (b) square of the ordinate [✓ (c) abscissa] (d) none

35. If $s=5 \sin 2t$, velocity at $t=\pi/4$ is
(a) 10 [✓ (b) 0] (c) 5 (d) none

36. The angle between the curves $y^2=2x^2+1$ and $y=2x-7$ at (2, –3) is
(a) $\tan^{-1}\frac{1}{2}$ [✓ (b) $\tan^{-1} 2$] (c) $\tan^{-1}\frac{3}{2}$ (d) none

37. The extreme points of the function $e^{\sin x}$ in $[-\pi, \pi]$ are
(a) 0, π (b) $\pm\pi$ (c) $\pm\pi/2$ (d) none

38. The length of the subnormal at $(c/2, c)$ to the curve $2xy=c^2$, $c>0$ is (a) $2c$ (b) c [✓ (c) $c/2$] (d) $4c$

39. The angle between the curves $y=x^2$ and $x=y^2$ at (0, 0) is
(a) not defined (b) 0 (c) $\pi/2$ (d) none

40. If $s=\frac{1}{3}t^3$ and $v^3=ks^2$, then $k=$
(a) 3 (b) 1 (c) 9 (d) none

41. If $v=\sqrt{s^2+1}$, then the acceleration is
(a) $s/\sqrt{s^2+1}$ (b) s (c) 1 (d) none

42. If $s=t^3-6t+1$, find the time when the velocity is 0
(a) 2 (b) 1 (c) $\sqrt{2}$ (d) none

43. The length of the subnormal to the curve $y=x \sin x$ at $x=\pi/2$ is
(a) $\pi/2$ (b) $2/\pi$ (c) 1 (d) none

44. If the curves $y=ax^2-8$ and $y=bx^3-4$ have a common tangent $y=12x-20$ at $x=2$, then (a, b) is
(a) (1, 1) (b) (3, 2) (c) (1, 2) (d) (3, 1)

45. The equation of the normal to the curve $x=4t^2$, $y=4t$ at $t=2$ is
(*a*) $x-4y+16=0$ (*b*) $4x+y-72=0$ (*c*) $4x-y+72=0$ (*d*) none

46. The acute angle between the curve $y=4-\frac{x^2}{2}$ and the line a $y=4-4x$ at (2, 2) is
(*a*) $\pi/4$ (*b*) $\tan^{-1} 1/3$ (*c*) $\tan^{-1} 1/2$ (*d*) none

47. The length of the subnormal to the curve $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ at $\theta=\pi/2$ is
(*a*) a (*b*) $2a$ (*c*) $a/\sqrt{2}$ (*d*) none

48. The equation of normal to the curve $x^3+3xy+y^3=1$ at $(2,-1)$ is
(*a*) $x-y+1=0$ (*b*) $x+y+1=0$ (*c*) $x-y-3=0$ (*d*) $x+y=1$

49. The maximum value of the function $x^3-12x^2+45x-7$ is
(*a*) 47 (*b*) 39 (*c*) 43 (*d*) none

50. A bullet is shot horizontally and its distance s in time t is given by $s=1280\,t-16t^2$. What is the velocity with which it is shot?
(*a*) 1280 (*b*) 32 (*c*) 1148 (*d*) none

51. The equation of motion of two particles starting from the same point are given by $s_1=9-t^2$ and $s_2=t^2-3t$. At what time they meet again?
(*a*) 4 (*b*) 2 (*c*) 3 (*d*) none

52. The angle between the curves $4x^2+9y^2=72$ and $x^2-y^2=5$ at (3, 2) is (*a*) 0 (*b*) $\tan^{-1} 2$ (*c*) $\tan^{-1}\frac{1}{2}$ (*d*) 90°

53. When a, b and x are $+ve$, the least value of $ax+b/x$ is
(*a*) ab (*b*) $2\sqrt{ab}$ (*c*) $\sqrt{ab}$ (*d*) none

54. The angle between the curves $y^2=4x$ and $x^2=4y$ at (4, 4) is
(*a*) $\tan^{-1}\frac{3}{2}$ (*b*) 90° (*c*) $\tan^{-1}\frac{3}{4}$ (*d*) none

55. The slope of the normal to the curve $y^3+3y=x$ at (4,1) is
(*a*) $\frac{1}{6}$ (*b*) $-\frac{1}{6}$ (*c*) 6 (*d*) -6

56. The normal to the curve $y=x+1/x$ at $x=a$ is parallel to y–axis. Then $a=$
(*a*) 1 (*b*) -1 (*c*) ± 1 (*d*) none

57. The length of the subtangent to the curve $x=4a\cos^3\theta$ and $y=4a\sin^3\theta$ at $\theta=\pi/4$ is
(*a*) a (*b*) $2a$ (*c*) $\sqrt{2}a$ (*d*) none

58. The extreme points of the function $x+1/x$ are

(*a*) 1, 2 (*b*) 1, 0 (*c*) –1, 0 (*d*) 1, –1

59. The minimum value of the function x^2e^x is

(*a*) $4/e^2$ (*b*) 1 (*c*) 0 (*d*) none

60. The points where the function $3x^4-10x^3+6x^2+5$ has extreme values are

(*a*) $x=0, 2$ (*b*) $x=0, 1, 2$ (*c*) $x=0, \frac{1}{2}, 1$ (*d*) $x=0, \frac{1}{2}, 2$

61. A manufacturer sells x articles at Rs. $(3-x/1000)$ per article, which costs in all Rs. $[100+x/2]$. For what value of x, the profit is maximum

(*a*) 2500 (*b*) 1250 (*c*) 5000 (*d*) none

62. The function $x^5-5x^4+5x^3-1$ attains minimum at $x=$

(*a*) 0 (*b*) 1 (*c*) 3 (*d*) none

63. The cost C of manufacturing a certain article is given by $C=5+48/x+3x^2$ where x is the number of articles produced. C is minimum when $x=$

(*a*) 2 (*b*) 3 (*c*) 8 (*d*) none

64. The rate of change of the surface area of a cube w.r.t. its side (*r*) is

(*a*) r (*b*) 12r (*b*) 8r (*d*) none

65. The function $f(x)=\frac{1}{\sqrt{1-x^2}}$ has the minimum value at $x=$

(*a*) ± 1 (*b*) 0 (*c*) 1 (*d*) none

Answers

1. (*d*) 2. (*b*) 3. (*a*) 4. (*c*) 5. (*b*) 6. (*b*) 7. (*a*) 8. (*c*) 9. (*c*) 10. (*c*)
11. (*b*) 12. (*d*) 13. (*d*) 14. (*b*) 15. (*a*) 16. (*d*) 17. (*a*) 18. (*a*) 19. (*a*)
20. (*b*) 21. (*b*) 22. (*c*) 23. (*a*) 24. (*c*) 25. (*a*) 26. (*d*) 27. (*b*) 28. (*b*)
29. (*b*) 30 (*a*). 31. (*b*) 32. (*c*) 33. (*b*) 34. (*c*) 35. (*b*) 36. (*b*) 37. (*c*)
38. (*a*) 39. (*c*) 40. (*c*) 41. (*b*) 42. (*c*) 43. (*a*) 44. (*d*) 45. (*b*) 46. (*b*)
47. (*a*) 48. (*c*) 49. (*a*) 50. (*a*) 51. (*c*) 52. (*d*) 53. (*b*) 54. (*c*) 55. (*d*)
56. (*c*) 57. (*c*) 58. (*d*) 59. (*c*) 60. (*d*) 61. (*b*) 62. (*c*). 63 (*a*) 64. (*b*)
65. (*b*).

5.44 Integration

1. The integral of $\dfrac{\sin x}{3+4\cos x}$ w.r.t. x is
(a) $\log(3+4\cos x)$ (b) $-\frac{1}{4}\log(3+4\cos x)$
(c) $-4\log(3+4\cos x)$ (d) $\frac{1}{4}\log(3+4\cos x)$

2. The integral of $(1+\sin^3 x)/\sin^2 x$ w.r.t. x is
(a) $\cot x-\cos x$ (b) $\cos x+\cot x$ (c) $\cos x-\cot x$ (d) $-\cos x-\cot x$

3. If $\int f(x)dx=f(x)+c$, then $f(x)=$
(a) $\log x$ (b) $\sin x$ (c) 2^x [✓ (d) e^x]

4. The integral of $\dfrac{1}{(1+\cos x)(1-\cos x)}$ w.r.t. x is
(a) $-\operatorname{cosec} x$ (b) $-\cot x$ (c) $\log(1-\cos^2 x)$ (d) none

5. $\int \sin x \sec^2 x\, dx=$
(a) $\sec x$ (b) $-\cos x+\sin x$ (c) $\sec x+x$ (d) none

6. $\int (\cot x+\tan x)^2\, dx=$
(a) $\tan x+\cot x$ [✓ (b) $\tan x-\cot x$] (c) $-\tan x+\cot x$ (d) none

7. The integral of $\dfrac{\tan^2 x-1}{\tan^2 x+1}$ w.r.t. x is
(a) $-2\sin 2x$ (b) $-\sin 2x$ (c) $-\frac{1}{2}\sin 2x$ (d) none

8. $\displaystyle\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\, dx=$
(a) $\sec x+\operatorname{cosec} x$ (b) $\sec x-\operatorname{cosec} x$
(c) $\cot x-\tan x$ (d) $\tan x-\cot x$

9. $\displaystyle\int \frac{1}{\sqrt{x^2-4}}\, dx=$
(a) $\sin^{-1} x/2$ (b) $\cosh^{-1} x/2$ (c) $\sinh^{-1} x/2$ (d) none

10. $\displaystyle\int \frac{dx}{4x^2+1}=$
(a) $\frac{1}{2}\tan^{-1} 2x$ (b) $\tan^{-1} 2x$ (c) $\frac{1}{4}\tan^{-1} 2x$ (d) $2\tan^{-1} 2x$

11. $\int \tan^6 x\sec^2 x\, dx=$
(a) $\dfrac{\tan^5 x}{5}+c$ (b) $\dfrac{\tan^6 x}{6}+c$ [✓ (c) $\dfrac{\tan^7 x}{7}+c$] (d) none

12. $\displaystyle\int \frac{1}{1-\cos 2x}\, dx$ is
(a) $\frac{1}{2}\cot x$ [✓ (b) $-\frac{1}{2}\cot x$] (c) $\frac{1}{2}\tan x$ (d) none

13. $\int \frac{1}{\sqrt{x^2+9}}\,dx$

(a) $\frac{1}{3}\tan^{-1}(x/3)$ (b) $2\sqrt{x^2+9}$ (c) $\frac{1}{3}\sinh^{-1}(x/3)$ (d) $\sinh^{-1}(x/3)$

14. $\int \frac{\log(\log x)}{x\log x}\,dx =$

(a) $\log(x\log x)$ [✓ (b) $\log\{\log(\log x)\}$] [✓ (c) $\frac{1}{2}\{\log(\log x)\}^2$]
(d) none

15. $\int \cos x \cot(\sin x)\,dx =$

(a) $\log\sin x$ (b) $\log\sin^2 x$ (c) $\log\{\log\sin x\}$ [✓ (d) $\log\sin(\sin x)$]

16. $\int \frac{1-x}{x^2}\,e^x\,dx =$

(a) $\frac{e^x}{x^2}$ [✓ (b) $-\frac{e^x}{x}$] (c) x^2e^x (d) $\frac{e^x}{x}$

17. $\int a^x e^x\,dx =$

(a) $\frac{e^x a^x}{\log a}$ [✓ (b) $\frac{e^x a^x}{\log(ae)}$] (c) $\frac{(ea)^x}{\log a}$ (d) none

18. $\int \sec^n x \tan x\,dx = \frac{\sec^n x}{n}$ is true

(a) for all n (b) for all $n > 0$ (c) for all $n \neq -1$ (d) none

19. $\int \frac{x+1}{\sqrt{x^2+2x+5}}\,dx =$

(a) $\sqrt{x^2+2x+5}$ (b) $2\sqrt{x^2+2x+5}$ (c) $\frac{1}{2}\sqrt{x^2+2x+5}$ (d) none

20. $\int \frac{1}{4-x^2}\,dx =$

(a) $\frac{1}{2}\log\frac{2+x}{2-x}$ (b) $\frac{1}{4}\log\frac{2+x}{2-x}$ (c) $\frac{1}{4}\log\frac{2-x}{2+x}$ (d) $\frac{1}{4}\log\frac{x-2}{x+2}$

21. $\int \frac{1}{\sqrt{1-x}}\,dx =$

(a) $\sin^{-1}\sqrt{x}$ (b) $\sqrt{1-x}$ [✓ (c) $-2\sqrt{1-x}$] (d) $2\sqrt{1-x}$

22. The integral of $\frac{1}{x\sqrt{1-\log x}}$ w.r.t. x is

(a) $2\sqrt{1-\log^2 x}$ [✓ (b) $-2\sqrt{1-\log x}$] (c) $\frac{1}{2}\sqrt{1-\log x}$ (d) $-\frac{1}{2}\sqrt{1-\log x}$

23. $\int \frac{dx}{\sqrt{x+2}-\sqrt{x-2}} =$

(a) $\frac{1}{6}\{(x+2)^{3/2}+(x-2)^{3/2}\}$ (b) $\frac{2}{3}\{(x+2)^{3/2}-(x-2)^{3/2}\}$

(c) $\frac{1}{6}\{(x+2)^{1/2}-(x-2)^{1/2}\}$ (d) none

24. $\int \frac{1+x\log x}{x}\, e^x\, dx =$

[✓ (a) $e^x \log x$] (b) $e^x x$ (c) $\frac{e^x \log x}{x}$ (d) none

25. $\int \frac{x}{\sin^2 x}\, dx =$

(a) $-x\cot x - \log\sin x$ [✓ (b) $-x\cot x + \log\sin x$]

(c) $x\cot x + \log\sin x$ (d) none

26. $\int \frac{x^4+1}{x^2}\, dx =$

(a) $\frac{x^3}{3} + \frac{1}{x}$ (b) $\frac{x^2}{2} + \frac{1}{x}$ [✓ (c) $\frac{x^3}{3} - \frac{1}{x}$] (d) none

27. $\int \frac{a+b\sin x}{\cos^2 x}\, dx =$

(a) $a\sec x + b\tan x$ (b) $a\tan x + b\sin x$

[✓ (c) $a\tan x + b\sec x$] (d) none

28. $\int \frac{x^4+x^2+1}{x^2+1}\, dx =$

(a) $x^3 + \tan^{-1} x$ (b) $\frac{x^2}{2} + \tan^{-1} x$

(c) $\frac{x^3}{3} - \cot^{-1} x$ (d) $\frac{x^3}{3} - \tan^{-1} x$

29. $\int \frac{2x^2\, dx}{(2+x^3)^2} =$

(a) $-\frac{1}{3(x^3+2)}$ (b) $\frac{2}{3(x^3+2)}$ (c) $\frac{-2}{3(x^3+2)}$ (d) none

30. $\int \frac{\sec x \operatorname{cosec} x}{\log\tan x}\, dx =$

(a) $\log\tan x$ (b) $\log(\log\tan x)$ (c) $\log(\sec x + \operatorname{cosec} x)$ (d) none

31. $\int \frac{\cos x \sin x}{5+\cos^2 x}\, dx=$

(a) $\log(5+\cos^2 x)$ (b) $-\frac{1}{5}\log(5+\cos^2 x)$

(c) $\frac{1}{2}\log(5+\cos^2 x)$ (d) $-\frac{1}{2}\log(5+\cos^2 x)$

32. $\int \frac{\sin x-\cos x}{\sin x+\cos x}\, dx=$

(a) $\frac{1}{\sin x+\cos x}$ (b) $\log(\sin x+\cos x)$

(c) $\log \frac{1}{\sin x+\cos x}$ (d) none

33. $\int \tan 4x\ dx=$

(a) $\log \sec 4x$ (b) $\log \sin 4x$ (c) $\frac{1}{4}\log \sin 4x$ (d) $-\frac{1}{4}\log \cos 4x$

34. The integral of $\frac{\sin x}{\sqrt{8+\sin^2 x}}$ w.r.t. x is

(a) $\frac{1}{2\sqrt{2}}\sin^{-1}x$ (b) $-\frac{1}{3}\sin^{-1}\left(\frac{\cos x}{3}\right)$

(c) $-\sin^{-1}\left(\frac{\cos x}{3}\right)$ (d) $\sin^{-1}\left(\frac{\cos x}{3}\right)$

35. The integral of $\sec^2 x \operatorname{cosec}^2 x$ w.r.t. x is

(a) $\tan x-\cot x$ (b) $\sec x+\operatorname{cosec} x$

(c) $\sec x-\operatorname{cosec} x$ (d) $\tan x+\cot x$

36. $\int \frac{1}{\sqrt{12+3x^2}}\, dx=$

(a) $\frac{1}{\sqrt{3}}\cosh^{-1}\frac{x}{2}$ (b) $\frac{1}{\sqrt{3}}\sinh^{-1}\frac{x}{3}$

(c) $\frac{1}{\sqrt{3}}\sinh^{-1}\frac{x}{2}$ (d) none

37. $\int x \sinh x\ dx=$

(a) $x\cosh x+\sinh x$ (b) $x\cosh x-\sinh x$

(c) $-x\cosh x+\sinh x$ (d) none

38. $\int \frac{\cos 2x}{(\cos x+\sin x)^2}\, dx=$

(a) $\log(\cos x+\sin x)$ (b) $-\log(\cos x+\sin x)$

(c) $\log(\cos x-\sin x)$ (d) none

39. $\int\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)dx=$

(a) $x^2-\frac{1}{x^2}$ (b) $x^3-\frac{1}{x^3}$ (c) $\frac{x^3}{3}-\frac{1}{x}$ (d) $\frac{x^3}{3}+\frac{1}{x}$

40. $\int\frac{x^5}{1+x^{12}}\,dx=$

(a) $\frac{1}{5}\tan^{-1}(x^5)$ (b) $\frac{1}{6}\tan^{-1}(x^6)$ (c) $\tan^{-1}(x^6)$ (d) none

41. $\int\frac{\sin x\cos x}{3\sin^2 x+2\cos^2 x}\,dx=$

(a) $\log(3\sin^2 x+2\cos^2 x)$ (b) $2\log(3\sin^2 x+2\cos^2 x)$
(c) $\frac{1}{2}\log(3\sin^2 x+2\cos^2 x)$ (d) none

42. $\int\frac{2}{x\{1+(\log x)^2\}}\,dx=$

(a) $\tan(\log x)$ (b) $\frac{1}{2}\tan^{-1}(\log x)$
(c) $2\tan^{-1}(\log x)$ (d) none

43. $\int\frac{\sin 2x}{1+\sin^2 x}\,dx=$

(a) $\tan^{-1}(\sin x)$ (b) $\log(1+\sin^2 x)$
(c) $2\log\cos x$ (d) none

44. $\int\frac{e^{mx}}{e^{mx}+1}\,dx=$

(a) $\frac{1}{m}(e^{mx}+1)$ (b) $\log(e^{mx}+1)$

(c) $m\log(e^{mx}+1)$ (d) $\frac{1}{m}\log(e^{mx}+1)$

45. The integral of $\frac{\cot x}{\log\sin x}$ w.r.t. x is

(a) $\log\sin x$ (b) $\log(\log\sin x)$ (c) $-\log(\log\sin x)$ (d) none

46. The integral of $\frac{\cos 2x}{\sin^2 x\cos^2 x}$ w.r.t. x is

(a) $\operatorname{cosec} x-\cot x$ (b) $-(\tan x+\cot x)$ (c) $\tan x-\cot x$ (d) none

47. $\int\frac{1}{1+\cos^2 x}\,dx=$

(a) $-\cot x$ (b) $\tan^{-1}\left(\frac{\tan x}{2}\right)$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)$ (d) none

48. $\int \frac{1+\tan x}{1-\tan x}\,dx=$

(a) $\log \sec (\pi/4+x)$ (b) $\log \sec (x-\pi/4)$

(c) $\log \sec x$ (d) none

49. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}\,dx=$

(a) $\log (e^x+x^e)$ (b) $e \log (e^x+x^e)$

(c) $1/e \log (e^x+x^e)$ (d) none

50. $\int \frac{\sec^2 x}{\sqrt{4-3\tan x}}\,dx=$

(a) $-\frac{1}{3}\sqrt{4-3\tan x}$ (b) $\frac{2}{3}\sqrt{4-3\tan x}$

(c) $\frac{1}{3}\sqrt{4-3\tan x}$ (d) $-\frac{2}{3}\sqrt{4-3\tan x}$

51. $\int \frac{dx}{9x^2+6x+17}=$

(a) $\frac{1}{3}\tan^{-1}\left(\frac{3x+1}{2}\right)$ (b) $\frac{1}{12}\tan^{-1}\left(\frac{3x+1}{4}\right)$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{4}\right)$ (d) none

52. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx=$

(a) $\tan^{-1}(e^{2x})$ (b) $\log (e^{2x}+e^{-2x})$

(c) $2\log (e^{2x}+e^{-2x})$ (d) $\frac{1}{2}\log (e^{2x}+e^{-2x})$

53. $\int \frac{x^3}{1+x^2}\,dx=$

(a) $\frac{1}{2}\{x^2-\log (1+x^2)\}$ (b) $\frac{1}{2}x^2-\log (1+x^2)$

(c) $\frac{1}{2}x^2-\tan^{-1} x$ (d) none

54. $\int \sec^5 x \tan x\,dx=$

(a) $\frac{\sec^5 x}{5}$ (b) $\frac{\sec^6 x}{6}$ (c) $\frac{\tan^6 x}{6}$ (d) none

55. $\int \frac{e^{1/x}}{x^2}\,dx=$

(a) $e^{1/x}$ (b) $-e^{1/x}$ (c) $e^{-1/x}$ (d) $-2e^{1/x}$

56. The integral of $\dfrac{\sin x+\cos x}{\sqrt{1+\sin 2x}}$ w.r.t. x is

(*a*) $\sqrt{1+\sin 2x}$ (*b*) $2\sqrt{1+\sin 2x}$ (*c*) x (*d*) none

57. $\displaystyle\int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}}\,dx=$

(*a*) $\sec^2(\sin^{-1}x)$ (*b*) $\log(\sin^{-1}x)$ (*c*) $\log\{\sec(\sin^{-1}x)\}$ (*d*) none

58. $\int \text{sech}\, x\, dx=$

(*a*) $\text{sech}\, x \tanh x$ (*b*) $2\tan^{-1}(e^x)$ (*c*) $\sin^{-1}(e^x)$ (*d*) none

59. $\displaystyle\int \frac{1-3\tan^2 x}{3\tan x-\tan^3 x}\,dx=$

(*a*) $\frac{1}{3}\log\sin 3x$ (*b*) $\frac{1}{3}\log\sec 3x$ (*c*) $-\frac{1}{3}\log\sin 3x$ (*d*) $\log\sec 3x$

60. $\int e^x\sqrt{1+e^x}\,dx=$

(*a*) $\frac{2}{3}\sqrt{1+e^x}$ [✓ (*b*) $\frac{2}{3}(1+e^x)^{3/2}$] (*c*) $\frac{1}{2}(1+e^x)^{-\frac{1}{2}}$ (*d*) none

61. $\displaystyle\int \frac{\sin(2+3\log x)}{x}\,dx=$

(*a*) $\cos(2+3\log x)$ (*b*) $-\frac{1}{2}\cos(2+3\log x)$
(*c*) $\frac{1}{3}\cos(2+3\log x)$ (*d*) $-\frac{1}{3}\cos(2+3\log x)$

62. $\displaystyle\int \frac{dx}{(1+x^2)\{9+(\cot^{-1}x)^2\}}$

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{3}\tan^{-1}\left(\dfrac{\cot^{-1}x}{3}\right)$
(*c*) $-\frac{1}{3}\tan^{-1}\left(\dfrac{\cot^{-1}x}{3}\right)$ (*d*) none

63. $\displaystyle\int \left\{\frac{\sec x}{(1-\tan x)^3}\right\}^2 dx=$

(*a*) $\frac{1}{5}(1-\tan x)^5$ (*b*) $\dfrac{1}{5(1-\tan x)^5}$ (*c*) $\dfrac{-1}{5(1-\tan x)^5}$ (*d*) none

64. $\displaystyle\int \frac{(1-x^2)^{-1/2}}{\sin^{-1}x}\,dx=$

(*a*) $\dfrac{(\sin^{-1}x)^{-2}}{-2}$ (*b*) $\log(\sin^{-1}x)$ (*c*) $(\sin^{-1}x)^{-1}$ (*d*) none

65. $\int \frac{\cot(2\log x)}{x}\,dx =$

(a) $\frac{1}{2}\sin(\log x)$ (b) $2\sin\left(\frac{\log x}{x}\right)$

(c) $\frac{1}{2}\log\sin(\log x)$ (d) $\frac{1}{2}\log\sin(2\log x)$

66. $\int \frac{e^x}{e^x+1}\,dx =$

(a) $x - \log(e^x+1)$ (b) $\frac{1}{e^x+1}$ (c) $\log(e^x+1)$ (d) none

67. $\int \frac{1}{\sqrt{x}\,\sqrt{1-x}}\,dx =$

(a) $4\sqrt{x(1-x)}$ (b) $2\sin^{-1}(\sqrt{x})$ (c) $\sin^{-1}\sqrt{x}$ (d) none

68. $\int e^x(\sin x + \cos x)\,dx =$

(a) $e^x \sin x$ (b) $-e^x \cos x$ (c) $-e^x \sin x$ (d) $e^x \cos x$

69. $\int \frac{\sec 3x \tan 3x}{2\sec 3x - 5}\,dx =$

(a) $\frac{1}{3}\log(2\sec 3x - 5)$ (b) $\frac{1}{3}\sec 3x$

(c) $-\frac{1}{6}\log(2\sec 3x - 5)$ (d) none

70. $\int x^2 e^x\,dx =$

(a) $(x^2+2x-2)e^x$ (b) $(x^2-x+1)e^x$ (c) $(x^2-2x+2)e^x$ (d) none

71. $\int \frac{x-1}{x+1}\,dx =$

(a) $x - \log(x+1)$ (b) $x + \log(x-1)$

(c) $x - \frac{1}{2}\log(x+1)$ (d) $x - 2\log(x+1)$

72. $\int \sec^{-1}\sqrt{x}\,dx =$

(a) $x\sec^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x-1}$ (b) $x\sec^{-1}\sqrt{x} + \sqrt{x-1}$

(c) $x\sec^{-1}\sqrt{x} - \sqrt{x-1}$ (d) none

73. $\int e^{\log(x-2)}\,dx =$

(a) $e^{\log(x-2)}$ (b) $x-2$ (c) x^2-2x (d) $\frac{x^2}{2}-2x$

74. $\int x\cos 2x\,dx =$

(a) $\frac{x\sin 2x}{2} - \frac{\cos 2x}{2}$ (b) $\frac{x\sin 2x}{2} + \frac{1}{2}\cos 2x$

(c) $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ (d) none

75. $\int \frac{(\sin^{-1}x)^2}{\sqrt{x^2-1}}\,dx=$

(*a*) $(\sin^{-1}x)^3/3$ (*b*) $-(\sin^{-1}x)^3/3$ (*c*) $(\cos^{-1}x)^3/3$ (*d*) none

76. $\int \frac{\pi^{1/x}}{x^2}\,dx=$

(*a*) doesn't exist (*b*) $\frac{-\pi^{1/x}}{\log \pi}$ (*c*) $\frac{\pi^{1/x}}{\log \pi}$ (*d*) $-\pi^{1/x}$

77. $\int \operatorname{cosec} x \; dx =$

(*a*) $\log \tan \frac{x}{2} + c$ (*b*) $-\log(\operatorname{cosec} x+\cot x)+c$

(*c*) $\log(\operatorname{cosec} x-\cot x)+c$ (*d*) all of them

78. $\int \frac{dx}{\sqrt{3-4x}} =$

(*a*) $2\sqrt{3-4x}$ (*b*) $-2\sqrt{3-4x}$ (*c*) $-\frac{1}{\sqrt{3-4x}}$ (*d*) $-\frac{1}{2}\sqrt{3-4x}$

79. $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$

(*a*) $\tan x+x$ (*b*) $x-\tan x$ (*c*) $\tan x-x$ (*d*) $2\sec^2 x$

80. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx =$

(*a*) $\tan(x\,e^x)$ (*b*) $-\sec(xe^x)$ (*c*) $-\cot(xe^x)$ (*d*) none

81. $\int \frac{\sin x}{(a+b\cos x)^2}\,dx=$

(*a*) $\frac{1}{b}\log(a+b\cos x)$ (*b*) $-\frac{1}{b}\log(a+b\cos x)$

(*c*) $\frac{1}{b\,(a+b\cos x)}$ (*d*) $-\frac{1}{b\,(a+b\cos x)}$

82. $\int \sqrt{\frac{1-\cos x}{1+\cos x}}\,dx=$

(*a*) $\log \sec\frac{x}{2}$ (*b*) $2\log\sec\frac{x}{2}$ (*c*) $\frac{1}{2}\log\sec\frac{x}{2}$ (*d*) none

83. $\int \frac{1}{2x^2-2x+1}\,dx =$

(*a*) $\tan^{-1}(2x-1)$ (*b*) $\frac{1}{2}\tan^{-1}(2x-1)$

(*c*) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{2}\right)$ (*d*) none

84. $\int x \sin\frac{x}{2}\,dx =$

(*a*) $2x\cos\frac{x}{2} - 4\sin\frac{x}{2}$ (*b*) $-2x\cos\frac{x}{2} - 4\sin\frac{x}{2}$

(*c*) $-2x\cos\frac{x}{2} + 4\sin\frac{x}{2}$ (*d*) none

85. $\int (\sin^{-1}x + \cos^{-1}x)\,dx =$

(*a*) $2x\sin^{-1}x - \sqrt{1-x^2}$ (*b*) $\frac{\pi x}{2}$ (*c*) $2x\sin^{-1}x + \sqrt{1-x^2}$ (*d*) none

86. $\int e^{-5}\,dx =$

(*a*) $-\frac{1}{5}e^{-5}$ (*b*) $-\frac{1}{5}e^{-5x}$ (*c*) $\frac{1}{5}e^{-5}$ (*d*) xe^{-5}

87. $\int \cos 4x \cos 2x\,dx =$

(*a*) $-\frac{1}{4}\left[\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right]$ (*b*) $\frac{1}{4}\left[\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right]$

(*c*) $\frac{1}{4}\left[\frac{\sin 6x}{3} + \sin 2x\right]$ (*d*) none

88. $\int (2-6x^2)^{-1/2}\,dx =$

(*a*) $\frac{1}{\sqrt{2}}\sin^{-1}(3x)$ (*b*) $\frac{1}{\sqrt{6}}\sin^{-1}(\sqrt{3}x)$

(*c*) $\frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{3x})$ (*d*) none

89. $\int \frac{e^x(1+x\log x)}{x}\,dx =$

(*a*) $e^x \log x$ (*b*) $e^x\frac{\log x}{x}$ (*c*) $e^x/\log x$ (*d*) none

90. If $\int \frac{1}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}}\log\tan\frac{x+\alpha}{2}$, then $\alpha =$

(*a*) 0 (*b*) $\pi/2$ (*c*) $\pi/4$ (*d*) none

91. $\int \frac{x}{(x-3)(x+1)}\,dx$

(*a*) $\frac{3}{4}\log(x-3)+\frac{1}{4}\log(x+1)$ (*b*) $\frac{1}{4}\log(x+1)+\frac{3}{4}\log(x-3)$

(*c*) $\frac{1}{4}\{\log(x-3)+\log(x+)\}$ (*d*) none

92. $\int \sec^6 x \sin^4 x\,dx=$

(*a*) $\frac{\sec^4 x}{4}$ (*b*) $\frac{\tan^5 x}{5}$ (*c*) can't find (*d*) none

93. $\int \frac{(e^{-x}+1)^3}{e^x}\,dx=$

(*a*) $-(e^{-x}+1)^4/4$ (*b*) $(e^x+1)^4/4$

(*c*) $(e^{-x}+1)^4/4$ (*d*) $\frac{1}{2}(e^{-x}+1)^2$

94. $\int \frac{2x^2}{\sqrt{1-x^6}}\,dx=$

(*a*) $\frac{2}{3}\sin^{-1}(x^3)$ (*b*) $\frac{1}{3}\sin^{-1}(x^3)$ (*c*) $\sin^{-1}(x^3)$ (*d*) none

95. $\int (4\tan^2 x-5\cot^2 x)\,dx=$

(*a*) $4+\operatorname{cosec} x$ (*b*) $4\tan x-5\cot x-x$

(*c*) $4\tan x+5\cot x+x$ (*d*) none

96. $\int \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx=$

(*a*) $x+\log x$ (*b*) $x^2/2+\log x+2$

(*c*) $2x+x^2/2+\log x$ (*d*) none

97. $\int \sec x(\sec x-\tan x)\,dx=$

(*a*) $\sec x-\tan x$ (*b*) $\sec x+\tan x$ (*c*) $\tan x-\sec x$ (*d*) none

98. $\int 2^{x+1}\,dx=$

(*a*) 2^{x+1} (*b*) $2^x/\log 2$ (*c*) $2^{x+1}\log 2$ (*d*) $2^{x+1}/\log 2$

99. $\int \sqrt{3x+5}\,dx=$

(*a*) $\frac{1}{2}(3x+5)^{3/2}$ (*b*) $\frac{2}{3}(3x+5)^{3/2}$ (*c*) $\frac{2}{9}(3x+5)^{3/2}$ (*d*) none

100. $\int \cot x(\tan x-\operatorname{cosec} x)\,dx=$

(*a*) $x-\cot x$ (*b*) $1-\operatorname{cosec} x$ (*c*) $x-\operatorname{cosec} x$ (*d*) $x+\operatorname{cosec} x$

101. $\int \frac{e^x-e^{-x}}{e^x}\,dx=$

(*a*) e^x-e^{-2x} (*b*) $x+\frac{e^{-2x}}{2}$ (*c*) $x-\frac{e^{-2x}}{2}$ (*d*) $x+2e^{-2x}$

102. $\int \{x^{-1}+(\frac{1}{5})^{-x}\}\, dx=$

(*a*) $\log x+5^x \log 5$ (*b*) $\log x+5^x/\log 5$

(*c*) $\log x-5^x/\log 5$ (*d*) none

103. $\int \sin^2 (x+1)\, dx=$

(*a*) $\frac{1}{2}\{x+\frac{1}{2}\sin(2x+1)\}$ (*b*) $\frac{1}{2}\{x-\frac{1}{2}\sin(2x+2)\}$

(*c*) $\frac{1}{2}\{x+\frac{1}{2}\sin 2(x+1)\}$ (*d*) none

104. $\int (x^{-2}-1)(x+1/x)^5\, dx=$

(*a*) $(x+\frac{1}{x})^6$ (*b*) $\frac{(x+\frac{1}{x})^6}{6}$ (*c*) $-\frac{(x+\frac{1}{x})^6}{6}$ (*d*) none

105. $\int x^3 (2-x^4)^3\, dx=$

(*a*) $\frac{(2-x^4)^4}{4}$ (*b*) $-\frac{(x^4-2)^4}{16}$ (*c*) $\frac{(2-x^4)^4}{16}$ (*d*) none

106. $\int (3^x+1)^4\, 3^x\, dx=$

(*a*) not possible (*b*) $(3^x+1)^5/5$ (*c*) $(3^x+1)^5/\log 243$ (*d*) none

107. $\int x^2 e^{x^3}\, dx=$

(*a*) $\frac{e^{x^3}}{3}$ (*b*) $\frac{e^{x^3}}{9}$ (*c*) e^{x^3} (*d*) none

108. $\int x^5 \log x\, dx=$

(*a*) $\frac{x^6 \log x}{6}-\frac{1}{6}x^6$ (*b*) $x^6 \log x-\frac{1}{36}x^6$

(*c*) $\frac{x^6 \log x}{6}-\frac{1}{36}x^6$ (*d*) done

109. $\int \frac{\sec^5 x}{\operatorname{cosec} x}\, dx=$

(*a*) $\frac{\sec^4 x}{4}$ (*b*) $\frac{\sec^6 x}{6}$ (*c*) $\frac{\sec^5 x}{5}$ (*d*) none

110. $\int \{x \log x\}^{-1}\, dx=$

(*a*) $(\log x)^2$ (*b*) $\log(\log x)$ (*c*) $x \log x$ (*d*) none

111. $\int \frac{dx}{\sqrt{25-9x^2}}=$

(*a*) $\frac{1}{15}\sin^{-1}\frac{3x}{5}$ (*b*) $\frac{1}{5}\sin^{-1}\frac{3x}{5}$ (*c*) $\frac{1}{3}\sin^{-1}\frac{3x}{5}$ (*d*) none

Answers

1. (*b*) 2. (*d*) 3. (*d*) 4. (*b*) 5. (*a*) 6. (*b*) 7. (*c*) 8. (*b*) 9. (*b*) 10. (*a*)
11. (*c*) 12. (*b*) 13. (*d*) 14. (*c*) 15. (*d*) 16. (*b*) 17. (*b*) 18. (*d*) 19. (*a*)
20. (*b*) 21. (*c*) 22. (*b*) 23. (*a*) 24. (*a*) 25. (*b*) 26. (*c*) 27. (*c*) 28. (*c*)
29. (*c*) 30. (*b*) 31. (*d*) 32. (*c*) 33. (*d*) 34. (*c*) 35. (*a*) 36. (*c*) 37. (*b*)
38. (*a*) 39. (*d*) 40. (*b*) 41. (*c*) 42. (*c*) 43. (*b*) 44. (*d*) 45. (*b*) 46. (*b*)
47. (*c*) 48. (*a*) 49. (*c*) 50. (*d*) 51. (*b*) 52. (*d*) 53. (*a*) 54. (*a*) 55. (*b*)
56. (*c*) 57. (*c*) 58. (*b*) 59. (*a*) 60. (*b*) 61. (*d*) 62. (*c*) 63. (*b*) 64. (*b*)
65. (*d*) 66. (*c*) 67. (*b*) 68. (*a*) 69: (*d*) 70. (*c*) 71. (*d*) 72. (*c*) 73. (*d*)
74. (*c*) 75. (*d*) 76. (*b*) 77. (*d*) 78. (*d*) 79. (*c*) 80. (*a*) 81. (*c*) 82. (*b*)
83. (*a*) 84. (*c*) 85. (*b*) 86. (*d*) 87. (*c*) 88. (*b*) 89. (*a*) 90. (*c*) 91. (*a*)
92. (*b*) 93. (*a*) 94. (*a*) 95. (*c*) 96. (*c*) 97. (*c*) 98. (*d*) 99. (*c*)
100. (*d*) 101. (*b*) 102. (*b*) 103. (*c*) 104. (*c*) 105. (*b*) 106. (*c*)
107. (*a*) 108. (*c*) 109. (*a*) 110. (*b*) 111. (*c*)

5.45 Definite integrals and application

1. $\int_0^3 \frac{dx}{x^2+9} =$

(*a*) $\pi/4$ (*b*) $\pi/12$ (*c*) $\frac{1}{4}$ (*d*) $1/12$

2. $\int_0^\pi \sqrt{x}\, dx =$

(*a*) $\frac{2}{3}\sqrt{\pi}$ (*b*) $\frac{1}{2\sqrt{\pi}}$ (*c*) $\frac{2\pi\sqrt{\pi}}{3}$ (*d*) none

3. $\int_{\frac{1}{2}}^{1/\sqrt{2}} \frac{dx}{x^2\sqrt{1-x^2}} =$

(*a*) 1 (*b*) $\sqrt{3}-1$ (*c*) $1-\sqrt{3}$ (*d*) $-\sqrt{3}-1$

4. $\int_1^\infty \frac{dx}{x^2} =$

(*a*) ∞ (*b*) 1 (*c*) -1 (*d*) 0

5. $\underset{n\to\infty}{Lt} \left\{\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots + \frac{1}{n+n}\right\} =$

(*a*) $\frac{1}{2}$ (*b*) log 2 (*c*) $\frac{1}{2}$ log 2 (*d*) none

6. $\int_0^1 \log\left(\frac{1-x}{x}\right) dx =$

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) none

7. The area bounded by the curve $y=x^2-x-6$, the x-axis and the lines $x=0$ and $x=2$ is

(*a*) 11 (*b*) $\frac{34}{3}$ (*c*) $\frac{32}{3}$ (*d*) none

8. $\int_0^{\pi/4} \cos 4x \cos 2x \, dx=$

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{1}{12}$

9. $\int_2^4 \frac{dx}{\sqrt{16-x^2}}=$

(*a*) $\pi/2$ (*b*) $\pi/4$ (*c*) $\pi/3$ (*d*) $\pi/8$

10. $\int_0^{\pi} \sin \frac{x}{2} \cos \frac{x}{2} \, dx=$

(*a*) 1 (*b*) 0 (*c*) 3 (*d*) none

11. The area bounded by the curve $y=\sin x$ and the x-axis from $x=0$ to 2π is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

12. $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}=$

(*a*) 0 (*b*) $\pi/2$ (*c*) π (*d*) 2π

13. $\int_0^1 x \log x \, dx$ (assume $\underset{x \to 0}{Lt}\, x^2 \log x=0$) is equal to

(*a*) $\frac{1}{2}$ (*b*) $-\frac{1}{2}$ (*c*) $\frac{1}{4}$ (*d*) $-\frac{1}{4}$

14. $\int_0^{\pi/2} \sqrt{1+\sin x} \, dx$

(*a*) $\sqrt{2}-2$ (*b*) $2\sqrt{2}$ (*c*) 2 (*d*) -2

15. The area bounded by the parabola $x=4-y^2$ and y-axis is

(*a*) 64/3 (*b*) 16/3 (*c*) 32/3 (*d*) none

16. $\int_0^{\pi/4} \cos^3 x \, dx =$

(*a*) 2/3 (*b*) $\frac{5}{4\sqrt{2}}$ (*c*) $10\sqrt{2}$ (*d*) none

17. The area enclosed between the curve $y=(x-1)(2-x)$ and the x-axis is

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{4}{3}$

18. $\int_{-\pi/4}^{\pi/4} \tan^5 x \sec x \, dx =$

(*a*) 0 (*b*) $\frac{3}{5}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{1}{3}$

19. **The area of the region** $\{(x, y): b^2x^2+a^2y^2 \leqslant a^2b^2\}$ **is**

(*a*) 2π (*b*) $2\pi ab$ (*c*) πab (*d*) π/ab

20. $\int_0^{\pi/4} \frac{\sec^2 t \, dt}{\sqrt{4-\tan^2 t}} =$

(*a*) $\pi/6$ (*b*) $\pi/3$ (*c*) $\frac{1}{2}$ (*d*) none

21. $\int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} \, dx=$

(*a*) $\frac{1}{2}$ (*b*) $\pi/2$ (*c*) $\pi/4$ (*d*) none

22. **The area of the ellipse** $\frac{x^2}{9}+\frac{y^2}{4}=1$ **in square units is**

(*a*) 36π (*b*) 6π (*c*) 12π (*d*) none

23. $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}=$

(*a*) π/ab (*b*) $\pi/2ab$ (*c*) $2\pi/ab$ (*d*) none

24. $\int_0^{\pi/4} \frac{2 \tan x \sec^2 x}{\tan^4 x+1} \, dx=$

(*a*) $\pi/2$ (*b*) π (*c*) $\pi/4$ (*d*) none

25. $\int_0^{\pi} \sin^m x \cos^{2n+1} x \, dx=$

(*a*) $\frac{m^2}{2n}$ (*b*) $\frac{m+2n}{3}$ (*c*) 0 (*d*) none

26. $\int_{-2}^{2} \frac{8}{x^2+4} \, dx=$

(*a*) π (*b*) 2π (*c*) $\pi/2$ (*d*) none

27. $\int_0^1 \sqrt{x}(1-x) \, dx=$

(*a*) $\frac{4}{15}$ (*b*) $\frac{2}{15}$ (*c*) $\frac{8}{15}$ (*d*) none

28. $\int_{-1}^{1} (1-x^2+2x^3) \, dx=$

(*a*) 0 (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{4}{3}$

29. The area of the region bounded by the curves $y=2^x$ and $y=2x-x^2$ between $x=0$ and $x=2$ is

(*a*) $\dfrac{2}{\log 2}-\frac{4}{3}$ (*b*) $\dfrac{3}{\log 2}-\frac{4}{3}$ (*c*) $\dfrac{3}{\log\ 2}-\frac{2}{3}$ (*d*) none

30. $\displaystyle {}_0\!\int^5 \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{5-x}}=$

(*a*) 5 (*b*) $\sqrt{5}$ (*c*) 5/2 (*d*) $\sqrt{5}/2$

31. $\displaystyle {}_0\!\int^{\pi/4} t\sec^2 t\,dt=$

(*a*) $\dfrac{\pi}{4}$ (*b*) $\dfrac{\pi}{4}-\log\dfrac{1}{\sqrt{2}}$ (*c*) $\dfrac{\pi}{2}-\frac{1}{2}\log 2$ (*d*) none

32. $\displaystyle {}_0\!\int^{\pi/4} \tan^2 x\ dx=$

(*a*) $\pi/4$ (*b*) $1-\pi/4$ (*c*) $\pi/4-1$ (*d*) $1+\pi/4$

33. $\displaystyle {}_0\!\int^{\pi/3} \tan x\ dx=$

(*a*) log 2 (*b*) 0 (*c*) 1 (*d*) 1–log 2

34. The area bounded by the curve $\sqrt{x}+\sqrt{y}=2$, the x–axis and the lines $x=0$ and $x=4$ is

(*a*) $\frac{4}{3}$ (*b*) $\frac{8}{3}$ (*c*) $\frac{64}{3}$ (*d*) none

35. $\displaystyle \underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} \frac{n}{n^2+r^2}=$

(*a*) log 2 (*b*) $\pi/2$ (*c*) $\pi/4$ (*d*) none

36. $\displaystyle {}_0\!\int^{\pi/2} \cos^4 x \sin x\ dx=$

(*a*) $\frac{1}{3}$ (*b*) $\frac{-1}{5}$ (*c*) $\frac{1}{5}$ (*d*) none

37. $\displaystyle {}_3\!\int^{11} \sqrt{2x+3}\,dx=$

(*a*) 98/3 (*b*) 49/3 (*c*) 196/3 (*d*) none

38. The area bounded by $y=x^2$, $y=0$ and $x=3$ is

(*a*) 0 (*b*) 9 (*c*) 9/2 (*d*) none

39. $\int_{-\infty}^{-1} \frac{4}{1+t^2}\, dt =$

(*a*) $-\pi/4$ (*b*) $-\pi/2$ (*c*) π (*d*) $-\pi$

40. $\int_1^4 \frac{dx}{x^2-2x+10} =$

(*a*) $\tan^{-1} 3 - \tan^{-1} \frac{1}{3}$ (*b*) $\pi/4$ (*c*) $\pi/12$ (*d*) none

41. $\int_0^{\pi/4} \cos^2 x \sin^2 x\, dx =$

(*a*) $\pi/4$ (*b*) $\pi/32$ (*c*) $\pi/8$ (*d*) none

42. $\int_0^1 \frac{1}{(x+1)\sqrt{x+1}}\, dx =$

(*a*) $\sqrt{2}-2$ (*b*) $2-\sqrt{2}$ (*c*) $-2-\sqrt{2}$ (*d*) none

43. $\int_1^e \log x\, dx =$

(*a*) 0 (*b*) 1 (*c*) $e-1$ (*d*) e

44. $\underset{n \to \infty}{Lt} \sum_{r=1}^{n-1} \frac{1}{\sqrt{n^2-r^2}} =$

(*a*) $\pi/4$ (*b*) $\pi/2$ (*c*) π (*d*) none

45. **The area bounded by $y=x^3$, $x=0$ and $y=8$ is**

(*a*) 12 (*b*) 4 (*c*) $\frac{1}{4}$ (*d*) none

46. $\int_0^{\pi/2} \sin 2x \log \tan x\, dx =$

(*a*) 0 (*b*) π (*c*) 2 (*d*) none

47. $\int_0^{\pi/4} \sin^2 x \sec^4 x\, dx =$

(*a*) $\frac{1}{5}$ (*b*) 0 (*c*) $\frac{1}{3}$ [✓ (*d*) none]

48. $\int_0^3 e^3\, dx =$

(*a*) 0 [✓ (*b*) e^9] (*c*) $3e^3$ (*d*) none

49. $\int_1^e \frac{\sqrt{\log t}}{t}\, dt =$

(*a*) $\frac{3}{2}$ (*b*) 3 (*c*) $\frac{1}{2}$ (*d*) $\frac{2}{3}$

50. $\int_0^{\pi/6} \frac{\tan \theta}{1-\tan^2 \theta}\, d\theta =$

(*a*) $\frac{1}{4} \log 2$ (*b*) $\log 2$ (*c*) $\frac{1}{2} \log 2$ (*d*) none

51. $\int_{3\pi}^{5\pi} \cos \frac{\theta}{2} d\theta =$

(*a*) 2 (*b*) 4 (*c*) 0 (*d*) none

52. $\int \{x + \int_{-1}^{1} t^3 dt\} dx =$

(*a*) $x^2/2$ (*b*) $x^2/2 + 2x$ (*c*) $\frac{1}{2}(x^2+x)$ (*d*) none

53. The area bounded by the curve $x^2y = x^2 + 5$, the x-axis and the lines $x=1$ and $x=3$ is

(*a*) $\frac{5}{3}$ (*b*) $\frac{11}{3}$ (*c*) $\frac{16}{3}$ (*a*) none

54. $\underset{n\to\infty}{Lt} \sum_{r=1}^{n} \frac{1}{n} \log \frac{r}{n}$, assuming $\underset{x\to 0}{Lt}\, x \log x = 0$, is equal to

(*a*) 1 (*b*) 0 (*c*) e (*d*) –1

55. The slope of a curve at any point (x, y) is $8x$. What is the equation of the curve if it passes through (1, 3)?

(*a*) $y = 2x^2 + 1$ (*b*) $y = 4x^2 - 1$ (*c*) $3x^2 = y$ (*d*) none

56. $\int_0^{\pi/2} \frac{\cos x\, dx}{4 - \sin^2 x} =$

(*a*) $\frac{1}{2} \log 4$ (*b*) $\frac{1}{4} \log 3$ (*c*) $\frac{1}{4} \log 2$ (*d*) none

57. $\int_{-1}^{2} \frac{x}{\sqrt{x^2+9}} dx =$

(*a*) $\sqrt{13} - \sqrt{8}$ (*b*) $\sqrt{11} - \sqrt{10}$ (*c*) $\sqrt{13} - \sqrt{10}$ (*d*) none

58. $\int_0^{\pi/2} \frac{dx}{9\cos^2 x + 16 \sin^2 x} =$

(*a*) $\frac{\pi}{12}$ ✓(*b*) $\frac{\pi}{24}$ (*c*) $\frac{\pi^2}{12}$ (*d*) none

$\frac{\pi}{2\times4\times3}$

$2 \quad \frac{\pi}{2\times3\times4}$

59. The area of the region enclosed between $y = \cos x$ and $y = \sin x$ from $x=0$ to $x = \pi/4$ is

(*a*) $\sqrt{2}$ (*b*) $1 + \sqrt{2}$ (*c*) $\sqrt{2} - 1$ (*d*) none

60. $\int_0^{\pi/2} x \sin x\, dx =$

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) none

61. $\int_{-\pi/4}^{\pi/4} \sec^2 x \tan^2 x\, dx =$

(*a*) 0 (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) none

62. $\int_0^{\pi/2} x^2 \cos x \, dx =$

(a) π^2-1 (b) $\pi^2/4-2$ (c) $\pi^2/4-1$ (d) none

63. $\int_0^4 (2x+1)\sqrt{2x+1} \, dx =$

(a) $\frac{242}{5}$ (b) $\frac{249}{5}$ (c) $\frac{119}{5}$ (d) none

64. The area of region of the curve $y^2=4-x^2$ lying above x-axis is

(a) 2π (b) π (c) 4π (d) none

65. $\int_0^{\pi} \frac{x \, dx}{1+\sin x} =$

(a) 0 (b) $\pi/4$ (c) $\pi/2$ [✓ (d) π]

66. A body moving in a straight line has an acceleration $6t^2$ at any time t. If it starts from rest, what is its velocity when $t=2$? (a) 2 (b) 8 (c) 16 (d) none

67. The area bounded by the parabola $y^2=4x$ and its latus rectum is

(a) 8/3 (b) 4/3 (c) 16/3 (d) none

68. $\int_0^{\frac{1}{2}} \frac{4(\sin^{-1}x)^3}{\sqrt{1-x^2}} \, dx =$

(a) $\pi^3/6^3$ (b) $\pi^4/3^4$ (c) $\pi^3/3^3$ (d) $\pi^4/6^4$

69. $\int_0^{\pi/2} \cos 13x \sin 15x \, dx =$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) none

70. $\int_0^{\pi/4} \sin^2 x \, dx =$

(a) $\frac{\pi}{8}$ (b) $\frac{1}{8}(\pi-2)$ (c) $\frac{\pi}{4}-2$ (d) none

71. The area of the region bounded by the curves $y=x^2+1$ and $y=2x+1$ is (a) 1/3 (b) 4/3 (c) 8/3 (d) 16/3

72. $\int_{-\frac{1}{2}}^{1} \frac{1}{\sqrt{1-x^2}} \, dx =$

(a) $2\pi/3$ (b) $4\pi/3$ (c) $-\pi/3$ (d) none

73. $\int_1^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}} =$

(a) $\pi/2$ (b) $\pi/4$ (c) $-\pi/4$ (d) none

74. $\int_2^0 \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx =$

(*a*) doesn't exist (*b*) $\pi/6$ (*c*) 0 (*d*) none

75. $\int_0^{\pi/4} \tan x \sec^3 x\, dx =$

(*a*) $2\sqrt{2}/3$ (*b*) $1/2$ (*c*) $2/3-1/\sqrt{2}$ (*d*) none

76. $\int_0^{\pi} \sin \frac{3x}{4} \cos \frac{3x}{4}\, dx =$

(*a*) $\frac{4}{3}$ (*b*) 0 (*c*) $\frac{2}{3}$ (*d*) none

77. $\int_{-3}^{-1} \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx =$

(*a*) $\frac{10}{9}$ (*b*) $\frac{4}{9}$ (*c*) $-\frac{1}{9}$ (*d*) none

78. $\int_0^{\pi/2} \sin 3x \cos 5x\, dx =$

(*a*) $\frac{1}{2}$ (*b*) 0 (*c*) $-\frac{1}{2}$ (*d*) none

79. $\int_0^{\pi/2} (\cos x - \sin x)^2\, dx =$

(*a*) $\pi/2$ (*b*) $1-\pi/2$ (*c*) $\pi/2-1$ (*d*) none

80. $\int_0^{\pi/2} \sqrt{\sin x} \cos x\, dx =$

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*a*) $\frac{4}{3}$ (*d*) none

81. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\, dx =$

(*a*) 0 (*b*) $\pi/4$ (*c*) $\pi/2$ (*d*) none

82. $\int_0^1 x\sqrt{x^2+1}\, dx =$

(*a*) $2\sqrt{2}/3$ (*b*) $\frac{1}{3}(2\sqrt{2}-1)$ (*c*) $\frac{1}{3}(2\sqrt{2}+1)$ (*d*) none

83. $\int_0^{\pi/2} \frac{\cos x}{\sqrt{1+\sin 2x}}\, dx =$

(*a*) $\pi/4$ (*b*) $\pi/2$ (*c*) $1/4$ (*d*) none

84. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\, dx =$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) none

85. $\int_0^1 \frac{x}{x+1}\, dx =$

(*a*) $1-\log 2$ (*b*) $\log 2$ (*c*) $\log 2 - 1$ (*d*) none

86. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{9-x^2} =$

(*a*) $\frac{1}{3}\log\frac{7}{5}$ (*b*) $\frac{1}{4}\log\frac{7}{5}$ (*c*) $\log\frac{7}{5}$ (*d*) none

87. $\int_0^{\pi/2} \frac{\sin 2x}{4-\sin^2 x}\, dx=$

(*a*) $\log\frac{2}{3}$ (*b*) $\log\frac{4}{3}$ (*c*) $\log\frac{8}{3}$ (*d*) none

88. $\int_0^{\pi/2} \frac{\sin x}{(1+\cos x)^2}\, dx=$

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$ (*c*) $-\frac{1}{2}$ (*d*) none

89. $\int_0^{\pi/2} \sin 2x \cos x \; dx=$

(*a*) 0 (*b*) $\frac{1}{3}$ (*c*) 1 (*d*) $\frac{2}{3}$

90. $\int_0^{\pi} \sin 3x \sin x \; dx=$

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{4}$ (*d*) none

91. $\int_0^4 \sqrt{16-x^2}\; dx =$

(*a*) $\pi/4$ (*b*) 16π (*c*) 4π (*d*) none

92. $\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9-4x^2}} =$

(*a*) $\frac{\pi}{12}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{2}$ (*d*) none

93. $\int_0^{\sqrt{\log 2}} x\, e^{x^2}\, dx =$

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{2}(e-1)$ (*c*) $\frac{1}{2}(e^2-1)$ (*d*) none

94. $\int_0^1 (x^2+x-1)^{10}(2x+1)\, dx=$

(*a*) 0 (*b*) $\frac{1}{11}$ (*c*) $\frac{2}{11}$ (*d*) none

95. If $\frac{1}{5}(x^{10}-1) = \int_0^{f(x)} (t+1)^4 dt$, then $f(x)=$

(*a*) x^5-1 (*b*) x^2-1 (*c*) x^5 (*d*) x^2+1

96. $\int_0^1 (\sqrt{x^3}+\sqrt[3]{x^2})\, dx=$

(*a*) 1 (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) none

97. $\int_{-1}^{1} (x^3+1)^2 x^2 \, dx =$

(*a*) 0 (*b*) $\frac{8}{3}$ (*c*) $\frac{8}{9}$ (*d*) none

98. $\int_{0}^{1} \cosh x \, dx =$

(*a*) 0 (*b*) $\frac{1}{2}(e+e^{-1})$ (*c*) $\frac{1}{2}(e-e^{-1})$ (*d*) none

99. $\int_{-2}^{2} \sinh x \, dx =$

(*a*) 0 (*b*) e^2-e^{-2} (*c*) $2(e^2+e^{-2})$ (*d*) none

Answers

1. (*b*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*b*) 6. (*a*) 7. (*b*) 8. (*c*) 9. (*c*). 10. (*a*)
11. (*d*) 12. (*c*) 13. (*d*) 14. (*c*) 15. (*c*) 16. (*d*) 17. (*c*) 18. (*a*) 19. (*c*)
20. (*a*) 21. (*c*) 22. (*b*) 23. (*b*) 24. (*c*) 25. (*c*) 26. (*b*) 27. (*a*) 28. (*d*)
29. (*b*) 30. (*c*) 31. (*c*) 32. (*b*) 33. (*a*) 34. (*b*) 35. (*c*) 36. (*c*) 37. (*a*)
38. (*b*) 39. (*c*) 40. (*d*) 41. (*b*) 42. (*b*) 43. (*b*) 44. (*b*) 45. (*a*) 46. (*a*)
47. (*c*) 48. (*c*) 49. (*d*) 50. (*a*) 51. (*b*) 52. (*a*) 53. (*c*) 54. (*d*) 55. (*b*)
56. (*b*) 57. (*c*) 58. (*b*) 59. (*c*) 60. (*a*) 61. (*c*) 62. (*b*) 63. (*a*) 64. (*a*)
65. (*d*) 66. (*c*) 67. (*a*) 68. (*d*) 69. (*d*) 70. (*b*) 71. (*b*) 72. (*a*) 73. (*b*)
74. (*a*) 75. (*d*) 76. (*d*) 77. (*a*) 78. (*c*) 79. (*c*) 80. (*a*) 81. (*b*) 82. (*b*)
83. (*a*) 84. (*b*) 85. (*a*) 86. (*a*) 87. (*b*) 88. (*c*) 89. (*d*) 90. (*a*) 91. (*c*)
92. (*b*) 93. (*a*) 94. (*c*) 95. (*b*) 96. (*a*) 97. (*c*) 98. (*c*) 99. (*a*)

SECTION 6: LIST OF FORMULAE

6.1 ALGEBRA

6.11 Set theory

1. An ordered pair of two elements a and b is denoted by (a, b); two ordered pairs (a, b) and (c, d) are equal iff $a=c$ and $b=d$.
2. The **product** or **Cartesian product** or **cross product** of two sets A and B is $A \times B = \{(a, b) \mid a \in A, b \in B\}$; the Cartesian plane is the product set $R \times R$
3. If $n(A)=p$ and $n(B)=q$, then $n(A \times B)=pq$.
4. $A \times B \neq B \times A$ unless $A=B$ or one of A and B is empty.
5. A **relation** R from A to B is a subset of $A \times B$; if $(a, b) \in R$, we write a R b; if $B=A$, then R is a relation in A.
6. Let R be a relation in a set A. Then R is called (*i*) **reflexive** if $(a, a) \in R, \ \forall a \in A$ (*ii*) **symmetric** if $(a, b) \in R \Rightarrow (b, a) \in R$ (*iii*) **anti-symmetric** if $(a, b) \in R$ and $(b, a) \in R \Rightarrow a=b$ (*iv*) **transitive** if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ (*v*) **equivalence relation** if R is reflexive, symmetric and transitive.
7. The **domain of a relation** R from A to $B = \{a \mid a \in A, (a, b) \in R\}$, and the **range** of $R = \{b \mid b \in B, (a, b) \in R\}$.
8. A **function** f from A to B ($f: A \to B$) is a relation from A to B in which each $a \in A$ appears in one and only one ordered pair belonging to f.
9. Let $f: A \to B$ be a function. Then A is called its **domain**; B is its **codomain**; if $a \in A$, then $f(a) \in B$ is called the **image** of 'a' under f; $\{f(a) \mid a \in A, f(a) \in B\}$ is called the **image set** or **the range** of f.
10. Let $f: A \to B$ be a function, f is called (*i*) **one-one** if $f(a)=f(a')$ $\Rightarrow a=a'$ (i.e. $a \neq a' \Rightarrow f(a) \neq f(a')$) (*ii*) **onto** if every element of B is the image of at least one element of A (i.e. the range $=$ codomain) (*iii*) **identity** function if $f(a)=a, \ \forall a \in A$ (*iv*) a **constant** function it $f(a)=b \in B, \ \forall a \in A$ i.e. the range of f is a singleton.

11. Let $f: A \to B$ and $g: B \to C$ be two functions. The **composition function** of f and g i.e. $(gof): A \to C$ in defined by $(gof)(a) = g(f(a))$, $\forall a \in A$.
12. $gof = g(f) \neq f(g) = fog$.
13. Let $f: A \to B$ be *a* function and $b \in B$. Then $f^{-1}(b)$, the **inverse** of $b = \{a \mid a \in A, f(a) = b\}$; $f^{-1}(b) \subset A$.
14. Let $f: A \to B$ be *a* function. If $f^{-1}: B \to A$ assigns to each $b \in B$, a unique element $f^{-1}(b) \in A$, then it is called the **inverse function** of f.
15. If A $f: A \to B$ is both one-one and onto, then f^{-1} exists and it is also both one-one and onto; $fof^{-1} = f^{-1}of =$ identity function.

6.12 Number theory

1. Let $a \neq 0$ and $b \in Z$. Then $a \mid b$ if $\exists\ b \in Z \ni b = ka$; '$b$' is said to have '$a$' as *a* **factor**; '$a$' is *a* **factor** of '$b$'; '$b$' is *a* **multiple** of 'a'; 'a' is *a* **divisor** of 'b'.
2. $a \mid 0$, $a \neq 0$; $a \mid a$, $a \neq 0$; $a \mid b \Rightarrow a \mid bx$, $x \in Z$; $a \mid b$ and $a \mid c \Rightarrow a \mid (pb + qc)$ $p, q \in Z$.
3. Divisibility on N is reflexive and transitive but divisibility on Z is transitive but not reflexive.
4. If $a \neq 0, b \in Z$, then $\exists\ q, r \in Z \ni b = aq + r$, $0 \leqslant r < |b|$ (**Division Algorithm**).
5. An integer d is called *a* **greatest common divisor** (*g.c.d.*) of two integers a and b if (*i*) $d \mid a$ and $d \mid b$ and (*ii*) whenever $c \mid a$ and $c \mid b$, then $c \mid d$.
6. If d is *a* g.c.d. of a and b, then $-d$ is also *a* g.c.d.; the positive g.c.d. of a and b is denoted by (a, b).
7. For any two non-zero integers, there is always *a* g.c.d. and it can be obtained by repeated application of Division Algorithm (**Euclidean Algorithm**).
8. $(a, b) = pa + qb$, $p, q \in Z$ i.e. g.c.d. of two integers can be expressed as *a* linear combination of them (p and q are not unique).
9. An integer $p \neq 0, \pm 1$ is called *a* **prime** if its only divisors are ± 1 and $\pm p$; all other integers except 0 and ± 1 are called **composite numbers**; 0 and ± 1 are neither prime nor composite.

10. Two integers a and b are said to be **relatively prime** if their g.c.d. $(a, b)=1$
11. The number of primes is infinite.
12. If p is prime and $p|ab$, then $p|a$ or $p|b$; if $(a, b)=1$ and $a|bc$, then $a|c$.
13. Let $a, b, m>0, \in Z$. Then 'a' is said to be **congruent** to 'b' modulo m i.e. $a \equiv b \pmod{m}$ if $m|(a-b)$.
14. The congruence relation is an **equivalence relation** i.e. (*i*) $a \equiv a \pmod{m}$ (**reflexive**) (*ii*) $a \equiv b \pmod{m} \Rightarrow b \equiv a \pmod{m}$ (**symmetric**) (*iii*) $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m} \Rightarrow a \equiv c \pmod{m}$ (**transitive**).
15. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a \pm c = (b \pm d) \pmod{m}$ and $ac \equiv bd \pmod{m}$; $a \pm x \equiv (b \pm x) \pmod{m}$ and $ax \equiv bx \pmod{m}$; $a^n \equiv b^n \pmod{m}$.
16. $ca \equiv cb \pmod{m} \Rightarrow a \equiv b \pmod{m}$ only when $(c, m)=1$.
17. The congruence $ax \equiv b \pmod{m}$ will have (*i*) no solution it $(a, m) \nmid b$ (*ii*) many solutions if $(a, m)|b$ (*iii*) a unique solution if $(a, m)=1$.

6.13 Matrices and Determinants

1. A **matrix** is a rectangular arrangement of numbers enclosed within brackets; if a matrix has m **rows** and n **columns**, then it is called a **mxn matrix** or a matrix of **order (type)** $m \times n$.
2. If in a matrix, the number of rows = the number of columns $=n$, then it is called a **square** matrix of order n.
3. A matrix having only one row of n numbers (elements) is called a **row matrix** and its order is $n \times 1$; a matrix having only one column of n numbers (elements) is called a **column matrix** of order $1 \times n$.
4. If the elements of a matrix are all zeroes, then it is called a **null matrix** or **zero matrix** and is denoted by O.
5. In a square matrix, the left (top)–to–right diagonal is called its **principal** diagonal and the elements of the principal diagonal are called the **diagonal elements**.
6. If in a **square** matrix, all the non-diagonal elements are zeroes, then it is called a **diagonal matrix**; if in a **diagonal** matrix all the diagonal elements are equal, then it is called a **scalar**

matrix; if in a **scalar** matrix, all the diagonal elements are equal to 1, then it is called a **unit matrix** or **identity matrix** and it is denoted by I

7. Two matrices are said to be **equal** if they have the same order and if the corresponding elements are also equal.
8. Let $A=(a_{ij})$ and $B=(b_{ij})$ be two matrices of order $m \times n$ (*i*) kA, the scalar (k) multiple of the matrix $A=(ka_{ij})$ where $k \in R$ (*ii*) $A+B$, the **sum** of A and $B=(a_{ij}+b_{ij})$; $A-B=(a_{ij}-b_{ij})$ and $A \pm B$ are also of order $m \times n$. If A and B are two matrices of different orders, $A+B$ or $A-B$ is not defined.
9. Matrix addition is **commutative** i.e. $A+B=B+A$ and **associative** i.e. $A+(B+C)=(A+B)+C$.
10. A' or A^T, the **transpose** of $A=(a_{ji})$ i.e. it is the matrix obtained from A by interchanging its rows and columns. If A is a matrix of order $m \times n$, then A' is of order $n \times m$.
11. If A is a matrix of order $m \times n$ and B is a matrix of order $p \times q$, then the product AB exists iff $n=p$ i.e. the number of columns in the first matrix is equal to the number of rows in the second matrix.
12. Let $A=(a_{ij})$ and $B=(b_{ij})$. Then $AB=(c_{ij})$ where c_{ij} is the sum of the products of the corresponding elements of ith row of A and jth column of B. If A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$, then AB is a matrix of order $m \times p$.
13. Matrix multiplication is **not commutative** $AB \neq BA$, $\forall$ A, B but **associative**, i.e. $A(BC)=(AB)C$ if $A(BC)$ and $(AB)C$ exist.
14. If a, b, c are real numbers, then (*i*) $ab=0 \Rightarrow a=0$ or $b=0$ (*ii*) $ab=ac \Rightarrow b=c$ when $a \neq 0$ (*iii*) $ab=ba$. These results do not hold good in the case of matrix multiplication.
15. If $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is a square matrix of order 2, then $a_{11}a_{22}-a_{21}a_{12}$ is defined as the **determinant** of A and it is written as det. A or $|A|=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$.

Similarly if

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \text{ then } |A|=a_{11}(a_{22}a_{33}-a_{32}a_{23})$$

$$-a_{12}(a_{21}a_{33}-a_{31}a_{23})+a_{13}(a_{21}a_{32}-a_{31}a_{22}).$$

16. A determinant has *a* value (but *a* matrix has no value) and it can be expanded along any row (or column).

17. The determinant got by suppressing the row and column containing an element is called the **minor** of that element. The **cofactor** of an element a_{ij} in $A=(a_{ij})$ is $(-1)^{i+j}$. the minor of a_{ij}.

18. If the elements of any row (or column) are multiplied by their own cofactors and added, the sum is the value of the determinant. If the elements of a row (or column) are multiplied by the cofactors of any other row (or column) and added, the sum is zero.

19. *adj* A, the **adjoint** of a square matrix is obtained by replacing the elements by their cofactors and taking the transpose.

20. $A\,(adj\,A)=(adj\,A)\,A=|A|\,I$

21. If $|A|=0$, then A is called a **singular** matrix and if $|A|\neq 0$, then A is called a **non-singular** matrix.

22. If A is a non-singular matrix, then A^{-1}, the **inverse** of $A=\dfrac{1}{|A|}\,adj\,A$; $AA^{-1}=A^{-1}A=I$.

23. (*i*) The value of a determinant is unaltered if its rows and columns are interchanged (*ii*) If two rows (or columns) of a determinant are interchanged, the determinant is multiplied by -1 (*iii*) If in a determinant two rows (or columns) are identical, then the value of the determinant is zero (*iv*) If the elements of any row (or column) are multiplied by k, the value of the determinant is multiplied by k (*v*) If in a determinant, one row (or column) is a multiple of another, then the value of the determinant is zero (*vi*) If each element in any row (or column) of a determinant is a sum of two terms, then the determinant can be expressed as a sum of two determinants (*vii*) If to the elements of any row (or column) of a determinant, the same multiples of the corresponding elements of other rows (or columns) of the determinant are added, then value of the determinant is unaltered. (*viii*) The determinant of product of two square matrices (of same order) is equal to the product of the determinants of the two matrices.

24. **Cramer's Rule:** (*i*) The solution of the system of equations $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ is given by $x=\dfrac{\Delta_1}{\Delta}$ and $y=\dfrac{\Delta_2}{\Delta}$ where $\Delta=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $\Delta_1=\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $\Delta_2=\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ (*ii*) the solution of the system of equations $a_ix+b_iy+c_iz=d_i$, $i=1, 2, 3$ is given by $x=\dfrac{\Delta_1}{\Delta}$, $y=\dfrac{\Delta_2}{\Delta}$ and $z=\dfrac{\Delta_3}{\Delta}$ where

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta_1=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$\Delta_2=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \quad \Delta_3=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

(*iii*) The solution of the above system will be unique if $\Delta \neq 0$.

6.14 Groups

1. A **binary operation** * on a non-empty set S is a rule which assigns to each ordered pair (a, b) of elements a and b, a unique element $a*b \in S$.

2. Let S be a non-empty set and *, a binary operation on it
   (*i*) **closure law:** $a*b \in S, \ \forall a, b \in S$
   (*ii*) **Commutative law:** $a*b=b*a, \ \forall a, b \in S$
   (*iii*) **Associative law:** $a*(b*c)=(a*b)*c, \ \forall a, b, c \in S$
   (*iv*) **Identity law:** $\exists \ e \in S \ni e*a=a*e=a, \ \forall a \in S$.

   Remark:—e is called the identity element of S under *. If $* \equiv +$, the identity element is called **zero** or additive identity and it is denoted by O and if $* \equiv .$, the identity is called **unity** or multiplicative identity and it is denoted by 1.

   (*v*) **Inverse law:** For each $a \in S$, $\exists \ a' \in S \ni a*a'=a'*a \equiv e$

   Remark:—a' is called the inverse of a. If $* \equiv +$, the inverse of a is called the additive inverse and it is denoted by $-a$ and if $* \equiv .$, the inverse of a is called the multiplicative inverse and it is denoted by a^{-1}.

3. A non-empty set with one or more binary operations defined on it is called an **Algebraic Structure.** An algebraic structure whose elements satisfy certain laws w.r.t. the binary operations defined on it is called an **Algebraic System.**

4. (*i*) **Z_n, the set of residue classes of integers modulo *n*.**

Let $n \in Z$ and $n \geqslant 2$, say $n=4$. Consider the relation congruence mod 4. Let

$\overline{0}$=the set of integers$\equiv 0 \pmod 4$=$\{0, \pm 4, \pm 8, ..\}$

$\overline{1}$=the set of integers$\equiv 1 \pmod 4$=$\{1, 1 \pm 4, 1 \pm 8, ..\}$

$\overline{2}$=the set of integers$\equiv 2 \pmod 4$=$\{2, 2 \pm 4, 2 \pm 8, ..\}$

$\overline{3}$=the set of integers$\equiv 3 \pmod 4$=$\{3, 3 \pm 4, 3 \pm 8, ..\}$

The subsets $\overline{0}$, $\overline{1}$, $\overline{2}$ and $\overline{3}$ are called **equivalence classes** and each integer belongs to exactly one of them. The set $\{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ or simply $\{0, 1, 2, 3\}$ is called the set of residue classes of integers modulo 4 and it is denoted by Z_4.

In general $Z_n = \{0, 1, 2, \ldots, (n-1)\}$.

(*ii*) **Binary operations in Z_n**

Let $a, b \in Zn$. $a \oplus_n b$=the remainder got by dividing $a+b$ by n and $a \otimes_n b$=the remainder got by dividing ab by n. $\oplus_n$ or $\oplus$ or simply '+' is called the addition modulo n and $\otimes_n$ or $\otimes$ or simply '×' is called the multiplication modulo n. Addition modulo n and multiplication modulo n are binary operations in Z_n.

5. **Binary operation table:** If $S=\{a_1, a_2, a_3, \ldots, a_n\}$ is a finite set, then a binary operation '*' on it can be defined by means of a table known as '* operation table' as given below:—

*	a_1	a_2	a_3	....	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$	$a_1 * a_3$	....	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$	$a_2 * a_3$	....	$a_2 * a_n$
a_3	$a_3 * a_1$	$a_3 * a_2$	$a_3 * a_3$	....	$a_3 * a_n$
.	.	.	.	....	.
.	.	.	.	....	.
.	.	.	.	....	.
a_n	$a_n * a_1$	$a_n * a_2$	$a_n * a_3$	....	$a_n * a_n$

6. Let G be a non-empty set and $*$, a binary operation defined on it. Then (G, $*$) is called a **group** if G satisfies **closure, associative, identity** and **inverse axioms.**

   Remark: closure law is redundant.

7. In a group (G, $*$), if $*$ is commutative, then G is called a **commutative** or **an Abelion group.**

8. In a group (G, $*$), if n(G) is **finite**, then it is called a **finite group**; otherwise **infinite.** n(G) is called the **order** of the group G.

9. In a group (*i*) the identity element is unique (*ii*) the inverse of an element is unique (*iii*) the inverse of the inverse of an element is itself (*iv*) the inverse of a product is equal to the product of the inverses in the reverse order (*v*) cancellation laws hold good i.e. $ab=ac \Rightarrow b=c$ and $ab=cb \Rightarrow a=c$, a, b, $c \in G$ (*vi*) the equations $ax=b$ and $ya=b$, $a, b \in G$, have unique solutions in G (*vii*) if the order of G is 3, then it is Abelion.

10. Let (G, $*$) be a group and H, a non-empty subset of G. If (H, $*$) is also a group, then H is called a **subgroup** of G. The identity element of G is the identity element of H.

11. A non-empty subset H of G will be a subgroup of G
    (*a*) iff the closure and inverse axioms are true in H.
    (*b*) iff $a*b^{-1} \in H$, $\forall a, b \in H$.

6.15 Vectors

1. An **ordered pair** (a_1, a_2) of real numbers is called a **2–D Vector** and a **3–D Vector** is an **ordered triple** (a_1, a_2, a_3) of real numbers. The real numbers a_1, a_2, a_3 are called the **components** of the vector.

2. Obviously a vector is a **row matrix.** If $\vec{a}$ and $\vec{b}$ are vectors of same dimension, then $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ and $k\vec{a}$, $k \in R$ are defined as in the case of matrices. For example $2(a_1, a_2, a_3)+3(b_1, b_2, b_3)=(2a_1+3b_1, 2a_2+3b_2, 2a_3+3b_3)$.

3. The **null vector** or **zero vector** is a vector whose components are all zeroes and it is denoted by $\vec{O}$.

4. In 2–D, $\hat{i}=(1, 0)$ and $\hat{j}=(0, 1)$ and in 3–D, $\hat{i}=(1, 0, 0)$, $\hat{j}=(0, 1, 0)$ and $\hat{k}=(0, 0, 1)$ are called **base vectors.**

5. Any vector can be expressed as a linear combination of the base vectors i.e. $(a_1, a_2, a_3)=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$.

6. A **directed line segment** is a segment of a line with an initial point, say A and a terminal point, say B. It is denoted by $\overrightarrow{AB}$. The direction of the ray AB is its direction and AB is its length. Two directed segments $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are said to be equal if AB=CD and have the same direction.

7. A vector can be represented **geometrically** by means of a directed segment (suppose $\vec{a}=(a_1, a_2, a_3)$. Let P be the point with coordinates (a_1, a_2, a_3). Then the directed segment $\overrightarrow{OP}$ represents $\vec{a}$). The length of the directed segment is the **length (magnitude or modulus) of the vector** and the direction of the directed line is the **direction of the vector.**

8. If $\vec{a}=(a_1, a_2, a_3)$, then the length of $\vec{a}=|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$. If the length of a vector is unity, then it is called a **unit vector.** A unit vector along $\vec{a}=\hat{a}=\frac{1}{|\vec{a}|}\vec{a}$. The base vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors.

9. (*i*) If two vectors $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and if OACB is a parallelogram, then the diagonal vector $\overrightarrow{OC}$ is the sum of $\vec{a}$ and $\vec{b}$ **(parallelogram law of addition).**

   (*ii*) If two vectors $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{OA}$ and $\overrightarrow{AC}$, then vector $\overrightarrow{OC}$ is the sum of $\vec{a}$ and $\vec{b}$ **(triangular law of addition).**

   (*iii*) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ represents the three sides a triangle in order, then $\vec{a}+\vec{b}+\vec{c}=\vec{o}$.

10. The scalar multiple $k\vec{a}$ of a vector $\vec{a}$ is a vector whose magnitude is $|k|$ times that of $\vec{a}$. Its direction is that of $\vec{a}$ if $k>0$ and if $k>0$, its direction is opposite to that of $\vec{a}$. $(-1)\vec{a}$ is called

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the **opposite** or **negative** of vector $\vec{a}$ and it is denoted by $-\vec{a}$ and $\vec{a}-\vec{b}=\vec{a}+(-1)\,\vec{b}$.

11. The **position vector** of a point P w.r.t. another point O is $\vec{OP}$. If A and B are any two points, then $\vec{AB}=\vec{OB}-\vec{OA}$ = P.V. of terminal point—P.V. of initial point.

12. Vectors lying on parallel lines are known as **collinear vectors.** Two vectors $\vec{a}$ and $\vec{b}$ will be collinear if $\vec{b}=k\,\vec{a}$. Three points A, B and C will be collinear if $\vec{AB}=k\,\vec{AC}$.

13. (*i*) The **scalar product** or **dot product** of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta$ where θ is the angle between them.

(*ii*) **Geometrically** $\vec{a}\cdot\vec{b}=|\vec{a}|\times$projection of $\vec{b}$ on $\vec{a}$ $=|\vec{b}|\times$ projection of $\vec{a}$ on $\vec{b}$.

(*iii*) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (*iv*) $\vec{a}\cdot\vec{a}=|\vec{a}|^2$ (*v*) $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$

(*vi*) $\vec{a}\cdot\vec{b}=0\Longleftrightarrow\vec{a}\perp\vec{b}$ (*vii*) $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1;\ \hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{i}=0$ $=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{j}=\hat{k}\cdot\hat{i}=\hat{i}\cdot\hat{k}$ (*viii*) $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(*ix*) If $\vec{a}=(a_1, a_2, a_3)$ and $\vec{b}=(b_1, b_2, b_3)$, then $\vec{a}\cdot\vec{b}=a_1 b_1+a_2 b_2+a_3 b_3$

14. (*i*) The **vector product** or **cross product** $\vec{a}\times\vec{b}=|\vec{a}|\,|\vec{b}|\sin\theta\ \hat{n}$ where $\hat{n}$ is a unit vector ⊥ both $\vec{a}$ and $\vec{b}$ and θ is the angle between them (*ii*) If $\vec{a}=\vec{OA}$ and $\vec{b}=\vec{OB}$, then $|\vec{a}\times\vec{b}|$ is the area of the parallelogram with OA and OB as co-terminal sides and $\frac{1}{2}|\vec{a}\times\vec{b}|$ is the area of $\triangle$OAB.

(*iii*) $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$ (*iv*) $\vec{a}\times\vec{a}=0$ (*v*) $\vec{a}\times\vec{b}=0\Longleftrightarrow\vec{a}\parallel\vec{b}$

(*vi*) $\vec{a}\times\vec{b}$ is a vector ⊥ both $\vec{a}$ and $\vec{b}$ and therefore $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ is a unit vector ⊥ both $\vec{a}$ and $\vec{b}$ (i.e. ⊥ the plane containing $\vec{a}$ and $\vec{b}$) (*vii*) $\sin\theta=\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$ (*viii*) $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$

and $\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k}$; $\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i}$; $\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$

(*ix*) $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ (*x*) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

15. (*i*) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, $\vec{a}\cdot(\vec{b}\times\vec{c})$, $(\vec{a}\times\vec{b})\cdot\vec{c}$ and $(\vec{a}\times\vec{c})\cdot\vec{b}$ are called **scalar triple products** of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(*ii*) $|\vec{a}\cdot(\vec{b}\times\vec{c})|$ represents the **volume of a parallelopiped** whose coterminal edges are represented by $\vec{a}$, $\vec{b}$ and $\vec{c}$. $\frac{1}{6}[\vec{a}\cdot(\vec{b}\times\vec{c})]$ is the volume of the tetrahedron whose terminal edges are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(*iii*) $\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$ i.e. *a* cyclic permutation of the three vectors does not change the value of the triple product.

(*iv*) $\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\cdot\vec{c}$ i.e. in a scalar triple product, the 'dot' and the 'cross' are interchangable. Either of this product is denoted by $[\vec{a}\ \vec{b}\ \vec{c}\,]$ or $[\vec{a}\ ,\ \vec{b},\ \vec{c}]$.

(*v*) An anti-cyclic permutation changes the value in sign but not in magnitude i.e. $[\vec{a}\ \vec{b}\ \vec{c}]=-[\vec{a}\ \vec{c}\ \vec{b})]=-[\vec{b}\ \vec{a}\ \vec{c}]=-[\vec{c}\ \vec{b}\ \vec{a}\,]$.

(*vi*) If $\vec{a}=(a_1, a_2, a_3)$, $\vec{b}=(b_1, b_2, b_3)$ and $\vec{c}=(c_1, c_2, c_3)$, then

$$\vec{a}.(\vec{b}\times\vec{c})=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

16. (*i*) Vectors lying in the same plane or parallel planes are called **coplanar** vectors; otherwise **non-coplanar.**

(*ii*) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ will be coplanar if $[\vec{a}\ \vec{b}\ \vec{c}]=0$.

(*iii*) Four points A, B, C and D will be coplanar if $[\overrightarrow{AB}\ \overrightarrow{AC},\overrightarrow{AD}]=0$.

6.2 TRIGONOMETRY

6.21 Properties of triangles and solution of triangles

1. **Sine rule:** $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$; $a=2R \sin A$, $b=2R \sin B$; $c=2R \sin C$; $\frac{a}{b}=\frac{\sin A}{\sin B}$, $\frac{b}{c}=\frac{\sin B}{\sin C}$, $\frac{c}{a}=\frac{\sin C}{\sin A}$, etc. Given two sides and an angle opposite to one of them, then the angle opposite to the other side can be obtained using this rule. Similarly given two angles and a side opposite to one of them, then the other side can be obtained.

2. **Cosine rule:** $a^2=b^2+c^2-2bc \cos A$, $b^2=c^2+a^2-2ca \cos B$, $c^2=a^2+b^2-2ab \cos C$; $\cos A=\frac{b^2+c^2-a^2}{2bc}$, $\cos B=\frac{c^2+a^2-b^2}{2ca}$, $\cos C=\frac{a^2+b^2-c^2}{2ab}$. Given three sides of a triangle, the three angles can be obtained using this rule. Also given two sides and the included angle, the third side can be obtained

3. **Projection rule:** $a=b \cos C+c \cos B$, $b=c \cos A+a \cos C$, $c=a \cos B+b \cos A$. Given two sides and the angles of a triangle, the third side can be obtained using this rule.

4. **Napier rule:** $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$; $\tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2}$; $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$. Given two sides and the included angle, the difference between the other two angles (and hence the two angles) can be obtained using this rule.

5. **Half angle formulae:** $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$, $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ where $2s=a+b+c$ and similar expressions for other half angles. Given the three sides, the angles can be found using this rule.

6. **If A is an angle of a triangle, then $\sin A=\sin\alpha \Rightarrow A=\alpha$ or $180°-\alpha$; $\cos A=\cos\alpha \Rightarrow A=\alpha$; $\tan A=\tan\alpha \Rightarrow A=\alpha$**

7. **In the ambiguous case of solving a triangle (i) when the given angle is acute, there will be only one solution if the side opposite to the given angle $\geqslant$ the other given side (ii) when the given angle is obtuse, there will be no solution if the side opp. to the given angle $\leqslant$ the other given side, otherwise one solution.**

6.22 Inverse trigonometric functions

1. (*i*) $\sin^{-1}x=y$ iff $\sin y=x$, where $-1\leqslant x\leqslant 1$ and $-\pi/2\leqslant y\leqslant\pi/2$
(*ii*) $\cos^{-1}x=y$ iff $\cos y=x$, where $-1\leqslant x\leqslant 1$ and $o\leqslant y\leqslant\pi$
(*iii*) $\tan^{-1}x=y$ iff $\tan y=x$, where $-\infty<x<\infty$ & $-\pi/2\leqslant y\leqslant\pi/2$
(*iv*) $\operatorname{cosec}^{-1}x=y$ iff $\operatorname{cosec} y=x$, where $|x|\geqslant 1$ and $-\pi/2\leqslant y\leqslant\pi/2$
(*v*) $\sec^{-1}x=y$ iff $\sec y=x$, where $|x|\geqslant 1$ and $o\leqslant y\leqslant\pi$
(*vi*) $\cot^{-1}x=y$ iff $\cot y=x$, where $-\infty<x<\infty$ & $-\pi/2\leqslant y\leqslant\pi/2$

2. **(i) $\sin^{-1}(-x)=-\sin^{-1}x$** **(ii) $\cos^{-1}(-x)=\pi-\cos^{-1}x$**
(iii) $\tan^{-1}(-x)=-\tan^{-1}x$ **(iv) $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$**
(v) $\sec^{-1}(-x)=\pi-\sec^{-1}x$ **(vi) $\cot^{-1}(-x)=-\cot^{-1}x$.**

3. **(i) $\sin^{-1}(\sin x)=x,\ -\pi/2\leqslant x\leqslant\pi/2$; $\sin(\sin^{-1}x)=x,\ -1\leqslant x\leqslant 1$**
(ii) $\cos^{-1}(\cos x)=x,\ 0\leqslant x\leqslant\pi$; $\cos(\cos^{-1}x)=x,\ -1\leqslant x\leqslant 1$
(iii) $\tan^{-1}(\tan x)=x,\ -\pi/2\leqslant x\leqslant\pi/2$; $\tan(\tan^{-1})\,x=x,\ \forall\, x\in R$

4. **(i) $\operatorname{cosec}^{-1}x=\sin^{-1}\dfrac{1}{x},\ |x|\geqslant 1$** **(*ii*) $\sec^{-1}x=\cos^{-1}\dfrac{1}{x},\ |x|\geqslant 1$**

(iii) $\cot^{-1}x=\tan^{-1}\dfrac{1}{x},\ \forall\, x\in R$

5. **(i) $\sin^{-1}x+\cos^{-1}x=\pi/2$,** **(ii) $\tan^{-1}x+\cot^{-1}x=\pi/2,\ x\geqslant 0$**
(iii) $\sec^{-1}x+\operatorname{cosec}^{-1}x=\pi/2$

6. **$\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\dfrac{x}{\sqrt{1-x^2}}=\operatorname{cosec}^{-1}\dfrac{1}{x}=\sec^{-1}\dfrac{1}{\sqrt{1-x^2}}$**

$=\cot^{-1}\dfrac{\sqrt{1-x^2}}{x}$, $0\leqslant x\leqslant 1$ and similarly for others. i.e. one inverse trigonometric function can be expressed interms of others

Remark:

When $-1 \leqslant x < 0$, $\sin^{-1} x \neq \cos^{-1}\sqrt{1-x^2}$. In fact $\sin^{-1}(-vc) \neq \cos^{-1}(\ )$ because the LHS lies in IV quadrant and RHS cannot lie in IV quadrant. However $\sin^{-1}(-x) = -\sin^{-1} x = -\cos^{-1}\sqrt{1-x^2}$, $(x>0)$.

7. (*i*) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$, $x, y \geqslant 0$, $xy \leqslant 1$

(*ii*) $\tan^{-1} x-\tan^{-1} y=\tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$, $x, y \geqslant 0$

(*iii*) $2\tan^{-1}x=\tan^{-1}\dfrac{2x}{1-x^2}$, $0 \leqslant x \leqslant 1$

6.23 General solutions of trigonometric equations

1. (*i*) If $\sin x=\sin\alpha$, then $x=n\pi+(-1)^n\alpha$, $n\in Z$
   (*ii*) If $\cos x=\cos\alpha$, then $x=2n\pi\pm\alpha$, $n\in Z$
   (*iii*) $\tan x=\tan\alpha$ then $x=n\pi+\alpha$, $n\in Z$
   (*iv*) The other three equations can be converted into the above forms and solved

2. (*i*) $\sin x+\sin y=0$ can be written as $\sin x=\sin(-y)$
   (*ii*) $\cos x+\cos y=0$ can be written as $\cos x=\cos(\pi-y)$
   (*iii*) $\tan x+\tan y=0$ can be written as $\tan x=\tan(-y)$
   (*iv*) $\cos x+\sin y=0$ can be written as $\cos x=\cos\left(\frac{\pi}{2}+y\right)$
   (*v*) $\tan x+\cot y=0$ can be written as $\tan x=\tan\left(\frac{\pi}{2}+y\right)$
   (*vi*) $\cos x=\sin y$ can be written as $\cos x=\cos\left(\frac{\pi}{2}-y\right)$
   (*vii*) $\tan x=\cot y$ can be written as $\tan x=\tan\left(\frac{\pi}{2}-y\right)$

3. $a\cos x+b\sin x=c$, when $|c| \leqslant \sqrt{a^2+b^2}$, can be reduced to

(*i*) $\cos(x-\alpha)=\cos\beta$ where $\cos\alpha=\dfrac{a}{\sqrt{a^2+b^2}}$

$\sin\alpha=\dfrac{b}{\sqrt{a^2+b^2}}$, and $\cos\beta=\dfrac{c}{\sqrt{a^2+b^2}}$

(*ii*) $\sin(x+\alpha)=\sin\beta$ where $\cos\alpha=\dfrac{b}{\sqrt{a^2+b^2}}$

$\sin\alpha=\dfrac{a}{\sqrt{a^2+b^2}}$ and $\sin\beta=\dfrac{c}{\sqrt{a^2+b^2}}$

6.24 Complex numbers—De Moivre's theorem

1. (*i*) To **add** two complex numbers, add the real parts and add the imaginary parts; similarly for subtraction.

   (*ii*) To **multiply** two complex numbers, multiply as if the numbers were ordinary binomial expressions and replace i^2 by -1.

   (*iii*) To **divide** a complex number by another non-zero complex number, multiply both numerator and denominator of the quotient by the conjugate of the denominator.

2. A complex number $x+iy$ can be represented **graphically** by a point whose cartesian coordinates are (x, y). The origin represents the complex number $0=0+i0$. All the points on the x–axis have coordinates of the form $(x, 0)$ and correspond to the numbers $x+i.0=x$ (real). The x–axis is called the **axis of reals.** Similarly points on the y–axis correspond to complex numbers $0+i.\ y=iy$ (imaginary). The y–axis is called the **axis of imaginaries.** The plane on which the complex numbers are represented is called the **complex plane** or **Argand plane.** The diagram representing the points corresponding to a set of complex numbers is called the **Argand diagram.**

3. The conjugate $\bar{z}$ is the reflection of z in the x–axis and $-z$ is the point symmetric to z w.r.t. origin.

4. If P is the point representing a complex number $x+iy$, in the Argand plane, OP$=r=$**modulus** of $z=|z|$ and $X\hat{O}P=\theta$, the angle made by OP with the x–axis$=$**amplitude** of z or **argument** of z. $r\geqslant 0$ and $-\pi<\theta\leq\pi$.

5. $x=r\cos\theta$, $y=r\sin\theta$ (useful to find x and y, given r and θ)

   $r=\sqrt{x^2+y^2}$ and θ is given by $\cos\theta=\dfrac{x}{r}$ and $\sin\theta=\dfrac{y}{r}$ (useful to find r and θ, given x and y).

6. $z=r(\cos\theta+i\sin\theta)$ is called the **polar form** of the complex number z and $z=x+iy$ is called the **rectangular form** of z.

7. The amplitude of a +ve real number is 0 and that of a negative real number is π. The amplitude of a number of the form $iy (y>0)$ is $\pi/2$ and that of $-iy (y>0)$ is $-\pi/2$.

8. $|\bar{z}|=|z|$ and $\arg\bar{z}=-\arg z$.

9. Let P be the point representing a complex number z in the complex plane Let $\alpha = \sin^{-1}\frac{|y|}{r}$

   (*i*) If P lies in I quadrant, $\arg z = \alpha$
   (*ii*) If P lies in II quadrant, $\arg z = \pi - \alpha$
   (*iii*) If P lies in III quadrant, $\arg z = \alpha - \pi$
   (*iv*) If P lies in IV quadrant, $\arg z = -\alpha$

10. (*i*) $|z_1 z_2| = |z_1||z_2|$ and $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

    (*ii*) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

11. Let P and Q be the two points representing two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ respectively.

    (*i*) Complete the parallelogram OPRQ. Then the point R represents the sum $z_1 + z_2$.

    (*ii*) Let S be the point on x-axis such that $|OS| = 1$. Construct $\triangle$ OQR directly similar to $\triangle$ OSP with $S\hat{O}P = Q\hat{O}R$ and $O\hat{P}S = O\hat{R}Q$. Then the point R represents the product $z_1 z_2$.

12. **De Moivre's theorem:** If n is a rational number, the value or one of the values of $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

13. (*i*) $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$
    (*ii*) $(\cos\theta - i\sin\theta)^{n} = \cos n\theta - i\sin n\theta$
    (*iii*) $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

14. $\cos\theta + i\sin\theta$ and $\cos\theta - i\sin\theta$ are reciprocals to each other; so are $\sin\theta + i\cos\theta$ and $\sin\theta - i\cos\theta$.

15. **n^{th} root of a complex number $z = x + iy$:** First find the polar form of $z = r(\cos\theta + i\sin\theta)$. Then $z = r\{\cos(2k\pi + \theta) + i\sin(2k\pi + \theta)\}$; $z^{1/n} = r^{1/n}\left\{\cos\left(\frac{2k\pi + \theta}{n}\right) + i\sin\left(\frac{2k\pi + \theta}{n}\right)\right\}$.

    The n^{th} roots are the values of $z^{1/n}$ when $k = 0, 1, 2, \ldots (n-1)$. For cube roots, put $n = 0, 1, 2$ and for fourth roots put $n = 0, 1, 2, 3$ and so on.

16. The n^{th} root of unity are $\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$

    where $k = 0, 1, 2, \ldots (n-1)$ or $1, \omega, \omega^2, \ldots, \omega^{n-1}$

    where $\omega = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$.

17. If $1, \omega$, and ω^2 are the **cube roots of unity,** then $1+\omega+\omega^2=0$; $\omega^3=1$. The relation $1+\omega+\omega^2=0$ holds only when ω is an imaginary cube root of unity.

6.3 ANALYTICAL GEOMETRY

6.31 Pair of straight lines

1. **A homogeneous equation** of degree 2 in x and y i.e. $ax^2+2hxy+by^2=0$ represents a pair of straight lines through origin. These straight lines are real or imaginary according as $h^2 \geqslant ab$ or $h^2<ab$. The point of intersection of the pair of lines $ax^2+2hxy+by^2=0$ is origin.
2. If m_1 and m_2 are the slopes of the lines represented by $ax^2+2hxy+by^2=0$, then $m_1+m_2=-\dfrac{2h}{b}$ and $m_1 m_2=\dfrac{a}{b}$.
3. The **acute angle** between the pair of lines given by $ax^2+2hxy+by^2=0$ is $\tan^{-1}\left|\dfrac{2\sqrt{h^2-ab}}{a+b}\right|$. If $a+b=0$, then the lines are **perpendicular** and if $h^2=ab$, then the lines are **coincident.**
4. **The condition for the general quadratic equation** in x and y i.e. $ax^2+2hxy+by^2+2gx+2fy+c=0$ **to represent a pair of lines** is $abc+2fgh-af^2-bg^2-ch^2=0$ i.e. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$
5. The **acute angle** between the straight lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ is the same as the acute angle between the lines $ax^2+2hxy+by^2=0$ and hence it is $\tan^{-1}\left|\dfrac{2\sqrt{h^2-ab}}{a+b}\right|$. If $a+b=0$, the lines are **perpendicular** and if $h^2=ab$, then the lines are **parallel.**
6. The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will represent a pair of parallel lines if $abc+2fgh-af^2-bg^2-ch^2=0$ and $h^2=ab$.

6.32 The circle

1. **A circle is a locus** of a point which moves such that it is always at a constant distance from a fixed point. The fixed point is called the **centre** and the constant distance is called (the length of) the **radius** of the circle.

2. The **standard form** of the equation of the circle with centre at (h, k) and radius r is $(x-h)^2+(y-k)^2=r^2$. In particular $x^2+y^2=r^2$ is the equation of a circle with centre at (0, 0) and radius r.
3. The **general form** of the equation of a circle is $x^2+y^2+2gx+2fy+c=0$. The centre of the circle is $(-g, -f)$ and radius$=\sqrt{g^2+f^2-c}$. The equation represent a real circle, a point (circle) or an imaginary circle according as g^2+f^2-c is $>, =$ or <0.
4. The general quadratic equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will represent a circle if $a=b$ and $h=0$ i.e. the coefficients of x^2 and y^2 must be same and the coefficient of $xy=0$.
5. The equation $ax^2+ay^2+2gx+2fy+c=0$ also represents a circle. It can be reduced to the general form by dividing the equation by 'a'.
6. In both the standard and general forms, the equation of a circle has three unknowns and hence a circle can be determined, given three independent conditions.
7. Two circles with radii r_1 and r_2 will **touch** each other **internally** or **externally** according as the distance between their centres is equal to $r_1 \sim r_2$ or r_1+r_2. If they touch **internally** the **point of contact** of the two circles **divides** the line of centres **externally**; if they touch **externally**, then the **division** is **internal**.
8. The **equation of a circle** having the points (x_1, y_1) and (x_2, y_2) as the **extremities of a diameter** is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.
9. The points of intersection of a circle and a straight line are obtained by solving their equations.
10. A line will be a **tangent** to a circle if it meets the circle at only **one point** (which is the point of contact).
11. **A line will be a tangent** to a circle if the **length of the perpendicular** from the centre of the circle to the line is **equal** to the **radius** of the circle.
12. **The equation of the tangent** to a circle $x^2+y^2+2gx+2fy+c=0$ at (x_1, y_1) is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$.
13. **The length of the tangent** to the circle $x^2+y^2+2gx+2fy+c=0$ from an external point (x_1, y_1) is $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

14. The **power of a point** P w.r.t. a circle with centre C and radius r is CP^2-r^2. A point will be **inside,on or outside a circle** according as its power w.r.t. the circle is $<$, $=$ or >0. The power of a point (x_1, y_1) w.r.t. the circle $x^2+y^2+2gx+2fy+c=0$ is $x_1^2+y_1^2+2gx_1+2fy_1+c$. If the point lies outside the circle, then its power is equal to the square of the length of the tangent from that point.

15. The **radical axis** of two circles is the locus of a point, powers of which w.r.t. the two circles are equal.

16. If the equations of the two circles in the general form are $S_1=0$ and $S_2=0$, then their radical axis is $S_1-S_2=0$. The radical axis is a line perpendicular to the line of centres of the two circles. If two circles cut at P and Q, then the common chord PQ is the radical axis. If two circles touch each other at P, then the common tangent at P is the radical axis.

17. The points of intersection of two circles are the same as the points of intersection of any one of the circles and their radical axis.

18. The point of concurrence of the three radical axes of three circles taken in pairs is called the **radical centre** of the three circles. The powers of the radical centre to the three circles are equal. If the radical centre lies outside the three circles, the lengths of the tangents to the three circles from the radical centre are equal.

19. If the angle between the tangents to two circles at a point of intersection is 90°, then the two circles are said to cut **orthogonally**. The angle between the tangents at a common point of two circles is the same as the angle between the two radii at that point.

20. The condition for two circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ to cut orthogonally is $2g_1g_2+2f_1f_2=c_1+c_2$.

21. (i) A circle with any point as centre and the length of the tangent from it to a given circle as radius cuts the given circle orthogonally. (ii) A circle with any point on the radical axis of two circles as centre and the length of the tangent from the point to either of the two circles as radius cuts both the circles orthogonally. (iii) The circle with the radical centre of three circles

as centre and the length of the tangent from it to any of the three circles as radius cuts the three circles orthogonally.

6.33 Conic Section

1. **A conic section** is a locus of a point which moves such that its distance from a fixed point (**focus**) bears a constant ratio (**eccentricity**) to its distance from a fixed line (**directrix**). The conic is called a **parabola, ellipse** or **hyperbola** according as the eccentricity, e is $=$, $<$ or $>$ 1.
2. If the points (x, y) and $(x, -y)$ lie on a curve, then the curve is symmetric w.r.t. x–axis. Similarly if the points (x, y) and $(-x, y)$ lie on a curve, then it is symmetric w.r.t. y–axis. If the points (x, y) and $(-x, -y)$ lie on a curve, then it is symmetric w.r.t. origin.
3. The parabola $y^2=4ax$ is symmetric w.r.t. x–axis. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are symmetric w.r.t. x–axis and y–axis. The lines to which a conic is symmetric are called the axes of the conic. For **ellipse,** x–axis is called the **major axis** and y–axis is called the **minor axis.** The length of the segment intercepted by a conic on an axis is called the length of the axis. For hyperbola, x–axis is called the transverse axis and y–axis is called the conjugate axis.
4. The parabola $y^2=4ax$ is not symmetric w.r.t. origin. The **ellipse** and **hyperbola** are symmetric w.r.t. origin which is called the **centre** of the conic. The **parabola has no centre.**
5. The points of intersection of the axes and the conic are called the **vertices** of the conic.
6. A line through the focus of a conic is called **focal chord.** If a focal chord is parallel to a directrix, then it is called a **latus rectum.** Since the central conics (ellipse and hyperbola) have two foci, there are two latera recta. The portion of a latus rectum intercepted by the conic is called the **length of the latus rectum.**
7. If P is any point on the ellipse (hyperbola), then the sum (difference) of its distances from the two foci is equal to twice the length of major axis (transverse axis).
8. The axis of a parabola is perpendicular to both latus rectum and directrix. It is also the line joining the focus and vertex. The mid point of latus rectum is focus.

9. Table of properties of standard (canonical) forms of conics

PROPERTY	PARABOLA	ELLIPSE	HYPERBOLA
Equation	$y^2=4ax$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Eccentricity	$e=1$	$e=\sqrt{1-b^2/a^2}<1$	$e=\sqrt{1+b^2/a^2}>1$
Centre	—	$(o,\ o)$	$(o,\ o)$
Vertices	$(o,\ o)$	$(\pm a,\ o)$; $(o, \pm b)$	$(\pm a,\ o)$
Axes: Length	—	Major axis $2a$ Minor axis $2b$	Transverse axis $2a$ Conjugate axis $2b$
Axes: Equation	$y=o$	$y=o$ $x=o$	$y=o$ $x=o$
LR: Length	$4\|a\|$	$2b^2/a$	$2b^2/a$
LR: Equation	$x=-a$	$x=\pm ae$	$x=\pm ae$
Foci	$(a,\ o)$	$(\pm ae,\ o)$ or $(\pm\sqrt{a^2-b^2},\ o)$	$(\pm ae, o)$ or $(\pm\sqrt{a^2+b^2}, o)$
Distance between foci	—	$2ae$	$2ae$
Directrices	$x=-a$	$x=\pm\frac{a}{e}$	$x=\pm\frac{a}{e}$

PROPERTY	PARABOLA	ELLIPSE	HYPERBOLA
Distance between directices	—	$2\frac{a}{e}$	$2\frac{a}{e}$
End points of LR	$(a, \pm 2a)$	$(\pm ae, \pm b^2/_a)$ or $(\pm\sqrt{a^2-b^2}, \pm b^2/a)$	$(\pm ae, \pm b^2/_a)$ or $(\pm\sqrt{a^2+b^2}, \pm b^2/a)$
Relation between a, b and e	—	$b^2=a^2(1-e^2)$	$b^2=a^2(e^2-1)$
Parametric equations	$x=at^2$, $y=2at$	$x=a \cos t$, $y=b \sin t$	$x=a \sec t$, $y=b \tan t$
OTHER FORMS			
I Type	$x^2=4ay$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $b>a$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$
II Type	$(y-k)^2=4a(x-h)$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, $a>b$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
III Type	$(x-h)^2=4a(y-k)$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, $b>a$	$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$

10. The properties of other forms of the conics are obtained as follows:

I Type: Interchange x and y AND interchange a and b if meaningful in the corresponding formulae of standard form

Example	Standard form		I Type	
(*i*) Foci	$(a, 0)$	→	$(0, a)$	—Parabola
	$(\pm ae, 0)$	→	$(0, \pm be)$	—Ellipse
	$(\pm ae, 0)$	→	$(0, \pm be)$	—Hyperbola
(*ii*) Length of LR	$4\lvert a\rvert$	→	$4\lvert a\rvert$	
	$2b^2/a$	→	$2a^2/b$	
	$2b^2/a$	→	$2a^2/b$	

II Type: Add 'h' to "x value" and 'k' to "y value" in the corresponding formulae of standard form.

Example	Standard form		II Type	
(*i*) End points of LR	$(a, \pm 2a)$	→	$(h+a, k\pm 2a)$	—Parabola
	$(\pm ae, \pm b^2/a)$	→	$(h\pm ae, k\pm b^2/a)$	—Ellipse
	$(\pm ae, \pm b^2/a)$	→	$(h\pm ae, k\pm b^2/a)$	—Hyperbola
(*ii*) Equations of directrices	$x=-a$	→	$x=h-a$	
	$x=\pm\frac{a}{e}$	→	$x=h\pm\frac{a}{e}$	
	$x=\pm\frac{a}{e}$	→	$x=h\pm\frac{a}{e}$	

III Type: Interchange x and y, interchange a and b, AND then add 'h' to "x value" and 'k' to "y value" in the corresponding formulae of standard form.

Example	Standard form				III Type
(*i*) Vertices	$(0, 0)$	→	$(0, 0)$	→	(h, k)
	$(\pm a, 0)$, $(0, \pm b)$	→	$(0, \pm b)$, $(\pm a, 0)$	→	$(h, k\pm b)$, $(h\pm a, k)$
	$(\pm a, 0)$	→	$(0, \pm b)$	→	$(h, k\pm b)$
(*ii*) Foci	$(a, 0)$	→	$(0, a)$	→	$(h, k+a)$
	$(\pm ae, 0)$	→	$(0, \pm be)$	→	$(h, k\pm be)$
	$(\pm ae, 0)$	→	$(0, \pm be)$	→	$(h, k\pm be)$

(*iii*) Equations of axes
- $y=0 \rightarrow x=0 \rightarrow x=h$
- maj: $y=0 \rightarrow x=0 \rightarrow x=h$
- trans: $y=0 \rightarrow x=0 \rightarrow x=h$

(*iv*) Relation between a, b and e
- — — —
- $b^2=a^2(1-e^2) \rightarrow a^2=b^2(1-e^2) \rightarrow a^2=b^2(1-e^2)$
- $b^2=a^2(e^2-1) \rightarrow a^2=b^2(e^2-1) \rightarrow a^2=b^2(e^2-1)$

Illustration

(*i*) To find the focus of $x^2+10x+6y+19=0$

The given equation reduces to $(x+5)^2=-6(y-1)$(III Type)

Focus$=(a, 0)\rightarrow(0, a)\rightarrow(h, a+k)=(-5, -3/2+1)=(-5,-1/2)$

(*ii*) To find the vertices of the ellipse $\frac{(x-1)^2}{9}+\frac{(y+1)^2}{5}=1$ (II Type)

Vertices: $(\pm a, 0), (0,\pm b)\rightarrow(h\pm a, k), (h, k\pm b)=(1\pm3, \sqrt{5}),$ $(1, -1\pm\sqrt{5})$.

(*iii*) To find the equations of directrices of $4x^2+3y^2-8x+12y-32=0$

The given equation reduces to $\frac{(x-1)^2}{12}+\frac{(y+2)^2}{16}=1$(III Type)

$a=2\sqrt{3}, b=4, h=1, k=-2, e=\sqrt{1-\frac{a^2}{b^2}}=1/2$

Equations of directrices: $x=\pm a/e\rightarrow y=\pm b/e\rightarrow y=k\pm b/e$ i.e. $y=-2\pm8$.

(*iv*) To find the coordinates of the foci of $9x^2-16y^2+144=0$.

The given equation reduces to $\frac{y^2}{9}-\frac{x^2}{16}=1$ (I Type)

$a=4, b=3, e=\sqrt{1+\frac{b^2}{a^2}}\rightarrow\sqrt{1+\frac{a^2}{b^2}}=\frac{5}{3}$

Foci: $(\pm ae, 0)\rightarrow(0,\pm be)=(0,\pm5)$

(*v*) To find the equation and length of transverse axis of $\frac{(y+1)^2}{16}-\frac{(x-2)^2}{9}=1$ (III Type)

Transverse axis:
- equation: $y=0\rightarrow x=0\rightarrow x=h$ i.e. $x=2$
- length: $2a\rightarrow2b\rightarrow2b$ i.e. 8

11. Tangent and Normal

Property	Parabola $y^2=4ax$	Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Equation of tangent			
at (x_1, y_1)	$yy_1=2a(x+x_1)$	$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$	$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$
at 't'	$yt=x+at^2$	$\frac{x\cos t}{a}+\frac{y\sin t}{b}=1$	$\frac{x\sec t}{a}-\frac{y\tan t}{b}=1$
Equation of normal			
at (x_1, y_1)	$y-y_1=-\frac{y_1}{2a}(x-x_1)$	$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$	$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$
at t	$y+tx=2at+at^3$	$\frac{ax}{\cos t}-\frac{by}{\sin t}=a^2-b^2$	$\frac{ax}{\sec t}+\frac{by}{\tan t}=a^2+b^2$
Condition for tangency $(y=mx+c)$	$c=\frac{a}{m},\quad m\neq 0$	$c^2=a^2m^2+b^2$	$c^2=a^2m^2-b^2$

6.4 CALCULUS

6.41 Limits and continuity

1. A **variable** (quantity) is a quantity which can take on different values. A **constant** is a quantity which can take only one value.
2. A variable y is called a **function** of a variable x if with every value of x we can associate a unique value of y. x is called the independent variable and y is called dependent variable. A function of a variable x is denoted by $f(x)$ and $f(a)$ is the value of the function at $x=a$.
3. **Basic elementary functions** are (*i*) power function x^n (*ii*) exponential function $a^x, a>0$ and $a \neq 1$ (*iii*) logarithmic function $\log_a x, a>0$ and $a \neq 1$ (*iv*) trigonometric functions (*v*) inverse trigonometric functions.
4. **The limit of a function** $f(x)$ as x tends to 'a' $= \underset{x \to a}{\mathrm{Lt}}\ f(x) = l$ means that when x is close to 'a', $f(x)$ is close to l.
5. The limit of a constant is itself. If u and v are functions of x,
$$\underset{x\to a}{\mathrm{Lt}}\ (u \pm v) = \underset{x\to a}{\mathrm{Lt}}\ u \pm \underset{x\to a}{\mathrm{Lt}}\ v;\quad \underset{x\to a}{\mathrm{Lt}}\ ku = k \underset{x\to a}{\mathrm{Lt}}\ u;$$
$$\lim_{x\to a} uv = \lim_{x\to a} u.\ \lim_{x\to a} v \text{ and } \lim \frac{u}{v} = \frac{\lim u}{\lim v},\ \lim v \neq 0.$$
6. (*i*) $\underset{n\to\infty}{\mathrm{Lt}}\ \dfrac{1}{n^p} = 0$ if $p>0$ and $\underset{n\to\infty}{\mathrm{Lt}}\ n^p = 0$ if $p<0$

   (*ii*) When x is measured in radian, $\underset{x\to 0}{\mathrm{Lt}}\ \dfrac{\sin x}{x} = 1$ and

   $\underset{x\to 0}{\mathrm{Lt}}\ \dfrac{\tan x}{x} = 1.$ (*iii*) $\underset{x\to a}{\mathrm{Lt}}\ \dfrac{x^n - a^n}{x-a} = na^{n-1}$

   (*iv*) $\underset{x\to 0}{\mathrm{Lt}}\ \dfrac{a^x - 1}{x} = \log a$ (*v*) $\underset{x\to 0}{\mathrm{Lt}}\ (1+x)^{1/x} = e.$
7. A function $f(x)$ is called **continuous** at $x=a$ if $\underset{x\to a}{\mathrm{Lt}}\ f(x) = f(a)$
8. A function $f(x)$ is **discontinuous** at $x=a$ if (*i*) $f(a)$ is not defined, (*ii*) $\underset{x\to a}{\mathrm{Lt}}\ f(x)$ doesn't exist or (*iii*) $\underset{x\to a}{\mathrm{Lt}}\ f(x) \neq f(a)$.

9. A polynomial function in x, $\sin x$, $\cos x$ and e^x are continuous for all values of x. A rational function in x, $\sin^{-1}x$, $\cos^{-1}x$ and $\log x$ are continuous wherever defined.

6.42 Differentiation

1 Let $y=f(x)$ be a continuous function. Let x be any point in the domain of the function. Let Δx be a small change (increment) in x. Then the function will receive a change $\Delta y=f(x+\Delta x)-f(x)$. $\underset{\Delta x\to 0}{\text{Lt}} \dfrac{\Delta y}{\Delta x}$ i.e.

$\underset{\Delta x\to 0}{\text{Lt}} \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$, if exists, is called the **differential coefficient** or the **derivative** of the function w.r.t. x and it is denoted by $f'(x)$, y' or $\dfrac{dy}{dx}$.

2. **Table of Derivatives**

(*i*) **Rules of differentiation:**

When u, v and y are functions of x,

$y=$	$\dfrac{dy}{dx}$	Remark
ku, k is a constant	$k\dfrac{du}{dx}$	Scalar multiple Rule
$u+v$	$\dfrac{du}{dx}+\dfrac{dv}{dx}$	Sum Rule
$u-v$	$\dfrac{du}{dx}-\dfrac{dv}{dx}$	Difference Rule
uv	$v\dfrac{du}{dx}+u\dfrac{dv}{dx}$	Product Rule
u/v	$\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$	Quotient Rule
$f(u)$	$f'(u)\dfrac{du}{dx}$ or $\dfrac{dy}{du}\cdot\dfrac{du}{dx}$	Chain Rule
$y=g^{-1}(x)$	$\dfrac{dy}{dx}=\dfrac{1}{\left(\dfrac{dx}{dy}\right)}$ or $\dfrac{1}{g'(y)}$	Inverse function

(*ii*) Derivatives of elementary functions

$y=f(x)$	$\dfrac{dy}{dx}=f'(x)$	$y=f(u)$	$\dfrac{dy}{dx}=f'(u)\dfrac{du}{dx}$
k, *a* constant	0	—	—
x^n	nx^{n-1}	u^n	$nu^{n-1}\dfrac{du}{dx}$
$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$	$\sqrt{u}$	$\dfrac{1}{2\sqrt{u}}\dfrac{du}{dx}$
$\dfrac{1}{x}$	$-\dfrac{1}{x^2}$	$\dfrac{1}{u}$	$-\dfrac{1}{u^2}\dfrac{du}{dx}$
$\dfrac{1}{x^n}$	$-\dfrac{n}{x^{n+1}}$	$\dfrac{1}{u^n}$	$-\dfrac{n}{u^{n+1}}\cdot\dfrac{du}{dx}$
$\sin x$	$\cos x$	$\sin u$	$\cos u\dfrac{du}{dx}$
$\cos x$	$-\sin x$	$\cos u$	$-\sin u\dfrac{du}{dx}$
$\tan x$	$\sec^2 x$	$\tan u$	$\sec^2 u\dfrac{du}{dx}$
$\cot x$	$-\operatorname{cosec}^2 x$	$\cot u$	$-\operatorname{cosec}^2 u\dfrac{du}{dx}$
$\sec x$	$\sec x\tan x$	$\sec u$	$\sec u\tan u\dfrac{du}{dx}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$	$\operatorname{cosec} u$	$-\operatorname{cosec} u\cot u\dfrac{du}{dx}$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$	$\cos^{-1} u$	$-\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$	$\tan^{-1} u$	$\dfrac{1}{1+u^2}\dfrac{du}{dx}$

$y=f(x)$	$\dfrac{dy}{dx}=f'(x)$	$y=f(u)$	$\dfrac{dy}{dx}=f'(u)\dfrac{du}{dx}$
$\cot^{-1} x$	$-\dfrac{1}{1+x^2}$	$\cot^{-1} u$	$-\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\sec^{-1} x$	$\dfrac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} u$	$\dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$
$\operatorname{cosec}^{-1} x$	$-\dfrac{1}{x\sqrt{x^2-1}}$	$\operatorname{cosec}^{-1} u$	$-\dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$
$a^x, a>0, a\neq 1$	$a^x \log a$	a^u	$a^u \log a \dfrac{du}{dx}$
e^x	e^x	e^u	$e^u \dfrac{du}{dx}$
$\log x$	$\dfrac{1}{x}$	$\log u$	$\dfrac{1}{u}\dfrac{du}{dx}$

3. Hyperbolic functions: (a) The six hyperbolic functions are

(*i*) $\sinh x=\frac{1}{2}(e^x-e^{-x})$ (*ii*) $\cosh x=\frac{1}{2}(e^x+e^{-x})$

(*iii*) $\tanh x=\dfrac{\sinh x}{\cosh x}$

(*iv*) $\coth x=\dfrac{1}{\tanh x}=\dfrac{\cosh x}{\sinh x}$ (*v*) $\operatorname{sech} x=\dfrac{1}{\cosh x}$

(*vi*) $\operatorname{cosech} x=\dfrac{1}{\sinh x}$.

(*b*) The fundamental relations satisfied by the hyperbolic functions are

(*i*) $\cosh^2 x-\sinh^2 x=1$ (*ii*) $1-\tanh^2 x=\operatorname{sech}^2 x$

(*iii*) $\coth^2 x-1=\operatorname{cosech}^2 x$.

(*c*) The six inverse hyperbolic functions are:

(*i*) $\sinh^{-1}x=\log(x+\sqrt{x^2+1})$ (*ii*) $\cosh^{-1}x=\log(x+\sqrt{x^2-1})$

(*iii*) $\tanh^{-1}x=\frac{1}{2}\log\dfrac{1+x}{1-x}$ (*iv*) $\coth^{-1}x=\tanh^{-1}\dfrac{1}{x}$

(*v*) $\operatorname{sech}^{-1}x=\cosh^{-1}\dfrac{1}{x}$ (*vi*) $\operatorname{cosech}^{-1}x=\sinh^{-1}\dfrac{1}{x}$.

(*d*) Derivatives of hyperbolic and inverse hyperbolic functions

$y=f(x)$	$\frac{dy}{dx}=f'(x)$	$f(u)$	$\frac{dy}{dx}$
$\sinh x$	$\cosh x$	$\sinh u$	$\cosh u\,\frac{du}{dx}$
$\cosh x$	$\sinh x$	$\cosh u$	$\sinh u\,\frac{du}{dx}$
$\tanh x$	$\operatorname{sech}^2 x$	$\tanh u$	$\operatorname{sech}^2 u\,\frac{du}{dx}$
$\coth x$	$-\operatorname{cosech}^2 x$	$\coth u$	$-\operatorname{cosech}^2 u\,\frac{du}{dx}$
$\operatorname{sech} x$	$-\operatorname{sech} x\tanh x$	$\operatorname{sech} u$	$-\operatorname{sech} u\tanh u\,\frac{du}{dx}$
$\operatorname{cosech} x$	$-\operatorname{cosech} x\coth x$	$\operatorname{cosech} u$	$-\operatorname{cosech} u\coth u\,\frac{du}{dx}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$	$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$	$\tanh^{-1} u$	$\frac{1}{1-u^2}\frac{du}{dx}$
$\coth^{-1} x$	$\frac{1}{1-x^2}$	$\coth^{-1} u$	$\frac{1}{1-u^2}\frac{du}{dx}$
$\operatorname{sech}^{-1} x$	$\frac{-1}{x\sqrt{1-x^2}}$	$\operatorname{sech}^{-1} u$	$\frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$
$\operatorname{cosech}^{-1} x$	$\frac{-1}{x\sqrt{1+x^2}}$	$\operatorname{cosech}^{-1} u$	$\frac{-1}{u\sqrt{1+u^2}}\frac{du}{dx}$

4. **Differentiation of implicit functions:** If a function $y=f(x)$ is defined by means of an equation in x and y, then $y=f(x)$ is said to be defined implicitly. The derivative of such function is obtained by differentiating every term in the equation w.r.t. x and then

solving for $\frac{dy}{dx}$. The process of obtaining $\frac{dy}{dx}$ of implicit functions is known as implicit differentiation.

5. **Differentiation of functions defined parametrically:** If a function y=f(x) is defined in terms of a third variable t (parameter), then it is said to be defined parametrically. Its derivative is obtained by using $\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}$. The process of obtaining derivatives of such functions is known as parametric differentiation.

6. **Logarithmic differentiation:** To find the derivatives of a function of the type (*i*) $y=\frac{u_1 u_2 u_3 \ldots u_n}{v_1 v_2 v_3 \ldots v_n}$ or (*ii*) $y=u^v$ where u's and v's are function of x, take logarithm before differentiation. By taking logarithm, product (quotient) is converted into sum (difference) and u^v is converted into product $v \log u$. Now differentiating the equation w.r.t. x and using $\frac{d}{dx}(\log u)=\frac{1}{u}.\frac{du}{dx}$, $\frac{dy}{dx}$ is obtained. This method of finding $\frac{dy}{dx}$ is known as logarithmic differentiation.

7. **Successive differentiation:** (*i*) Let $f'(x)$ be the derivative of a function $y=f(x)$. The derivative of $f'(x)$ is called the second derivative of the function $f(x)$ and it is denoted by $f''(x)$, y'', y_2 or $\frac{d^2y}{dx^2}$. (*ii*) If $y=f(x)$ is defined explicitly, then y'' is obtained by differentiating y' w.r.t. x (*iii*) If $y=f(x)$ is defined implicitly, then y' will be a function of x and y; Differentiating again w.r.t. x, we get y'' which will be interms of x, y and y'. Substituting for y', y'' is got interms of x and y (*iii*) If $y=f(x)$ is defined parametrically $\frac{dy}{dx}$ will be a function of the parameter t. Using $\frac{d^2y}{dx^2}=\dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ the second derivative is got.

6.43 Application of defferentiation:

1. $\frac{dy}{dx}$ is the slope of the tangent to a curve $y=f(x)$ at any point (x, y) on it and the slope of the normal at any point is $-1\Big/\left(\frac{dy}{dx}\right)$.

2. The equation of the tangent to a curve at any point (x_1, y_1) is $y-y_1=m(x-x_1)$ where $m=\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ and the equation of the normal at (x_1, y_1) is $y-y_1=-\frac{1}{m}(x-x_1)$

3. (i) The acute angle θ between two curves at a point of intersections P is given by $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$ where $m_1=\left(\frac{dy}{dx}\right)_P$ for I curve and $m_2=\left(\frac{dy}{dx}\right)_P$ for II curve.

   (*ii*) If $m_1=m_2$, then the two curves **touch each other** at P.

   (*iii*) If $m_1m_2=-1$, then the two curves cut each other orthogonally.

4. The length of the **subtangent** to a curve at any point $(x_1, y_1)=\frac{y_1}{y'}$ and the length of **subnormal**$=y_1y'$ where $y'=\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

5. $\frac{dy}{dx}$=Rate of change of y w.r.t. x.

   (*i*) $\frac{dy}{dx}=+\text{ve}\Rightarrow$ when x increases, y also increases;

   $\frac{dy}{dx}=-\text{ve}\Rightarrow$ when x increases, y decreases.

   (*ii*) With the usual notation, velocity $v=\frac{ds}{dt}$;

   acceleration $=\frac{d^2s}{dt^2}$ or $\frac{dv}{dt}$

6. (*i*) A function $f(x)$ has a **maximum** at a point $x=a$ if $f(x)<f(a)$ when x is close to 'a'; if $f(x)>f(a)$ when x is close to 'a' then $f(x)$ gas a **minimum** at $x=a$.

(*ii*) If a function $f(x)$ has **extremum** (a maximum or a minimum) at $x=a$, then $f'(a)=0$. The roots of the equation $f'(x)=0$ are called the **critical values** or **turning values** of $f(x)$.

(*iii*) A function $f(x)$ has a **maximum** at $x=a$, if $f'(a)=0$ and $f''(a)<0$; and $f(a)$ is a (local) maximum of the function $f(x)$.

(*iv*) A function $f(x)$ has a **minimum** at $x=a$, if $f'(a)=0$ and $f''(a)>0$; and $f'(a)$ is called a (local) minimum of the function $f(x)$.

(*v*) If $f'(a)=0=f''(a)$, then $x=a$ is called a **point of inflexion**

6.44 Integration:

1. If $y=f(x)$, $\frac{dy}{dx}=f'(x)$. We write this as $dy=f'(x)dx$ and call 'dy' as the **differential** of y. For example, if $y=x^2$, $dy=2xdx$ and if $y=\cos t$, $dy=-\sin t\, dt$.

2. Let $f(x)$ be the deviative of a function $F(x)$ i.e. $F'(x)=f(x)$. Then $f(x)dx$ is the differential of the function $F(x)$ i.e. $f(x)\,dx=d\,F(x)$. The function $F(x)$ is called the antiderivative or primitive of the function $f(x)$. For a function $f(x)$, there are infinitely many antiderivatives because $d[F(x)+c]=dF(x)=f(x)dx$ where c is any real number. The most general form of antiderivative is called the **indefinite integral.** The indefinite integral of $f(x)$ w.r.t. x is written as $\int f(x)dx$; $f(x)$ is called the **integrand** and x is the **variabe of integration.**

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3. Tables of Integration

Table 1

$f(x)$	$\int f(x)\,dx$
k, a constant	$kx+c$
$x^n, n \neq -1$	$x^{n+1}/_{n+1}+c$
$1/x$	$\log x+c$
$a^x, a>0, a \neq 1$	$a^x/\log a+c$
e^x	e^x+c
$\sin x$	$-\cos x+c$
$\cos x$	$\sin x+c$
$\tan x$	$\log \sec x+c$
$\cot x$	$\log \sin x+c$
$\sec x$	$\log(\sec x+\tan x)+c$
$\operatorname{cosec} x$	$\log(\operatorname{cosec} x-\cot x)+c$
$\sec^2 x$	$\tan x+c$
$\operatorname{cosec}^2 x$	$-\cot x+c$
$\sec x \tan x$	$\sec x+c$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x+c$

Table 2

$f(x)$	$\int f(x)\,dx$
$\sinh x$	$\cosh x+c$
$\cosh x$	$\sinh x+c$
$\tanh x$	$\log \cosh x+c$
$\coth x$	$\log \sinh x+c$
$\operatorname{sech}^2 x$	$\tanh x+c$
$\operatorname{cosech}^2 x$	$-\coth x+c$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x+c$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x+c$

Table 3

$f(x)$	$\int f(x)\,dx$ (add c)
$\frac{1}{\sqrt{1-x^2}}$; $\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}x$ or $-\cos^{-1}x$; $\sin^{-1}\frac{x}{a}$ or $-\cos^{-1}\frac{x}{a}$
$\frac{1}{\sqrt{x^2-1}}$; $\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}x$ or $\log(x+\sqrt{x^2-1})$ $\cosh^{-1}\frac{x}{a}$ or $\log(x+\sqrt{x^2-a^2})$
$\frac{1}{\sqrt{x^2+1}}$; $\frac{1}{\sqrt{x^2+a^2}}$	$\sinh^{-1}x$ or $\log(x+\sqrt{x^2+1})$; $\sinh^{-1}\frac{x}{a}$ or $\log(x+\sqrt{x^2+a^2})$
$\frac{1}{x^2+1}$; $\frac{1}{x^2+a^2}$	$\tan^{-1}x$ or $-\cot^{-1}x$; $\frac{1}{a}\tan^{-1}\frac{x}{a}$ or $-\frac{1}{a}\cot^{-1}\frac{x}{a}$
$\frac{1}{1-x^2}$; $\frac{1}{a^2-x^2}$	$\tanh^{-1}x$; $\frac{1}{a}\tanh^{-1}\frac{x}{a}$ or $\frac{1}{2a}\log\frac{a+x}{a-x}$
$\frac{1}{x^2-1}$; $\frac{1}{x^2-a^2}$	$-\coth^{-1}x$; $-\frac{1}{a}\coth^{-1}\frac{x}{a}$ or $\frac{1}{2a}\log\frac{x-a}{x+a}$
$\frac{1}{x\sqrt{x^2-1}}$; $\frac{1}{x\sqrt{x^2-a^2}}$	$\sec^{-1}x$ or $\operatorname{cosec}^{-1}x$; $\frac{1}{a}\sec^{-1}\frac{x}{a}$ or $-\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a}$

4. Rules of Integration

(*i*) $\int k f(x)\,dx = k\int f(x)\,dx$

(*ii*) $\int \{f(x) \pm g(x)\}\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

(*iii*) If $\int f(x)dx = g(x)+c$, then $\int f(ax+b)dx = \frac{1}{a}\,g(ax+b)+c.$

Using this rule, we get $\int (ax+b)^n\, dx = \frac{1}{a}\,\frac{(ax+b)^{n+1}}{n+1}, n \neq -1;$

$$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\log(ax+b);\ \int e^{ax+b}\, dx = \frac{1}{a}\, e^{ax+b};$$

$$\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b);$$

$$\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b);$$

$$\int \tan(ax+b)\, dx = \frac{1}{a}\log\sec(ax+b) \text{ and so on}$$

(*iv*) Substitution method

$F(x)$	Substitution	$\int F(x)\, dx$
$f(g(x)) . g'(x)$	$t=g(x)$	$\int f(t)dt$
$\{f(x)\}^n . f'(x)$	$t=f(x)$	$\int t^n dt$
$\frac{f'(x)}{f(x)}$	$t=f(x)$	$\int \frac{dt}{t} = \log f(x)$
$\frac{f'(x)}{\sqrt{f(x)}}$	$t=f(x)$	$\int \frac{dt}{\sqrt{t}} = 2\sqrt{f(x)}$
containing $\sqrt{a^2-x^2}$	$x=a\sin\theta$	—
containing $\sqrt{a^2+x^2}$	$x=a\tan\theta$	—
containing $\sqrt{x^2-a^2}$	$x=a\sec\theta$	—

(*v*) Method of partial fraction: If $f(x)$ is a rational function of x, resolve it into partial fraction and integrate each fraction.

The most often occurring integrals are $\int \frac{A}{ax+b}\, dx = \frac{A}{a}\log(ax+b);\ \int \frac{A}{(ax+b)^n}\, dx = \frac{A}{a}\,\frac{(ax+b)^{-n+1}}{(-n+1)}$ and

$$\int \frac{Ax+B}{x^2+a^2}\, dx = A\int \frac{x}{x^2+a^2}\, dx + B\int \frac{1}{x^2+a^2}\, dx = \frac{A}{2}\log(x^2+a^2) + \frac{B}{a}\tan^{-1}\frac{x}{a}.$$

(*vi*) Integration by parts

$$\int u\,dv = uv - \int v\,du \text{ or } \int f(x)g(x)dx = f(x)\int g(x)dx - \int\left\{f'(x)\int g(x)\,dx\right\}dx$$

$$\text{or } \int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

While integrating by parts, take u=log. function, inverse trig. function, algebraic function, trig. function and exponential function in the order of priority (LIATE).

5. Definite Integrals

(*i*) Fundamental theorem of integral calculus:

If $\int f(x)dx = F(x)+c$, then $\int_a^b f(x)dx = F(b)-F(a)$

(*ii*) While evaluating a definite integral by substitution method, change the limits of the integration.

(*iii*) The rule of integration by parts is

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$$

6. Properties of definite integrals

(*i*) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(*ii*) $\int_b^a f(x)dx = -\int_a^b f(x)\,dx$

(*iii*) $\int_a^a f(x)dx = 0$ (*iv*) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(*v*) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

7. Define integral as a limit of a sum

(*i*) $$\int_a^b f(x)\,dx = \underset{h\to 0}{\text{Lt}}\, h.\sum_{r=1}^{n} f(a+rh) \text{ where } h=\frac{b-a}{n}$$

$$= \underset{n\to\infty}{\text{Lt}}\,\frac{1}{n}\sum_{r=1}^{n} f\left(a+r\frac{b-a}{n}\right)$$

(*ii*) $$\underset{n\to\infty}{\text{Lt}}\,\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$$

8. Application of definite integral

(*i*) The (directed) area of the region enclosed between the curve $y=f(x)$, the x-axis and the lines $x=a$ and $x=b$ is $\int_a^b f(x)dx$ (assuming that the curve doesn't cross the x-axis between a and b).

(*ii*) The (directed) area of the region enclosed between the curve $x=f(y)$, the y-axis and the lines $y=a$ and $y=b$ is $\int_a^b f(y)\,dy$.

(*iii*) The (directed) area bounded by the parabola $y=ax^2+bx+c$ and the x-axis is $\int_\alpha^\beta (ax^2+bx+c)\,dx$ where α and β are the roots of the equation $ax^2+bx+c=0$.

(*iv*) The area bounded by the parabola $x=ay^2+by+c$ and the y-axis is $\int_\alpha^\beta (ay^2+by+c)\,dy$ where α and β are the roots of $ay^2+by+c=0$.

(*v*) The area of the region enclosed between the curves $y=f(x)$ and $y=g(x)$ and the lines $x=a$ and $x=b$ is $\int_a^b [f(x)-g(x)]\,dx$.

(*vi*) The area enclosed between two curves $y=f(x)$ and $y=g(x)$ is $\int_a^b [f(x)-g(x)]\,dx$ where a and b are the x-coordinates of the points of intersection (which lie on the same side of x-axis). If the two points of intersection lie on the same side of y-axis, then it is $\int_a^b [f(y)-g(y)]\,dy$ where a and b are the y-coordinates of the points of intersection of the curves $x=f(y)$ and $x=g(y)$.

6.45 Some more formulae (which can be used to answer objective type questions)

1. Let R be a relation in a set A. R is **not reflexive** if there is atleast one element $a \in A$ such that $(a, a) \notin R$. R is **not symmetric** if there are elements $a, b \in A$ such that $(a, b) \in R$, $(b, a) \notin R$. R is **not anti-symmetric** if there exists elements $a, b \in A$, $a \neq b$ such that $(a, b) \in R$ and $(b, a) \in R$. R is

not transitive if there exists elements $a, b, c \in A$ such that $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$. R is **both symmetric and anti-symmetric** if $(a, b) \in R \Rightarrow a=b$ i.e. R must be a subset "diagonal line" of $A \times A$.

2. If $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a matrix of order 2×2,

   then $\operatorname{adj} A=\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ and $A^{-1}=\dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \; |A| \neq 0$

3. From the binary operation table, closure, commutative, identity and inverse laws can be easily checked. If the table contains only elements of the set, then the closure law is true. If the table is symmetric w.r.t. the principal diagonal then the commutative law is true. **Associative law is NOT obvious** from the table. If there exists an element such that the row against (and the column below) it is the duplicate of the first row (and the first column), then that element is the identity element. If every row and every column have the identity element, then inverse exists for every element. Further, if in a binary operation table, each element of set occurs in each row and in each column only once, then it is a group table.

4. Among the four algebraic operations, $(+, -, \times, \div)$, **subtraction and division are neither commutative nor associative.**

5. Matrix subtraction and multiplication are not commutative.

6. Z_n is a group under **addition modulo n**. But **Z_n is not a group** under **multiplication** mod n. However, $Z_n-\{0\}$ is a **multiplicative group** when **n is prime.**

7. The set of n^{th} **roots of unity** is a **multiplicative group** but not additive group.

8. Consider the sets **N, Z, Q, R and C.** **Division is not a binary** operation in any of these sets. **Subtraction** is not a binary operation only **in N. N is not a** group under + or ×. The remaining are **additive groups** and they are **not multiplicative** groups because of **failure of inverse law.** However $Q-\{0\}$, $R-\{0\}$ and $C-\{0\}$ are **multiplicative groups** but $Z-\{0\}$ is **not.**

9. The groups in (6) and (7) are good examples of finite groups while the groups in (8) are infinite groups.

10. If H is a finite subset of a group G and if closure law is true in H, then H will be a subgroup of G.

11. The angle between two vectors $\vec{a}$ and $\vec{b}$ is the same as the angle between the unit vectors $\hat{a}$ and $\hat{b}$.

12. Three points (a_1, a_2, a_3), (b_1, b_2, b_3) and (c_1, c_2, c_3) are collinear if $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

13. Area of $\Delta ABC = \frac{1}{2}\, bc \sin A = \frac{1}{2}\, ca \sin B = \frac{1}{2}\, ab \sin C = \sqrt{s(s-a)(s-b)(s-c)}$

14. If the three sides of a triangle are given, to find the t–functions of half angles, use half angles formulae but if the sides are not integers, use cosine rule, get the angle and then find the t–functions of half angle.

15. The equation $a \cos x + b \sin x = c$ will have no solution if $|c| > \sqrt{a^2+b^2}$

16. Remember the amplitudes of the complex numbers $1+i$, $\sqrt{3}+i$, $1+\sqrt{3}i$, their multiples and their conjugates.

17. $(\sin \theta + i \cos \theta)^n = \sin n\theta + i \cos n\theta$ if $n \equiv 1 \pmod 4$.

18. Consider the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$. If this equation represents a pair of lines, then their point of intersection is $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$ or it can be obtained by solving the equations $ax_1+hy_1+g=0$ and $hx_1+by_1+f=0$ for x_1 and y_1. The condition for the above equation to represent a pair of parallel lines is $h^2=ab$ and $af^2=bg^2$ and the distance between the parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$

19. The distance between the focus and the vertex of a parabola is $|a|$. a is +*ve* or −*ve* according as the focus lies to the right (above) or left (below) the vertex. The distance between vertex and focus is equal to the distance of the vertex from the

directrix (This can be used to find one of the vertex, focus and directrix, given the other two).

20. Consider the equation $ax^2+by^2+2gx+2fy+c=0$ (observe that there is no mixed term). If it is an ellipse (a and b are of same sign), then the eccentricity e is given by $e=\sqrt{1-\frac{a}{b}}$ if $a<b$ and $\sqrt{1-\frac{b}{a}}$ if $b<a$. If it is a hyperbola $e=\sqrt{1+\frac{a}{b}}$ or $\sqrt{1+\frac{b}{a}}$. The equation of the tangent to this curve at any point (x_1, y_1) is obtained by replacing x^2 by xx_1, y^2 by yy_1, $2x$ by $x+x_1$, and $2y$ by $y+y_1$ in the equation of the curve.

21. If $y=mx+c$ is a tangent (*i*) the circle $x^2+y^2=a^2$ (*ii*) the parabola $y^2=4ax$ (*iii*) the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$. (*iv*) the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the point of contact is

(*i*) $\left(\frac{-a^2m}{c}, \frac{a^2}{c}\right)$ (*ii*) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ (*iii*) $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$

(*iv*) $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$.

22. $\underset{n\to\infty}{\text{Lt}}\ \frac{f(n)}{g(n)}$ is 0 or ∞ according as the degree of $f(n)$ < or > the degree of $g(n)$. If they are of same degree, then it is equal to the ratio of the coefficients of the highest degree terms in $f(n)$ and $g(n)$.

23. **L' Hospital's Rule:**

Lt $\frac{f(x)}{g(x)}$ is $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, then Lt $\frac{f(x)}{g(x)}=$ Lt $\frac{f'(x)}{g'(x)}$. If Lt $\frac{f'(x)}{g'(x)}$ is also $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, then it is equal to Lt $\frac{f''(x)}{g''(x)}$. Most of the problems can be easily solved with the help of this rule.

Illustration: (*i*) $\underset{x\to 0}{\text{Lt}}\ \dfrac{1-\cos x}{x}=\underset{x\to 0}{\text{Lt}}\ \dfrac{\dfrac{d}{dx}(1-\cos x)}{\dfrac{d}{dx}(x)}$

$=\underset{x\to 0}{\text{Lt}}\ \dfrac{\sin x}{1}=0.$

(*ii*) $\underset{x\to 0}{\text{Lt}}\ \dfrac{1-\cos px}{1-\cos qx}=\underset{x\to 0}{\text{Lt}}\ \dfrac{p\sin px}{q\sin qx}$ (which is $\dfrac{0}{0}$ form again)

$=\underset{x\to 0}{\text{Lt}}\ \dfrac{p^2\cos px}{q^2\cos qx}=\dfrac{p^2}{q^2}$ (*iii*) $\underset{x\to 2}{\text{Lt}}\ \dfrac{x^2-2x+5}{x^2-3x+5}\neq\underset{x\to 2}{\text{Lt}}\ \dfrac{2x-2}{2x-3}$

because the given limit is not of the form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$

24. $\dfrac{d}{dx}(y^2)=2yy'=2$ (subnormal); $\dfrac{d}{dx}(\log y)=\dfrac{1}{\text{subtangent}}$; the product of subtangent and subnormal=the square of the ordinate.

25. $\dfrac{d}{dx}(x^x)=x^x(1+\log x)=x^x(\log ex)$.

26. The following integrals are standard ones. However they have aternate answers.

$\int \sec x\,dx=-\log(\sec x-\tan x)=\log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

$\int \operatorname{cosec} x\,dx=-\log(\operatorname{cosec} x+\cot x)=\log\tan\dfrac{x}{2},$

$\int \tan x\,dx=-\log\cos x$; $\int \cot x=-\log\operatorname{cosec} x$.

27. $\int e^x\{f(x)+f'(x)\}\,dx=e^xf(x)$.

28. $\int udv=uv-u'v_1+u''v_2-u'''v_3+\ldots\ldots$ This rule (of repeated application of integration by parts) can be used if u=first function$=x^n$, $n\in\text{N}$; u', u'', u''' etc. are successive derivatives of u and v_1, v_2, v_3, etc. are sucessive integrals of v.

Examples

(*i*) $\int x^3\cos x\,dx=(x^3)(\sin x)-(\ \ )(\ \ )+(\ \ )(\ \ )-\ldots\ldots\ldots$

$=(x^3)(\sin x)-(3x^2)(-\cos x)+(\ \ )(\ \ )-\ldots\ldots$

$=(x^3)(\sin x)-(3x^2)(-\cos x)+(6x)(-\sin x)-(\ \ )(\ \ )$

$=(x^3)(\sin x)-(3x^2)(-\cos x)+(6x)(-\sin x)-(6)(\cos x)$

(*ii*) $\int x^2e^{2x}dx=(x^2)\left(\dfrac{e^{2x}}{2}\right)-(2x)\left(\dfrac{e^{2x}}{4}\right)+(2)\left(\dfrac{e^{2x}}{8}\right)$

29. Remember the following (additional) integrals.

$f(x)$	$\int f(x)\,dx$
$\sin^2 x$	$\frac{x}{2} - \frac{1}{4}\sin 2x$
$\cos^2 x$	$\frac{x}{2} + \frac{1}{4}\sin 2x$
$\tan^2 x$	$\tan x - x$
$\cot^2 x$	$-\cot x - x$
$\sin^3 x$	$-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$
$\cos^3 x$	$\frac{3}{4}\sin x + \frac{1}{12}\sin 3x$
$\sin mx \sin nx$	$\frac{-\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)}$
$\cos mx \cos nx$	$\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)}$
$\sin mx \cos nx$	$\frac{-\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)}$
$\frac{1}{a+bx^2}$, $a, b > 0$	$\frac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)$
$\frac{1}{\sqrt{a-bx^2}}$, $a, b > 0$	$\frac{1}{\sqrt{b}}\sin^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)$
$\frac{x^{n-1}}{a+bx^n}$	$\frac{1}{nb}\log(a+bx^n)$
$\frac{x}{\sqrt{x^2 \pm a^2}}$	$\sqrt{x^2 \pm a^2}$
$\frac{x}{\sqrt{a^2-x^2}}$	$-\sqrt{a^2-x^2}$
$\frac{1}{a^2\cos^2 x + b^2\sin^2 x}$	$\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$

$f(x)$	$\int f(x)\,dx$
$\log x$	$x \log x - x$
$x^n \log x,\ n \neq -1$	$\frac{x^{n+1}}{n+1} \log x - \frac{x^{n+1}}{(n+1)^2}$
$\frac{1}{x \log x}$	$\log (\log x)$

Add to the above, the integrals of inverse trigonometric functions.

30. A function $f(x)$ is odd if $f(-x)=-f(x)$ (example : $\sin x$, x^3 etc.) and even if $f(-x)=f(x)$ (example $\cos x$, x^2 etc.).

Now $\int_{-a}^{a} f(x)\,dx=0$, if $f(x)$ is odd and $\int_{-a}^{a} f(x)\,dx=$ $2\int_{0}^{a} f(x)\,dx$, if $f(x)$ is even.

31. Remember the "popular" problems (with answers) under the use of the rule $\int_{0}^{a} f(x)dx=\int_{0}^{a} f(a-x)dx$.

ADDITIONAL PROBLEMS

Algebra Chapter I. Page 18.

9a. R is the relation on the set of real numbers defined by aRb if (*i*) $a-b$ is zero or irrational (*ii*) $ab \geq 0$ (*iii*) $1+ab>0$. Prove that R is reflexive and symmetric but not transitive.

9b. On the set of integers the relation R is defined by

(*i*) $R=\{(a, b) \mid a, b \in Z \text{ and } a-b \text{ is even}\}$

(*ii*) $R=\{(a, b) \mid a, b \in Z \text{ and } a-b \text{ is a multiple of } 3\}$

(*iii*) $R=\{(a, b) \mid a, b \in Z \text{ and } |a|=|b|\}$.

Prove that R is an equivalence relation.

14a. Let f: $Z \to Z$ be defined by

(*i*) $f(x)=2x$ is injective (one–one) but not surjective (onto).

(*ii*) $f(x)= \begin{cases} x/2 \text{ if } x \text{ is even} \\ 0 \text{ if } x \text{ is odd} \end{cases}$

is surjective but not injective.

(*iii*) $f(x)=3x^2+2$ is neither surjective nor injective.

(*iv*) $f(x)=x+1$ is bijective (both one-one and onto).

16a. Let $f : R \to R$ be defined by

$$f(x)= \begin{cases} \dfrac{x-1}{x+1}, & x \neq -1 \\ 1, & x=-1 \end{cases}$$

Prove that $(fof)(x)=-\dfrac{1}{x}$, $x \neq -1$ and $(fof)(-1)=0$.

16b. $f(x) = \begin{cases} x^2, & x<1 \\ 3, & x=1 \\ x, & x>1 \end{cases}$ and $g(x)=3x^2+x-1$ are functions from $R \to R$. Verify that

(*i*) $(gof)(1)=29$ (*ii*) $(fog)(2)=13$

(*iii*) $(fof)(-2)=4$ (*iv*) $(gog)(0)=1$

Algebra Chapter 2 Page 21.

17. If a, b, x and y are integers such that $ax+by=1$, then prove that $(a, b)=1$

18. $(a, b)=1$ and $(a, c)=1$, then prove that $(a, bc)=1$.

19. If $(a, b)=d$, then prove that $\left(\dfrac{a}{d}, \dfrac{b}{d}\right)=1$.

11. Show that (*i*) 2^n+1 or 2^n-1 is divisible by 3

(*ii*) $2^{2n}+1$ or $2^{2n}-1$ is divisible by 5

(*iii*) $5^{2n}+1$ or $5^{2n}-1$ is divisible by 13

according as n is odd or even.

Algebra Chapter 3 Page 29.

8a. Show that

(*i*) $\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix}=0$ (*ii*) $\begin{vmatrix} x+2y & x+4y & x+6y \\ x+3y & x+5y & x+7y \\ x+4y & x+6y & x+8y \end{vmatrix}=0$

(*iii*) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}=(a-1)^3$ (*iv*) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}=0$

(*v*) $\begin{vmatrix} a-x & a+x & a+x \\ a+x & a-x & a+x \\ a+x & a+x & a-x \end{vmatrix}=4x^2(x+3a)$

Page 31.

12a. (*i*) If $\begin{vmatrix} 2x+7 & x+4 & x+3 \\ x+4 & 2x+6 & x+2 \\ x+3 & x+2 & 2x+5 \end{vmatrix}=0$, then prove that $x=-2, -3$ or -4.

(*ii*) If $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}=0$, then prove that $3x+7=0$.

(*iii*) If $\begin{vmatrix} x+2 & 3 & 4 \\ 2 & x+3 & 4 \\ 2 & 3 & x+4 \end{vmatrix}=0$, then prove that $x=0$ or -9

(*iv*) If $\begin{vmatrix} x+1 & x+2 & 3 \\ 3 & x+2 & x+1 \\ x+1 & 2 & x+3 \end{vmatrix}=0$, then prove that $x=0, 2$ or -3

Chapter 5 Page 45.

36a. (*i*) If $\vec{b}=3\vec{a}+\vec{c}$, prove that $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$

(*ii*) Prove that $(3\vec{a}-\vec{b})\times(2\vec{b}-\vec{a})=5(\vec{a}\times\vec{b})$

(*iii*) Prove that $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=2(\vec{a}\times\vec{b})$

(*iv*) Prove that $(2\vec{a}+\vec{b})\times(\vec{a}+2\vec{b})=3(\vec{a}\times\vec{b})$

(*v*) If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then prove that $(\vec{a}-\vec{d}) \,//\, (\vec{b}-\vec{c})$

Trigonometry Chapter 3 Page 57.

12. (*vi*) If $\sin^{-1} x+\sin^{-1} y+\sin^{-1} z=\pi/2$, prove that

$$x^2+y^2+z^2+2xyz=1$$

(*vii*) If $\sin^{-1} x+\sin^{-1} y+\sin^{-1} z=\pi$, prove that

$$x\sqrt{1-y^2}+y\sqrt{1-x^2}=z$$

(*viii*) Prove that $\sin(\sin^{-1} x-\sin^{-1} y)=x\sqrt{1-y^2}-y\sqrt{1-x^2}$

(*ix*) Prove that $\cos(\sin^{-1} x-\cos^{-1} y)=y\sqrt{1-x^2}+x\sqrt{1-y^2}$

(*x*) Prove that $\sin^{-1} x+\sin^{-1} y=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$ or $\pi-\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$, $x\geq 0, y\geq 0$.

(*xi*) Prove that $\cos^{-1} x+\cos^{-1} y=\cos^{-1}\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\}$, $x\geq 0$, $y\geq 0$

Page 57,

15. (*i*) If $\sin^{-1} x=\cos^{-1} x$, prove that $x=\dfrac{1}{\sqrt{2}}$

(*ii*) If $\tan^{-1} x=\cot^{-1} x$, prove that $x=\pm 1$

(*iii*) If $\sin^{-1} x+\sin^{-1} 2x=\pi/3$, prove that $28x^2=3$.

Chapter 5 Page 62.

5. ABCD is a rectangle whose sides are parallel to the coordinate axes and its centre is at origin. If A represents $2-3i$, find the coordinates of B, C and D.

Analytical Geometry Chapter 1 Page 71.

8a. (*i*) If the slope of one of the lines of by $ax^2+2hxy+by^2=$ is three times the other prove that $3h^2=4ab$.

(*ii*) If the slope of one of the lines of $ax^2+2hxy+by^2=$ is k times the other; prove that $4kh^2=ab(1+k)^2$.

(*iii*) If the sum of the slopes of the lines given by $ax^2+bxy+3y^2=0$ is equal to four times the product, prove that $4a+b=0$.

(*iv*) If the angle between the lines $ax^2+2hxy+by^2=0$ is 60°, prove that $(3a+b)(a+3b)=4h^2$.

Page 77.

8a. (*i*) Prove that the equation of the circle with centre a $(-1,1)$ and touching y-axis is $(x+1)^2+(y-1)^2=1$.

(*ii*) Prove that the equations of the circles touching the x-axis at $(1, 0)$ and having radius 2 units are $(x-1)^2+(y\pm2)^2=4$.

(*iii*) Prove that the equations of the circles which touch both the coordinate axes and pass through the point $(2, 1)$ are $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-10x-10y+25=0$.

(*iv*) Prove that the equations of the circles which touch both the axes and pass through the point $(8, 9)$ are $x^2+y^2-10x-10y+25=0$ and $x^2+y^2-58x-58y+841=0$.

Page 92.

2. (*d*) Prove that the equation of the ellipse with major axis parallel to x-axis,

(*i*) the centre at $(1, 2)$, the distance between foci $=2\sqrt{5}$ and the distance between directrices$=18/\sqrt{5}$ is

$$\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1.$$

(*ii*) the foci at $(1\pm4\sqrt{2}, 4)$ and $e=2\sqrt{2}/3$ is

$$\frac{(x-1)^2}{36}+\frac{(y-4)^2}{4}=1.$$

(*iii*) the centre at $(1, -1)$, latus rectum$=2$ and the length of minor axis$=6$ is $\frac{(x-1)^2}{81}+\frac{(y+1)^2}{9}=1$

(*iv*) the centre at $(-2, 1)$, LR$=8/3$ and the distance between foci$=2\sqrt{5}$ is $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1.$

3. (*d*) Prove that the equation of the hyperbola with transverse axis parallel to x–axis,

(*i*) the distance between vertices$=24$ and foci at $(-10, 2)$ and $(16, 2)$ is $\frac{(x-3)^2}{144}-\frac{(y-2)^2}{25}=1.$

(*ii*) the foci at $(-4, 1)$ and $(6, 1)$ and $e=5/4$ is $\frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$

(*iii*) the vertices at $(-4, 3)$ and $(8, 3)$ and $e=4/3$ is $\frac{(x-2)^2}{36}-\frac{(y-3)^2}{28}=1$

(*iv*) the centre at $(1, -2)$, $e=3/2$ and LR$=5$ is $\frac{(x-1)^2}{4}-\frac{(y+2)^2}{5}=1.$

Calculus Chapter 1 Page 101.

3. Prove that the function,

(*i*) $f(x)=\begin{cases} 3x-2, & x\leq 0 \\ x+1, & x>0 \end{cases}$ is not continuous at $x=0$

(*ii*) $f(x)=\begin{cases} x^2+1, & x<-1 \\ -2x, & x\geq -1 \end{cases}$ is continuous at $x=-1$

(*iii*) $f(x)=\begin{cases} -x-1, & x\leq -1 \\ -\sqrt{1-x^2}-1, & -1<x<1 \\ x-1, & x\geq 1 \end{cases}$ is discontinuous at $x=\pm 1$

(*iv*) $f(x)=\begin{cases} \frac{\sin x}{x}, & x<0 \\ 1, & x=0 \\ \frac{\tan x}{x}, & x>0 \end{cases}$ is continuous at $x=0$

(v) $f(x) = \begin{cases} x^2-1, & x \leq 0 \\ 2x+1, & 0 < x < 1 \\ 3x, & x \geq 1 \end{cases}$ is discontinuous at $x=0$ but continuous at $x=1$

(vi) $f(x) = \begin{cases} 0 \text{ when } x \text{ is rational} \\ 1 \text{ when } x \text{ is irrational} \end{cases}$

is not continuous anywhere.

SECOND YEAR PRE-UNIVERSITY EXAMINATION

QUESTION PAPERS—MARCH 1 89

Note: Answer *all* questions

Time: 3 hours] Part—A [Max: 100 marks

For each of the following questions four choices are provided. Select and write the correct answer. Such of the questions which are answered more than once will be given zero mark.

$20\times1=20$

1. If $A=\{x/x^2-5x+6=0\}$, $B=\{2, 4\}$, $C=\{4, 5\}$ then $(A-B)\times(B-C)$ is
(a) $\{(3, 2)\}$ (b) $\{(0, 3)\}$ (c) $\{(2, 3), (2, 4)\}$ (d) none of these

2. The digit in the unit place of 2^{12} is
(a) 2 (b) 4 (c) 6 (d) 8.

3. If $x\equiv3 \pmod 5$ then x is the set
(a) $\{0, \pm3, \pm8, \pm13, \ldots\}$ (b) $\{3, 8, 13, \ldots\}$
(c) $\{-2, -7, -12, \ldots\}$
(d) $\{3, 8, 13, \ldots\}\cup\{-2, -7, -12, \ldots\}$

4. If $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ then A^2 is
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
(d) none of these.

5. The value of $\begin{vmatrix} 4200 & 4201 \\ 4202 & 4203 \end{vmatrix}$ is
(a) 2 (b) -2 (c) 1 (d) -1

6. A binary operation $*$ on a set G is associative if for every $x, y, z\in G$
(a) $(x*y)*z=(y*z)*x$
(b) $(x*y)*z=(x*z)*y$
(c) $(x*y)*z=x*(y*z)$
(d) $(x*y)*z=z*(x*y)$

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$ad^2 + bc^2 = 0.$

$c^2 = d^2$